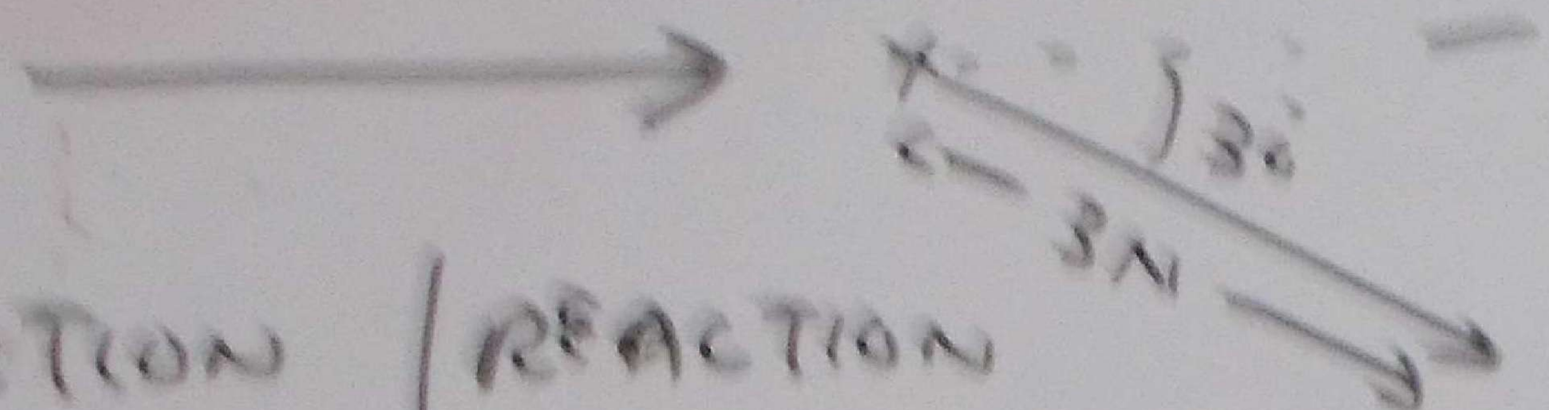
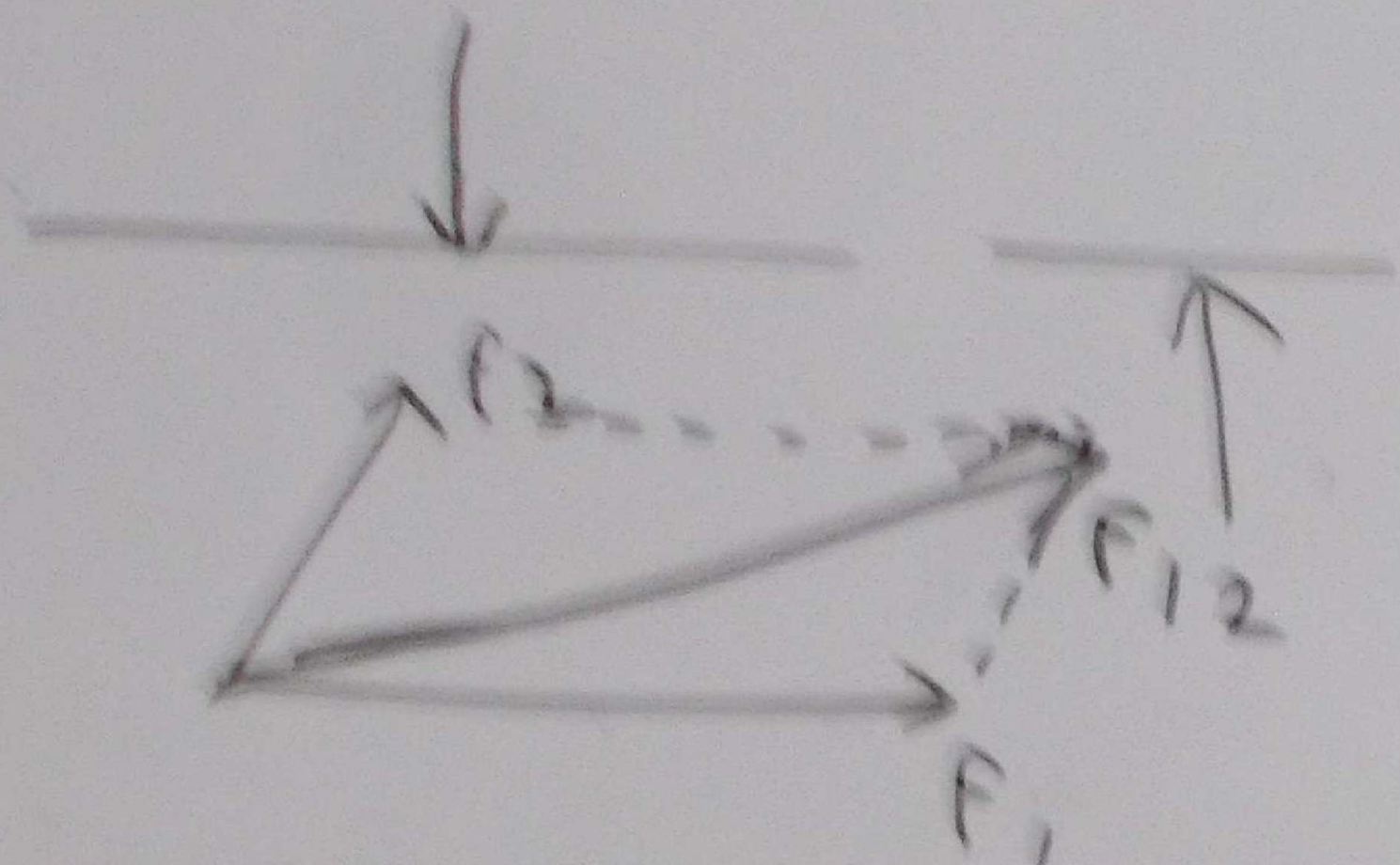
FORCE — VECTOR

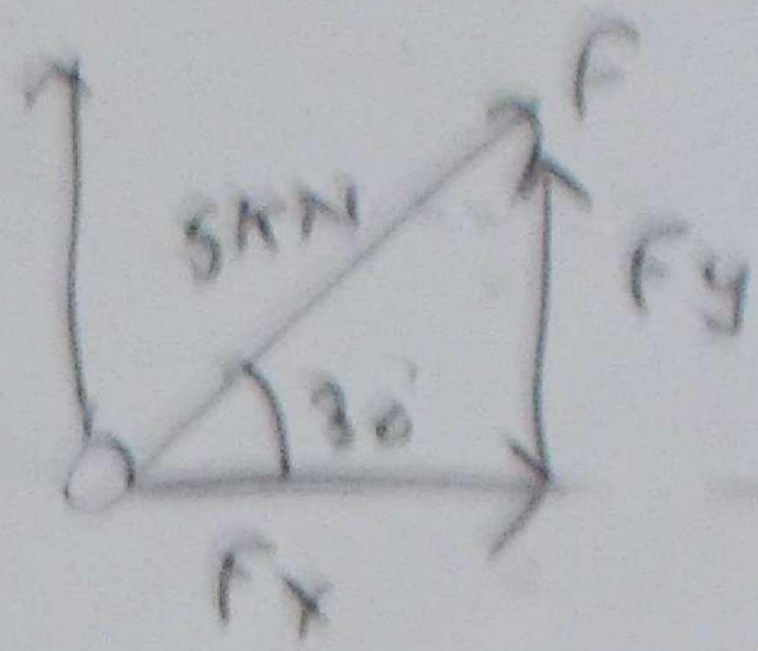
DIRECTION / MAGNITUDE



ACTION / REACTION



EX A FORCE OF 5KN ACTING UP AND TO THE RIGHT AT 30° TO THE HORIZONTAL, CALCULATE HORIZONTAL AND VERTICAL COMPONENTS.



$F_x = F\cos 30°$

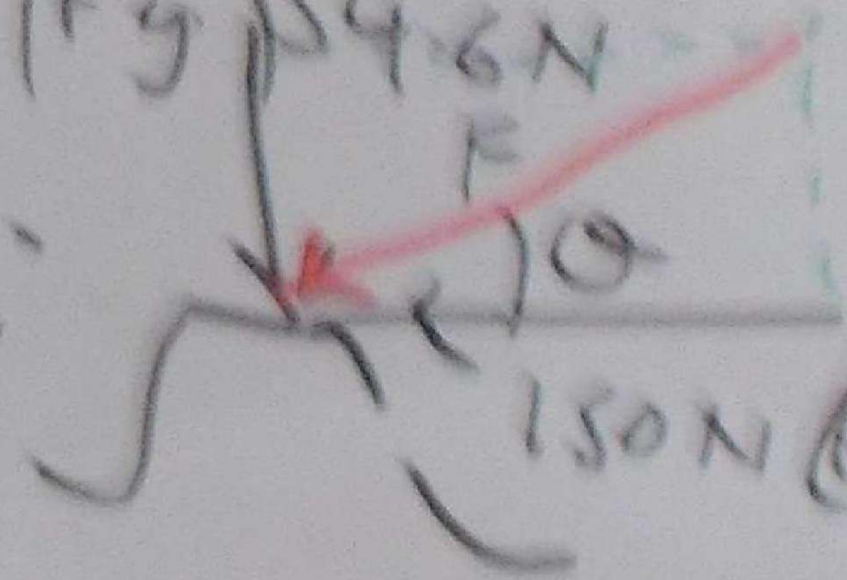
$= 5 \times 0.866$

$= 4.33\ kN$

$F_y = F\sin 30$

$= 5 \times 0.5 = 2.5\ kN$

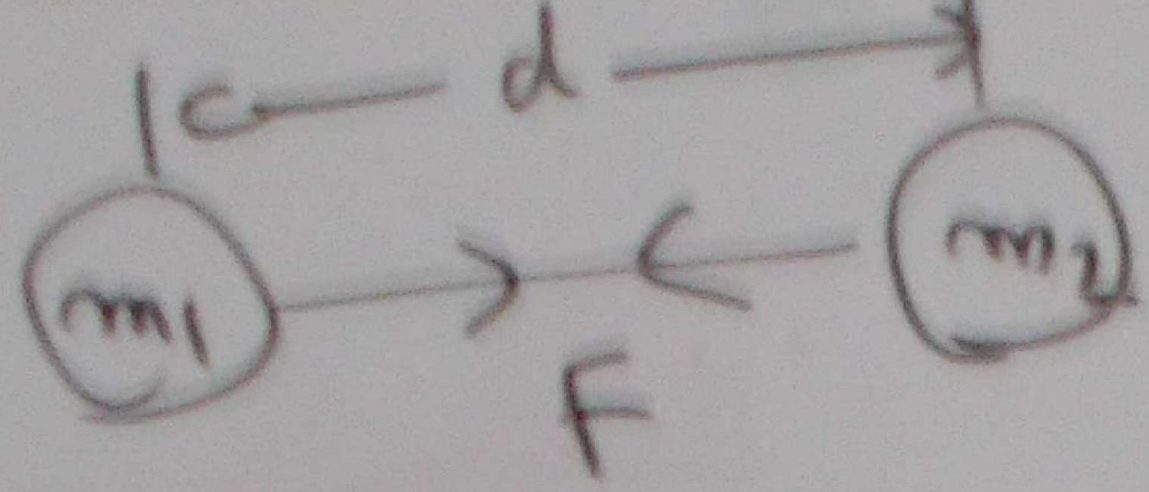
EX IF THE COMPONENTS OF FORCE ACTING ON A GEAR TOOTH ARE 54.6 N IN RADIAL DIRECTION AND 150 N IN TANGENTIAL DIRECTION, DETERMINE TOTAL FORCE AND THE ANGLE BETWEEN FORCE AND TANGENTIAL DIRECTION.



$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{150^2 + 54.6^2} = 159.6\ N$

$\theta = \tan^{-1}\frac{54.6}{150} = 20°$



$$F = G\frac{m_1 m_2}{d^2}$$

$$G = 66.7 \times 10^{-12} \frac{N \cdot m^2}{kg^2}$$

WEIGHT OF BODY (W)

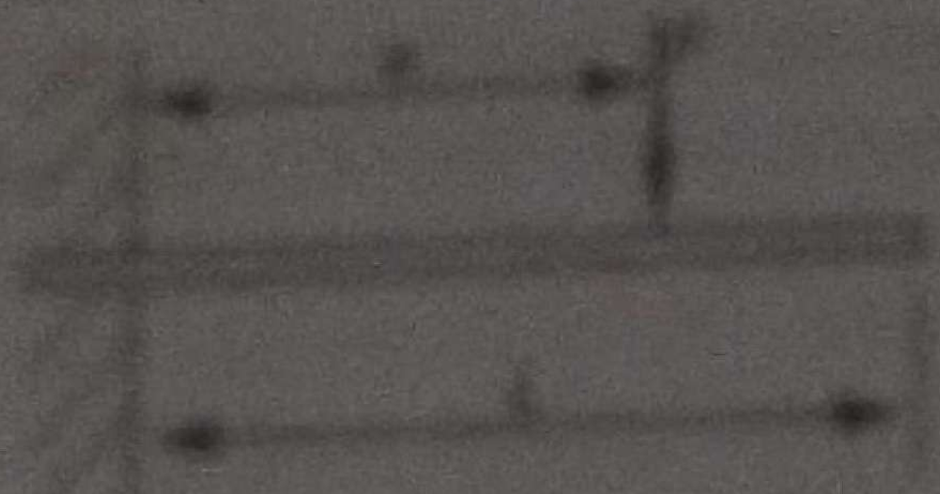
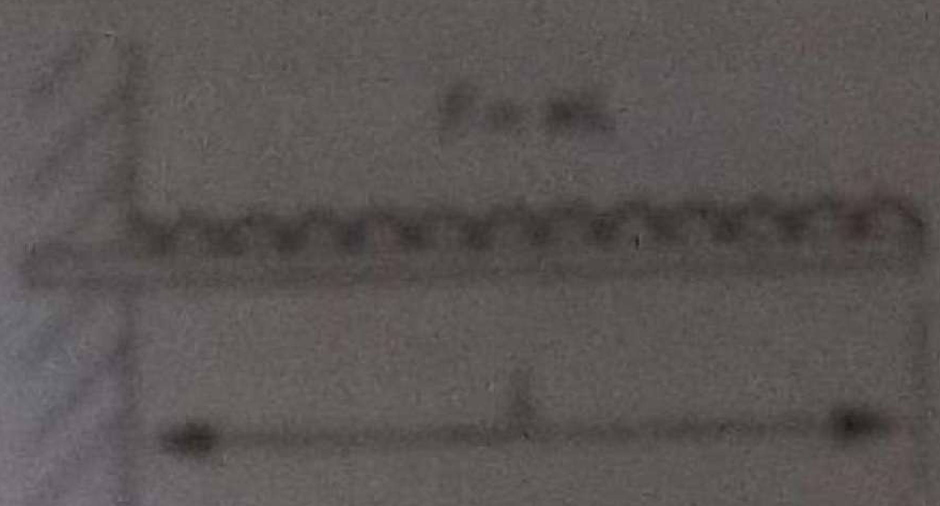
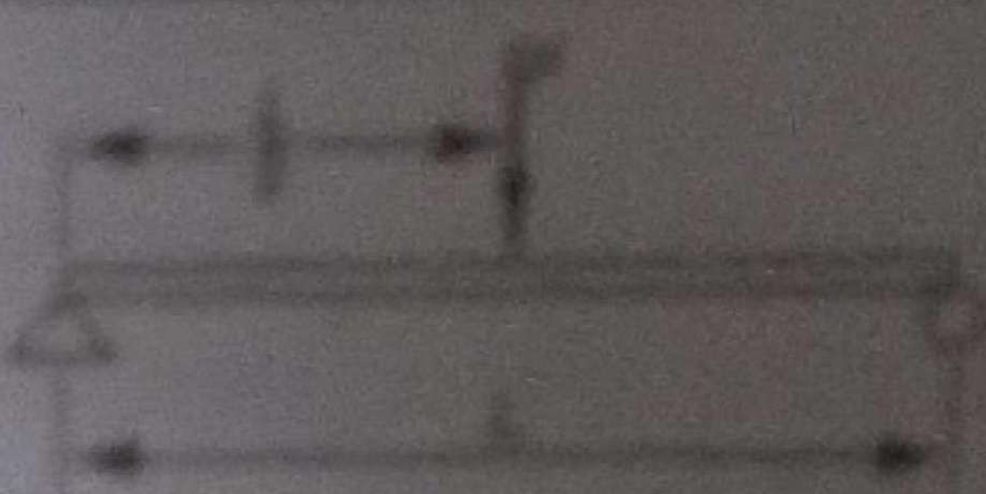
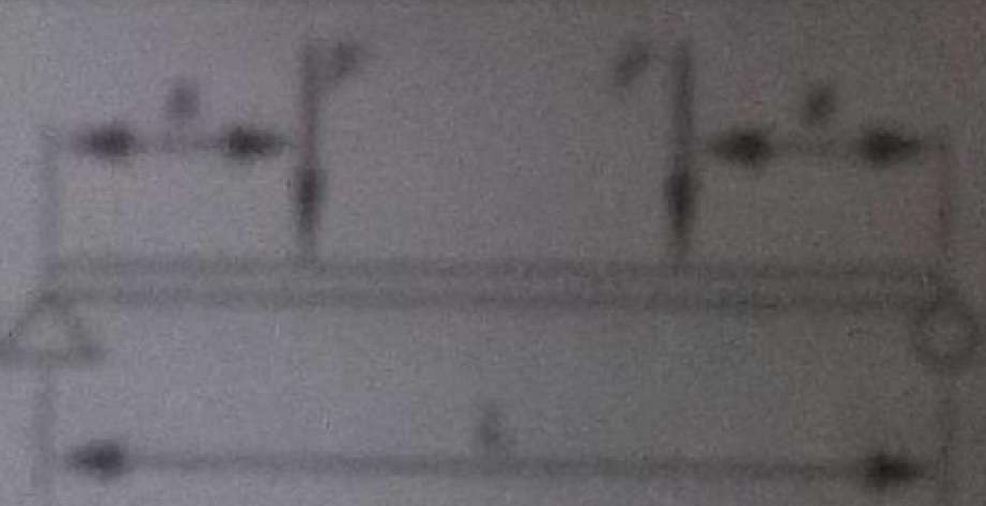
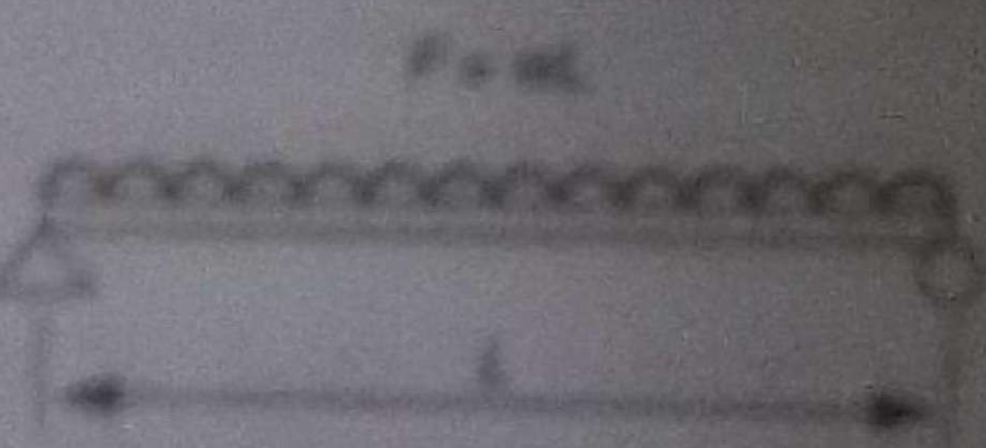
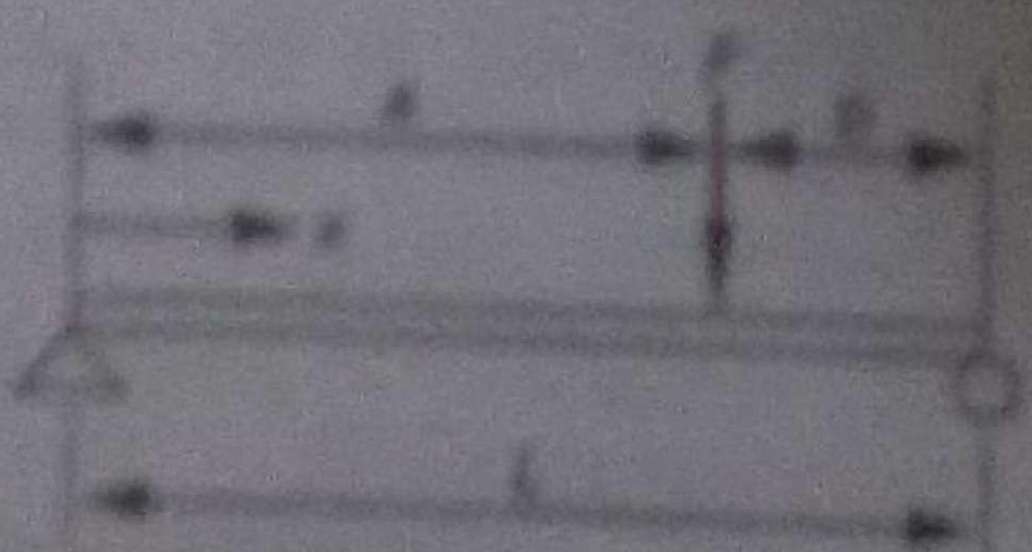
g = GRAVITATION = 9.81 N/kg

EARTH $W = mg$ (N)

Pb A MAN HAS A MASS OF 79kg WHAT IS THE WEIGHT?

$$F_W = mg = 79 \times 9.81$$
$$= 775 N$$

Table 36.1 *Deflection formulae for variously loaded beams*

Case	Beam and load	Maximum deflection	Occurs at
1		$y = \dfrac{FL^3}{3EI}$	free end
2		$y = \dfrac{Fa^2(3L - a)}{6EI}$	free end
3		$y = \dfrac{FL^3}{8EI}$	free end
4		$y = \dfrac{FL^3}{48EI}$	midspan
5		$y = \dfrac{Fa(3L^2 - 4a^2)}{24EI}$	midspan
6		$y = \dfrac{5FL^3}{384EI}$	midspan
7		$y = \dfrac{Fab(a + 2b)\sqrt{3a(a + 2b)}}{27EIL}$ (where $a > b$)	$x = \sqrt{\dfrac{a(a + 2b)}{3}}$

Note: To obtain deflection in mm, F must be in N, E must be in MPa, I must be in mm^4, and L, a and b must be in mm.

Example 30.6

A steel structural beam (E = 200 000 MPa and $I = 554 \times 10^6$ mm^4) is 12 m long between simple supports at each end and carries a concentrated load of 40 kN at the midspan point, as shown in Figure 30.5. Calculate the deflection due to this load.

Fig. 30.5

Solution

Here we have:

$$P = 40\,000 \text{ N}$$
$$L = 12\,000 \text{ mm}$$
$$E = 200\,000 \text{ MPa}$$
$$\text{and} \quad I = 554 \times 10^6 \text{ mm}^4$$

Substitute into the formula for the maximum deflection of a simply supported beam with a concentrated load at the midspan (case 4):

$$y = \frac{PL^3}{48EI}$$
$$= \frac{40\,000 \times 12\,000^3}{48 \times 200\,000 \times 554 \times 10^6}$$
$$= 13.0 \text{ mm}$$

This is the maximum deflection at the midpoint.

Example 30.7

If the same steel beam (E = 200 000 MPa, L = 12 000 mm and $I = 554 \times 10^6$ mm^4) carries two symmetrically located concentrated loads of 30 kN each, located 2.5 m from the supports as shown in Figure 30.6, what is the maximum deflection due to this loading?

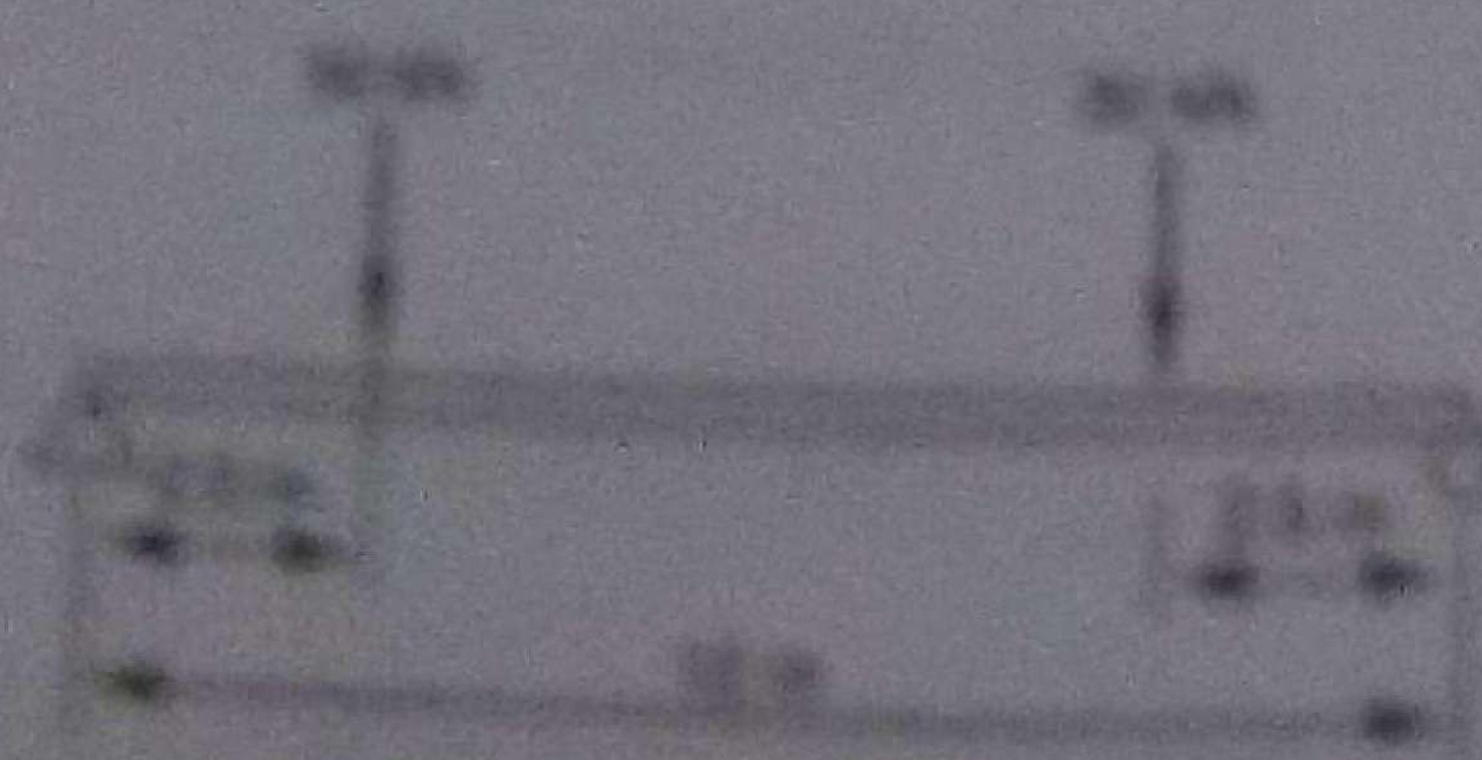


Fig. 30.6

SOLUTION

Substitute into the equation for case 3 (two symmetrical concentrated loads)

$$y = \frac{Pa(3L^2 - 4a^2)}{24EI}$$

$$= \frac{30\,000 \times 2500 \times (3 \times 12\,000^2 - 4 \times 2500^2)}{24 \times 200\,000 \times 334 \times 10^6}$$

$$= 13.3 \text{ mm}$$

This is the maximum deflection, which occurs at midspan.

Example 30.8

If the same simply supported steel beam ($E = 200\,000$ MPa, $L = 12\,000$ mm and $I = 334 \times 10^6$ mm^4) was used to carry a uniformly distributed load of 5 kN/m over its entire length (Fig. 30.7), what would be the midspan deflection?

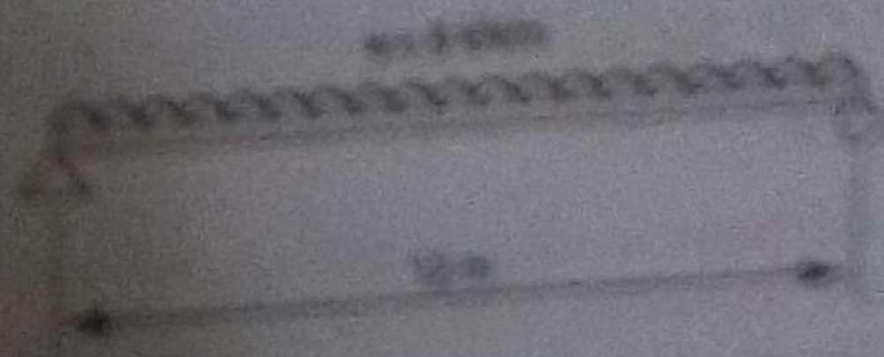


Fig. 30.7

SOLUTION

If the load intensity $w = 5$ kN/m, then the total load on the beam is

$$P = 5 \text{ kN/m} \times 12 \text{ m}$$
$$= 60 \text{ kN}$$
$$= 60\,000 \text{ N}$$

Substitute into the equation for case 5

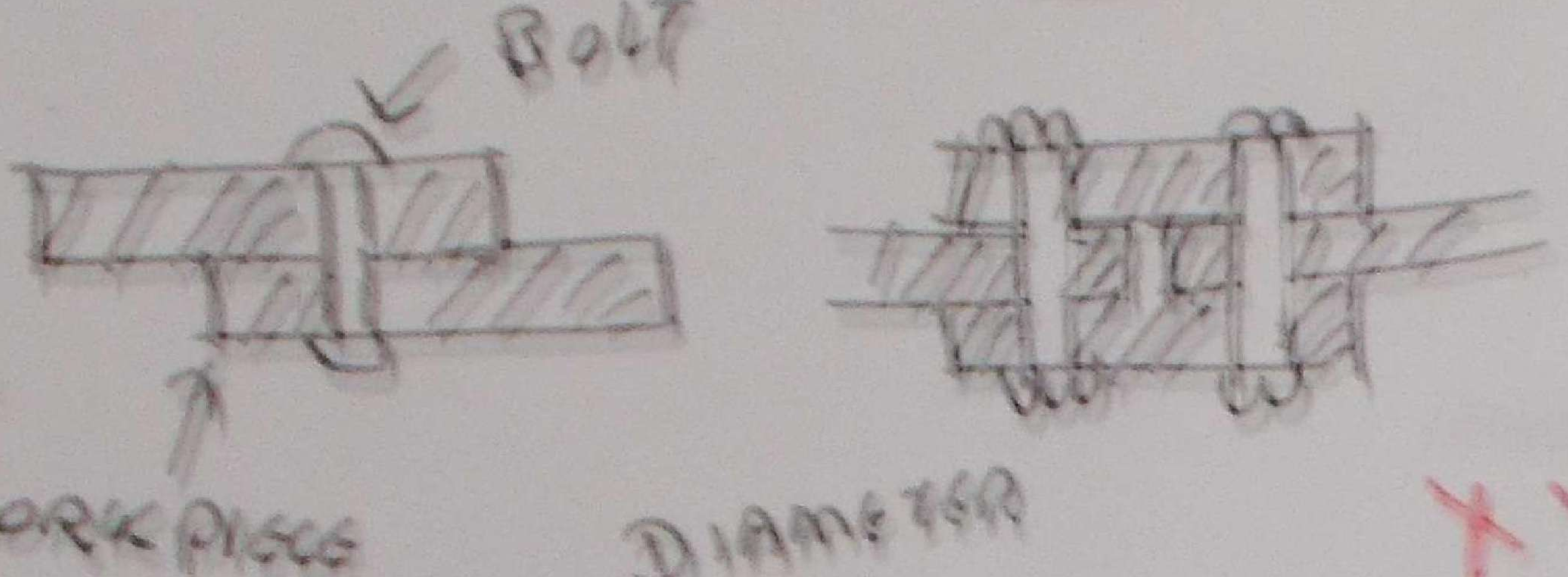
$$y = \frac{5PL^3}{384EI}$$

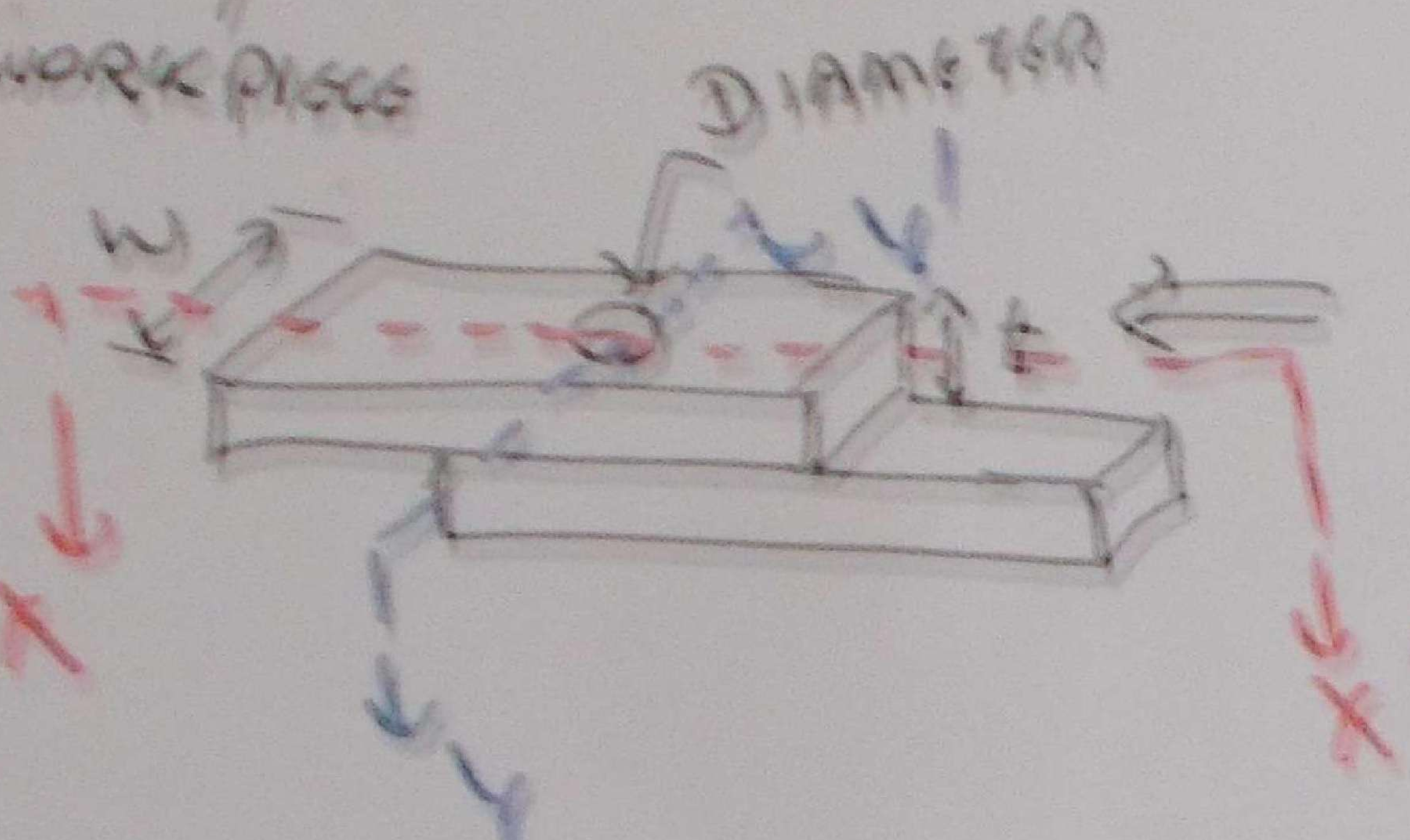
$$= \frac{5 \times 60\,000 \times 12\,000^3}{384 \times 200\,000 \times 334 \times 10^6}$$

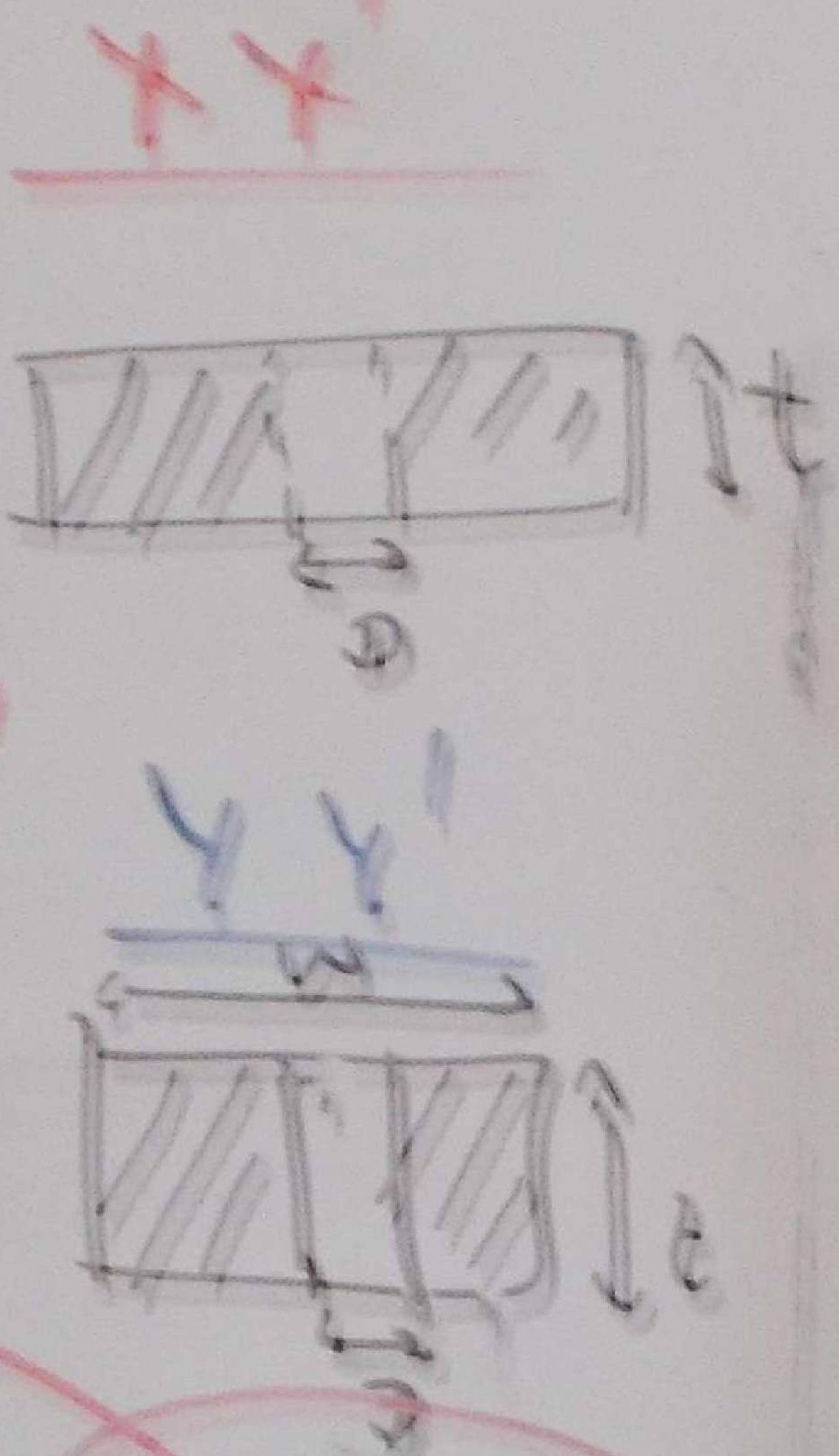
$$= 19.3 \text{ mm}$$

This is the maximum deflection, which occurs at midspan.

BOLT AND WELDED JOINTS





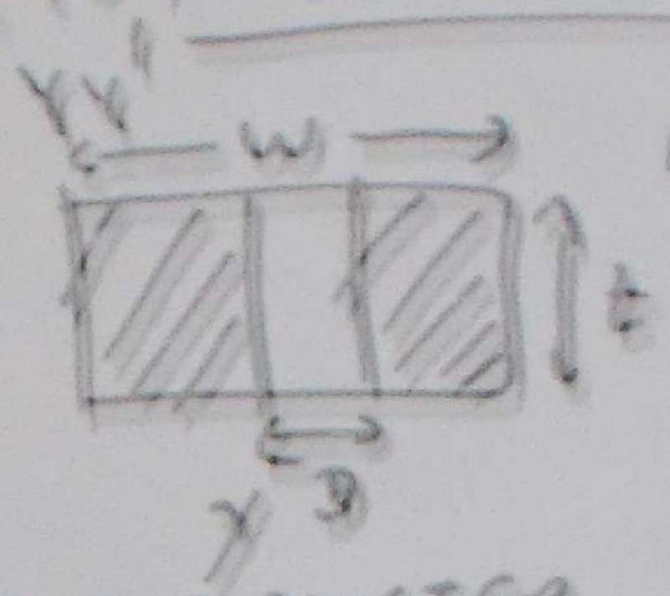


ALLOWABLE STRENGTH OF JOINT

(a) ALLOWABLE SHEAR STRESS × BOLT AREA = ALLOWABLE LOAD IN SHEAR

$90\,\text{MPa} \times \frac{\pi}{4} D^2 =$ ——— " ———

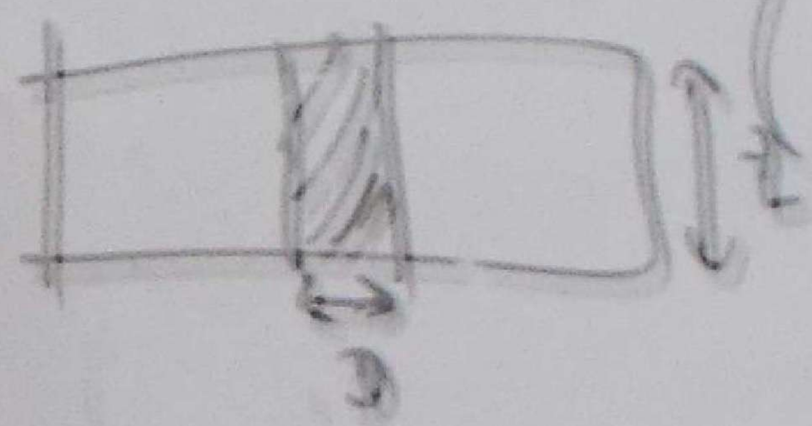
(a) TENSION



BOLT DIAMETER

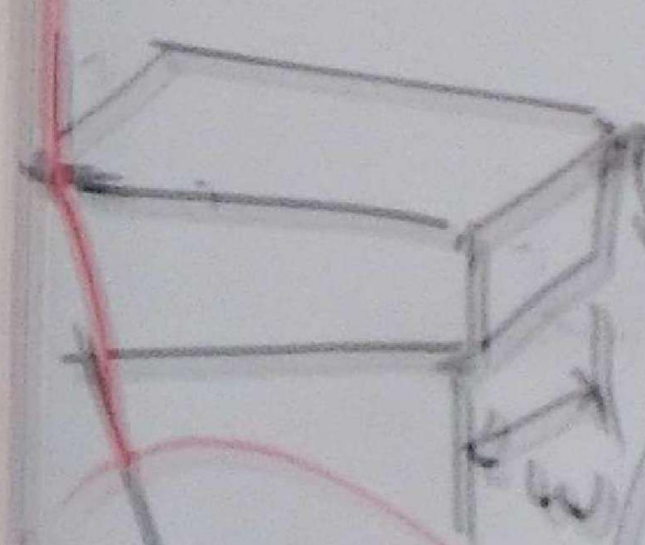
$A = (W - D)\,t \times$ ALLOWABLE TENSILE STRESS 110 MPa = ALLOWABLE LOAD IN TENSION

(b) BEARING STRENGTH



(BOLT DIAMETER × PLATE THICKNESS) × ALLOWABLE COMPRESSIVE STRESS 220 MPa = ALLOWABLE BEARING LOAD

JOINT EFFICIENCY = $\frac{\text{STRENGTH OF JOINT}}{\text{STRENGTH OF UNPUNCHED PLATE}}$

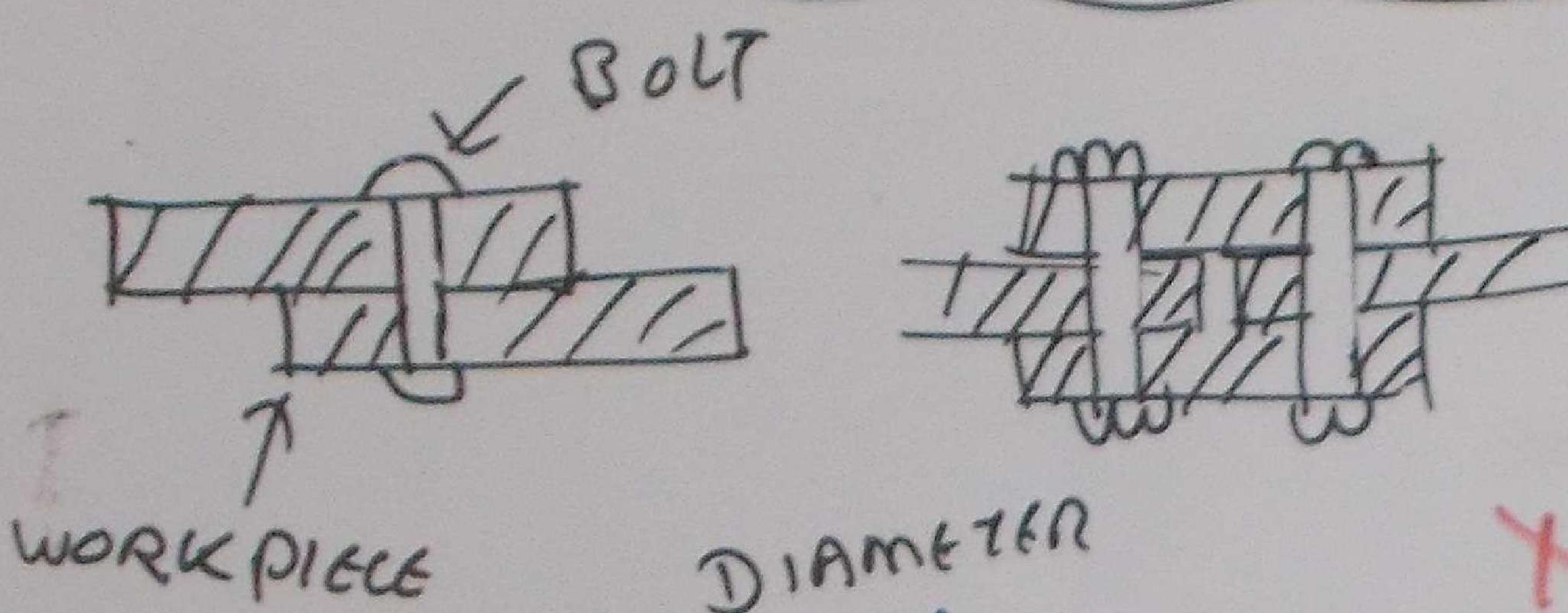


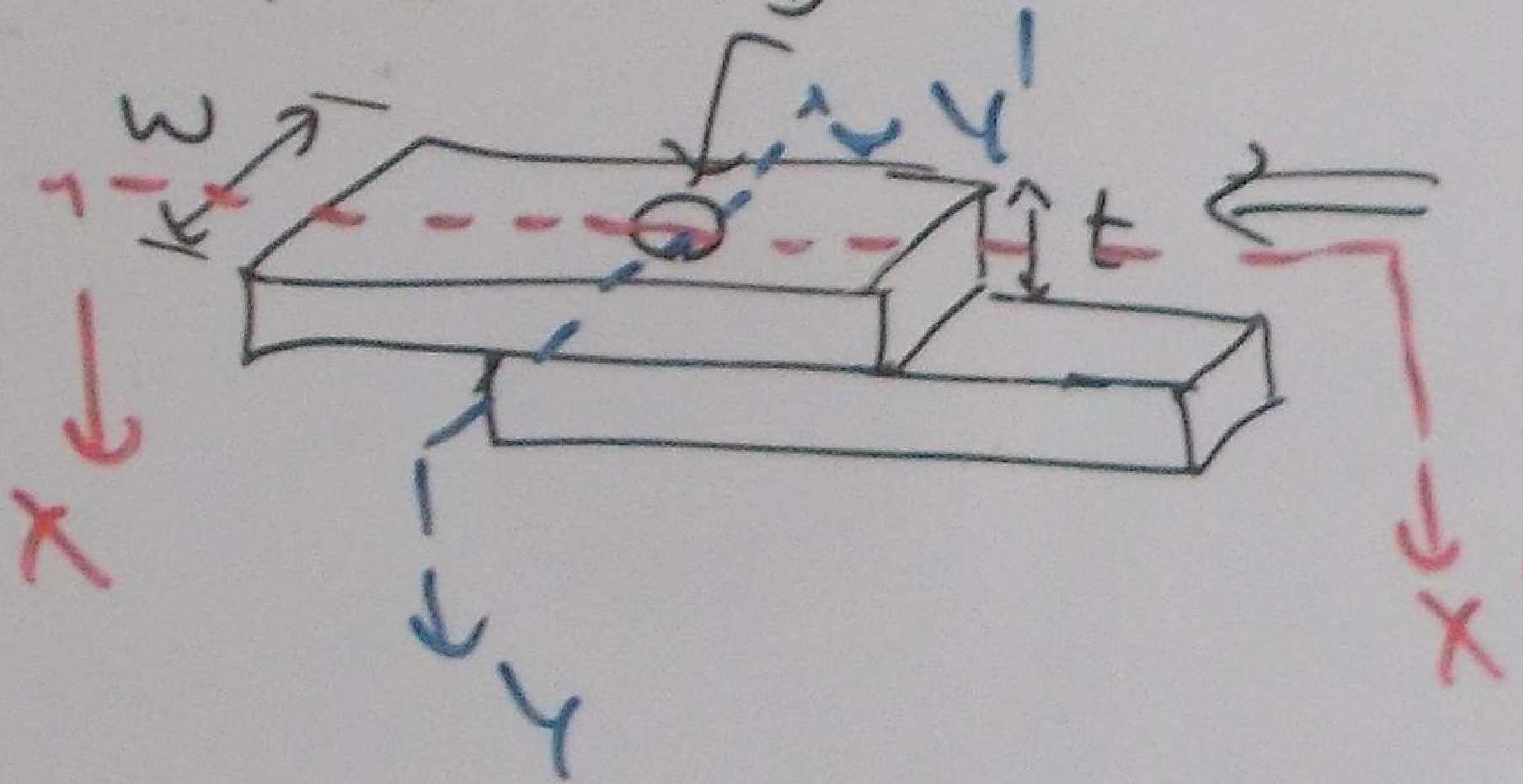
ALLOWABLE LOAD IN SHEAR

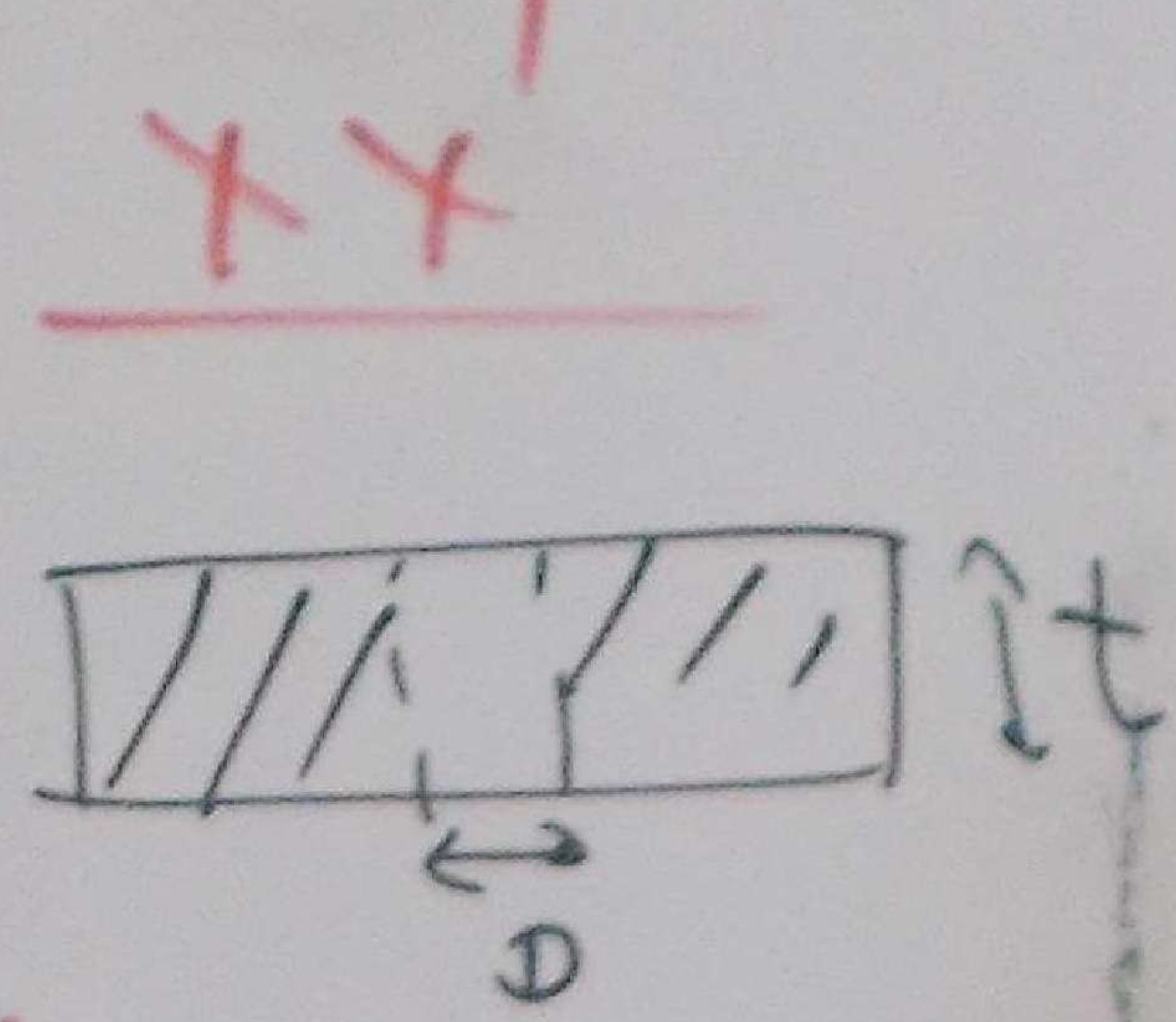
(PLATE WIDTH × PLATE THICKNESS) × ALLOWABLE TENSION 110 MPa

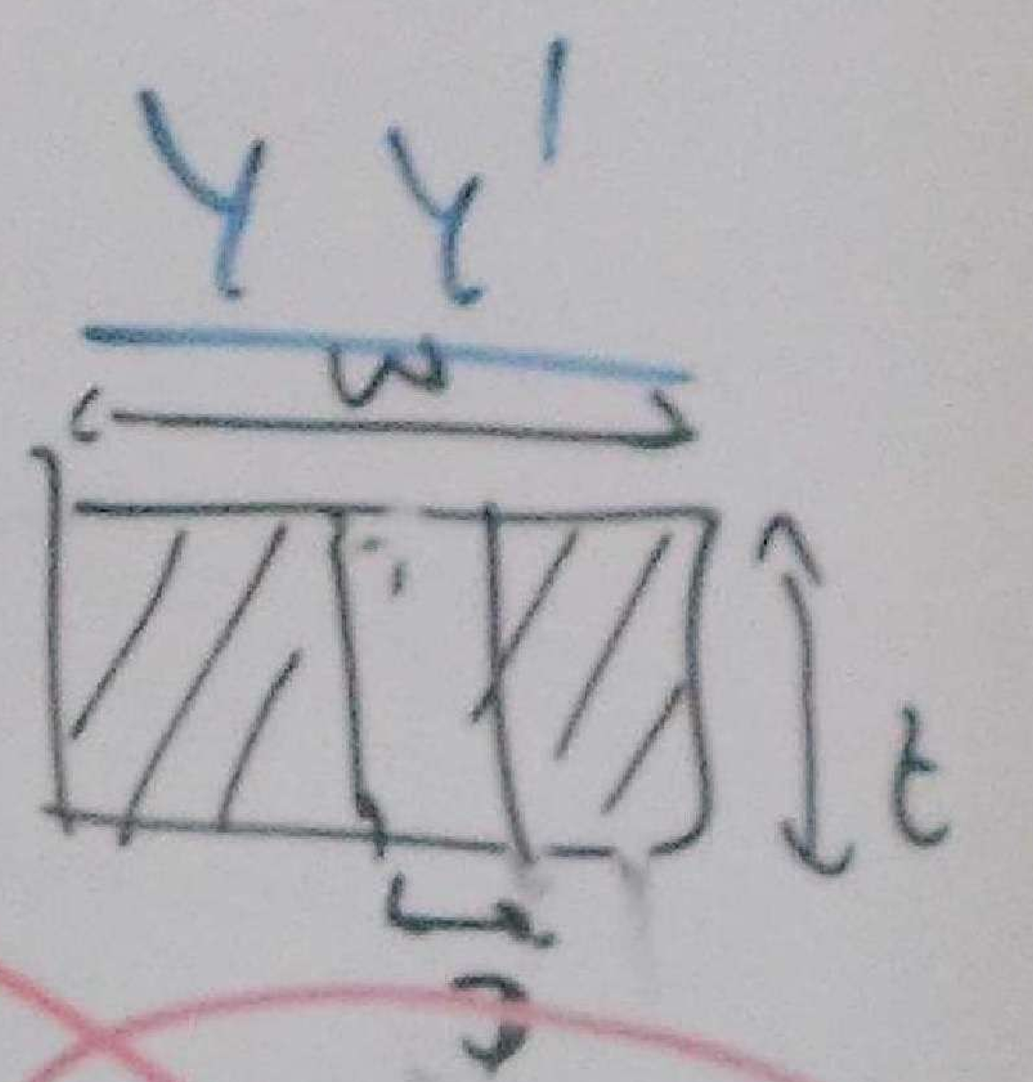
W × t

BOLT AND WELDED JOINTS







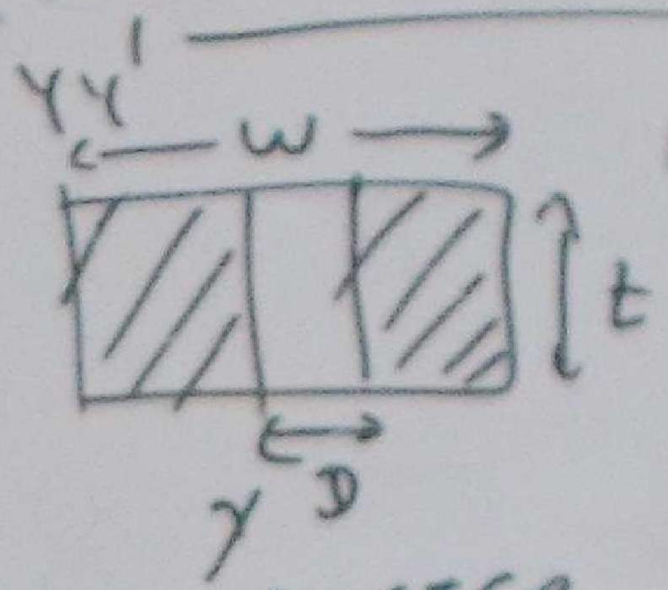


ALLOWABLE STRENGTH OF JOINT

(a) ALLOWABLE SHEAR STRESS × BOLT AREA = ALLOWABLE LOAD IN SHEAR

$$90\ \text{mpa} \times \frac{\pi}{4} D^2 = \text{— " —}$$

(b) TENSION

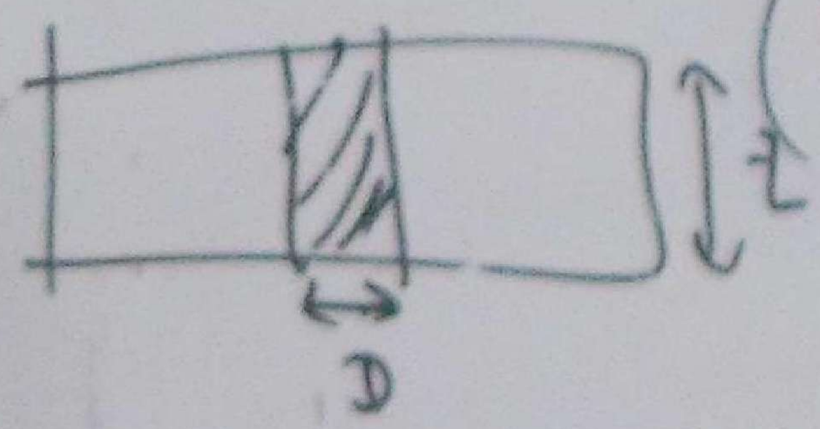


BOLT DIAMETER

$A = (W - D)\,t \times$ ALLOWABLE TENSILE STRESS $=$ ALLOWABLE LOAD IN TENSION

110 MPa

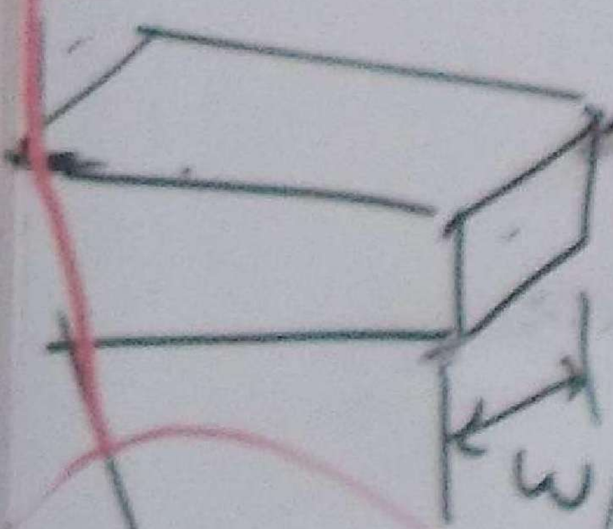
(1) BEARING STRENGTH



(BOLT DIAMETER $\times$ PLATE THICKNESS) $\times$ ALLOWABLE COMPRESSIVE STRESS $=$ ALLOWABLE BEARING LOAD

220 MPa

JOINT EFFICIENCY $=$ $\dfrac{\text{STRENGTH OF JOINT}}{\text{STRENGTH OF UN PUNCHED PLATE}}$

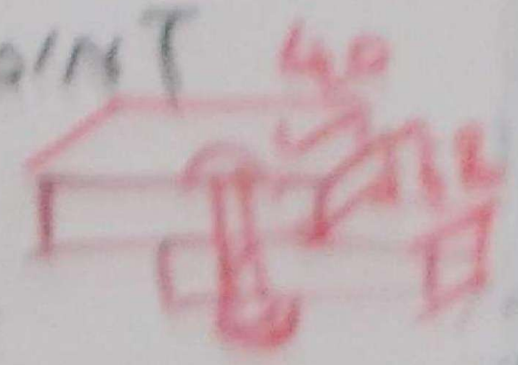


(PLATE WIDTH $\times$ PLATE THICKNESS) $\times$ ALLOWABLE TENSION

$W \times t$ 110 MPa

ALLOWABLE LOAD IN SHEAR

P3 SIMPLE LAP JOINT, TWO STEEL STRAPS
40 mm WIDE x 8 mm THICK
BOLT 16 mm DIAMETER
ALLOWABLE STRENGTH OF JOINT



(a) ALLOWABLE LOAD IN SHEAR

ALLOWABLE LOAD IN SHEAR = ALLOWABLE SHEAR STRESS × BOLT AREA

$= 90\,MPa \cdot \frac{\pi}{4} D^2$

$= 90\,MPa \times 0.7854 (16)^2$

$= 18.1\,kN$

(b) ALLOWABLE LOAD IN TENSION?

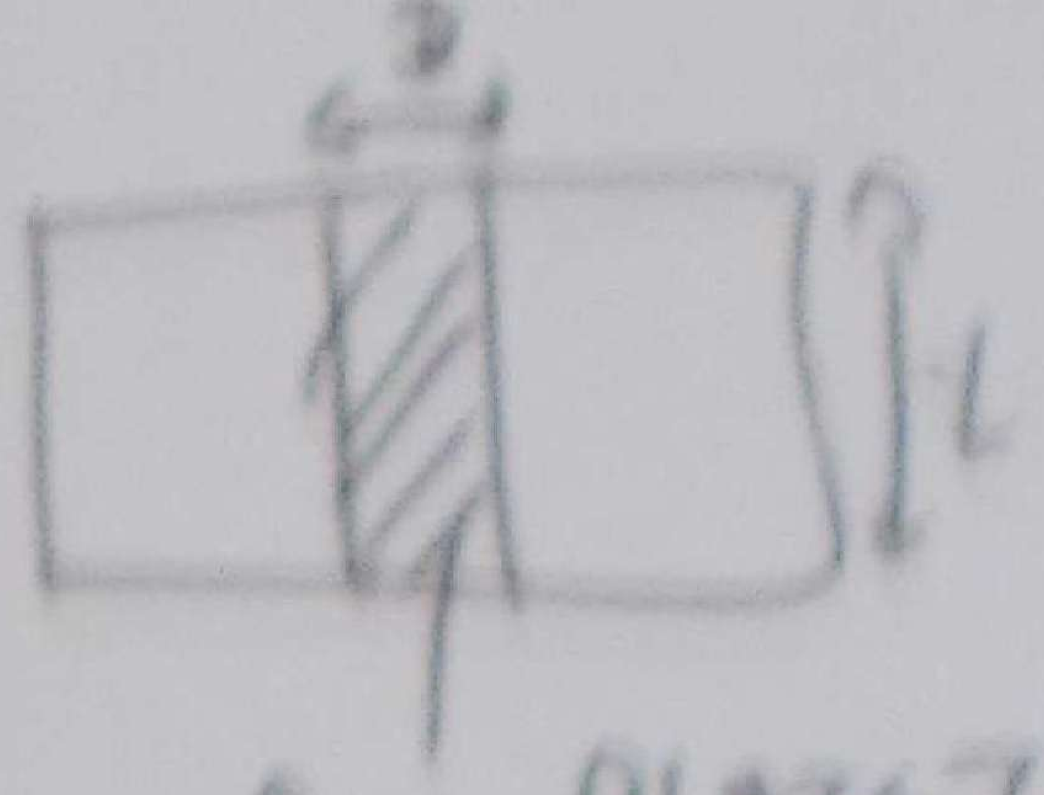
$A = (W - D)t = (40 - 16) \times 8$

$= 192\,mm^2$

A × ALLOWABLE TENSILE STRESS

$192 \times 110 = 21.1\,kN$

(i) BEARING STRENGTH = 128 × ALLOWABLE COMPRESSIVE STRESS 220 MPa



A_c = PLATE THICKNESS × BOLT DIAMETER

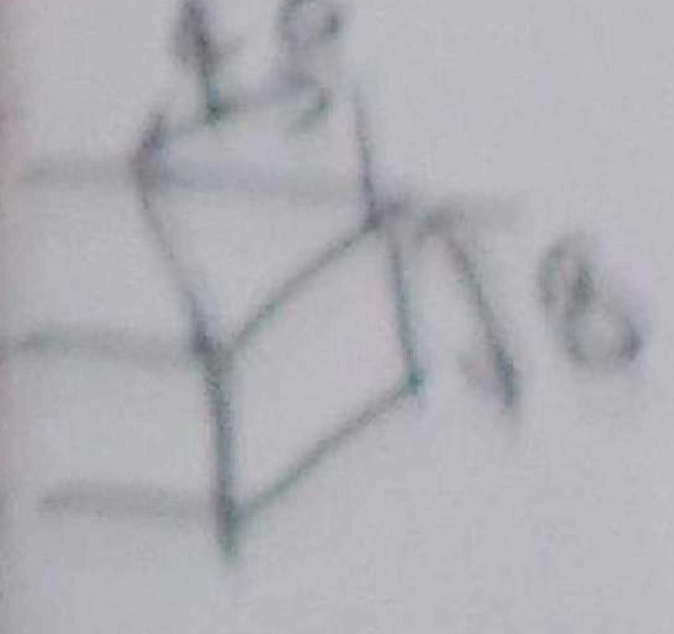
= 8 × 16 = 128 mm^2

→ = 128 × 220 = 28160 N

STRENGTH OF JOINT = 18.1 kN > 28.2 kN

P6 JOINT EFFICIENCY = ?

JOINT EFFICIENCY = $\frac{\text{STRENGTH OF JOINT}}{\text{STRENGTH OF UNPUNCHED PLATE}}$



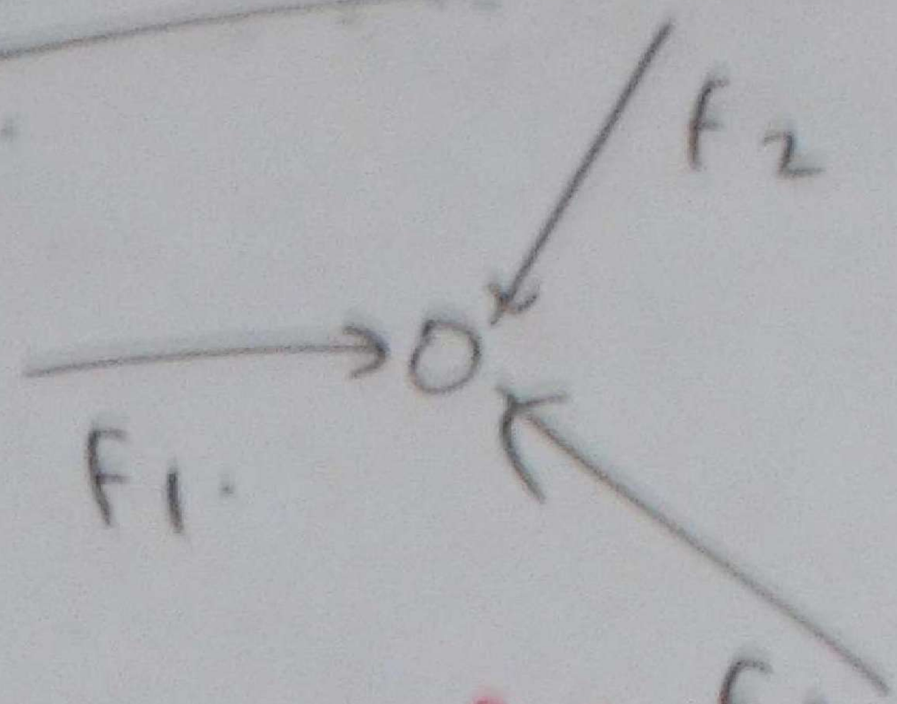
= $\frac{18.1 \text{ kN}}{35.2 \text{ kN}}$ = 0.514

= 51.4%

AREA OF UNPUNCHED PLATE × ALLOWABLE COMPRESSIVE STRESS = 35.2 kN

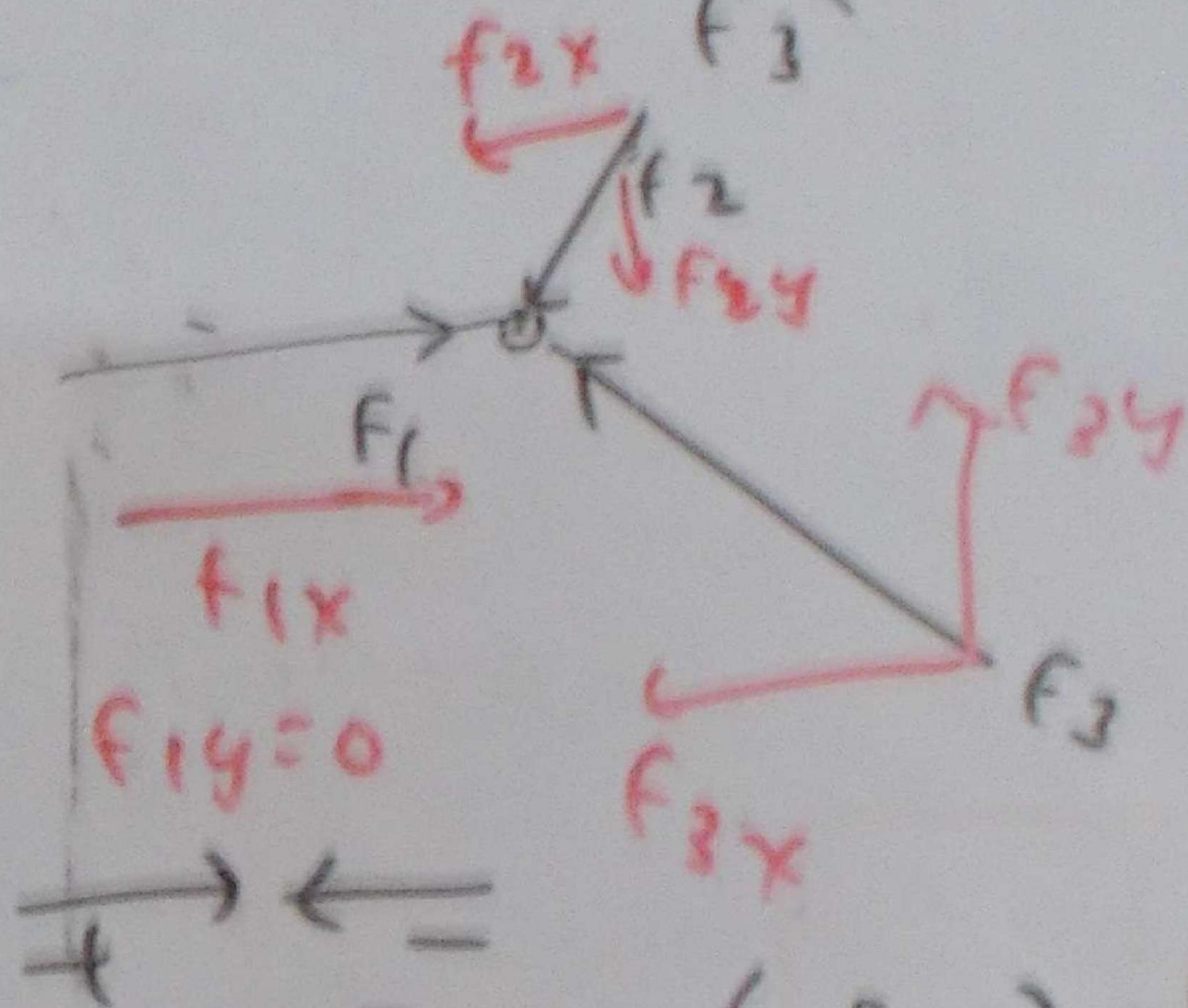
320 × 220 MPa

EQUILIBRIUM



$$\sum F_x = 0$$

$$\sum F_y = 0$$



$$\sum F_x = F_{1x} + (-F_{2x}) + (-F_{3x}) = 0$$

$$\sum F_y = \underbrace{F_{1y}}_{0} + (-F_{2y}) + (+F_{3y}) = 0$$

(9) SINGLE LAP JOINT, TWO STEEL STRIPS

40 mm WIDE x 8 mm THICK

BOLT 16 mm DIAMETER

ALLOWABLE STRENGTH OF JOINT

(a) ALLOWABLE LOAD IN SHEAR

ALLOWABLE LOAD IN SHEAR = ALLOWABLE SHEAR STRESS × BOLT AREA

$= 90\ MPa \times \frac{\pi}{4} D^2$

$= 90\ MPa \times 0.7854 (16)^2$

$= 18.1\ kN$

(b) ALLOWABLE LOAD IN TENSION?

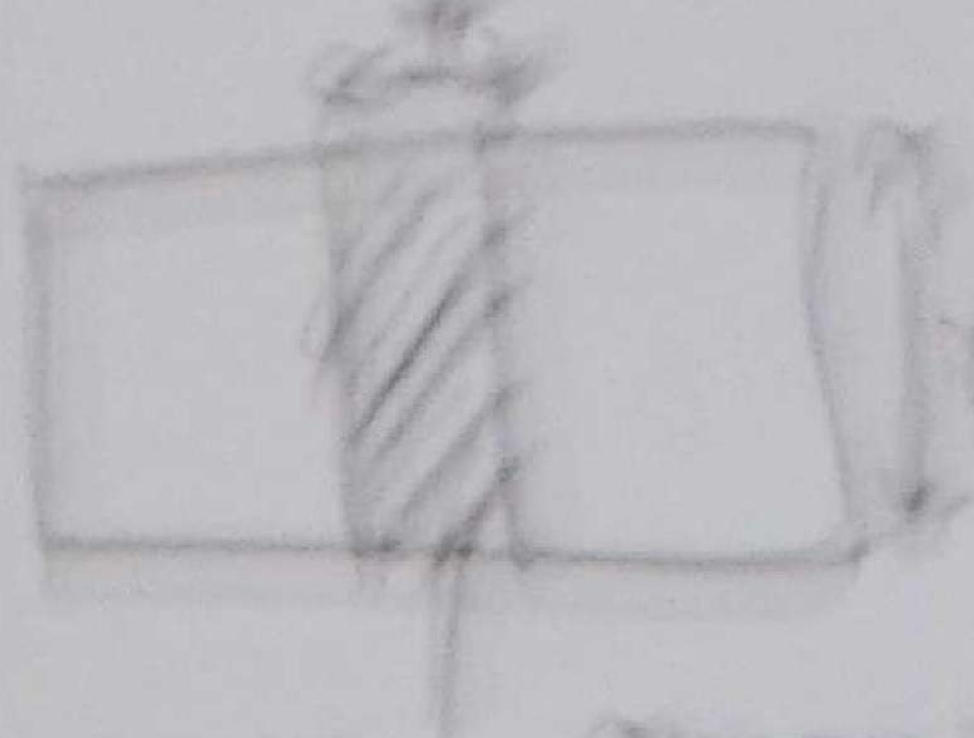
$A = (w - D)t = (40 - 16) \times 8$

$= 192\ mm^2$

= A × ALLOWABLE TENSILE STRESS

$192 \times 110 = 21.1\ kN$

(c) BEARING STRENGTH = 128 × ALLOWABLE COMPRESSIVE STRESS 220 MPa



A_c = PLATE THICKNESS × BOLT DIAMETER

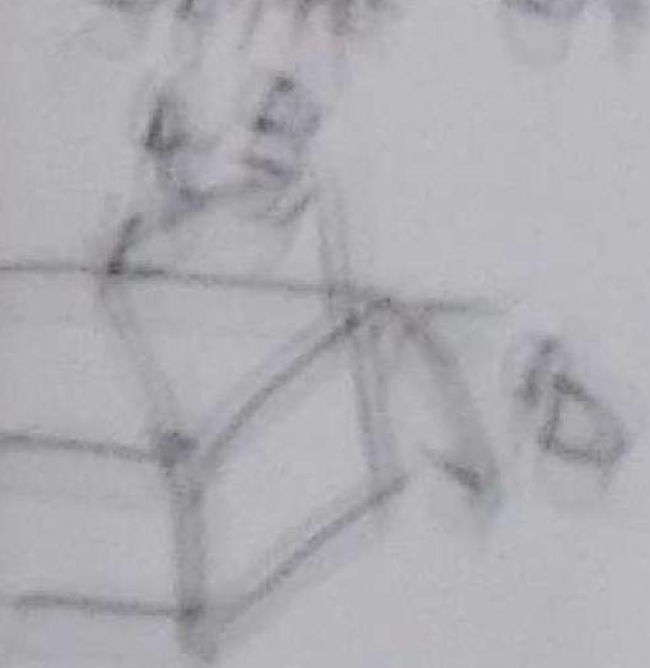
= 8 × 16 = 128 mm²

⟶ = 128 × 220 = 28160 N ≃ 28.2 kN

STRENGTH OF JOINT = 18.1 kN

(d) JOINT EFFICIENCY = ?

JOINT EFFICIENCY = $\frac{\text{STRENGTH OF JOINT}}{\text{STRENGTH OF UNPUNCHED PLATE}}$



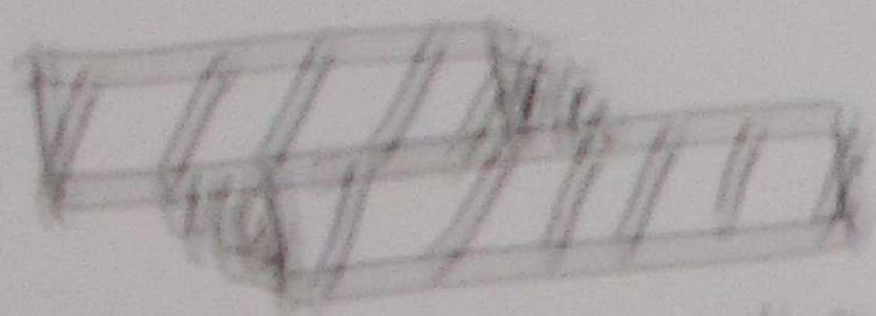
= $\frac{18.1 \text{ kN}}{35.2 \text{ kN}}$ = 0.514

= 51.4%

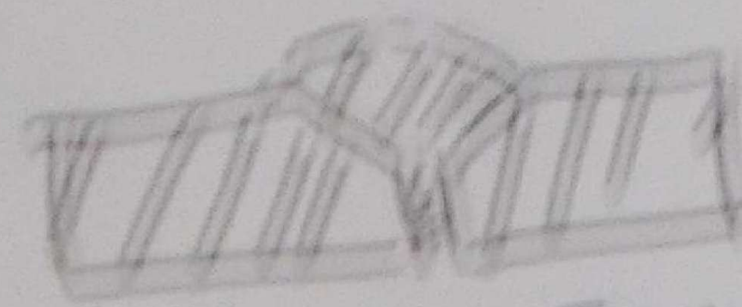
AREA OF UNPUNCHED PLATE × ALLOWABLE COMPRESSIVE STRESS = 35.2 kN

320 × 110 MPa =

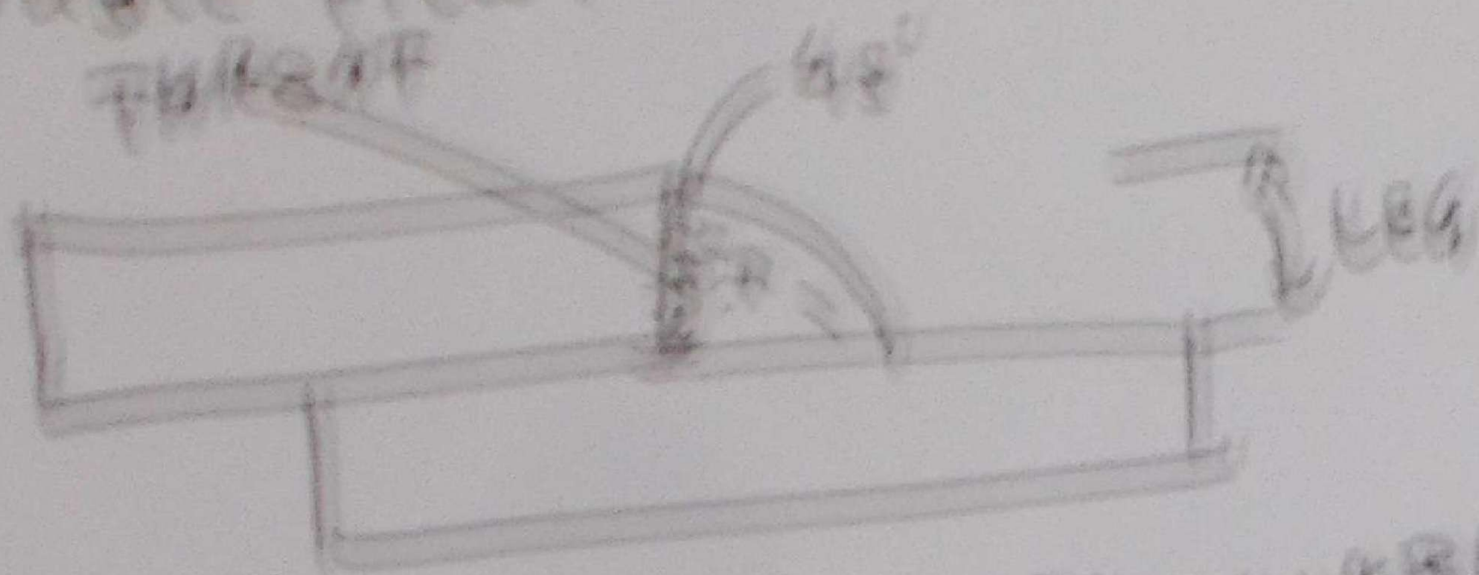
WELDED FASTENER



DOUBLE FILLET LAP JOINT



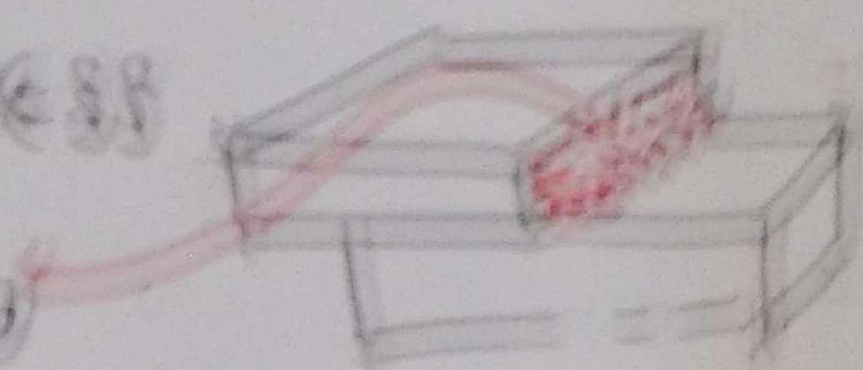
BUTT JOINT



P = MAXIMUM ALLOWABLE LOAD ON WELD

f = ALLOWABLE STRESS

l = LENGTH OF WELD



S = NOMINAL SIZE OF WELD

t = THROAT THICKNESS = 0.707 * S

$$f = \text{ALLOWABLE STRESS} = \frac{\text{ULTIMATE STRESS OF ELECTRODE}}{\text{SAFETY FACTOR (3)}}$$

$$\text{Allowable load/mm} = \text{Allowable stress} \times \text{Throat thickness}$$

$$\text{Length of weld (mm)} = \frac{\text{Maximum allowable load}}{\text{Allowable load/mm}}$$

P13 Fillet weld = 8 mm (Leg length side)

Electrode strength = 410 MPa

(a) Throat thickness (t) | Load: 56 kN

$$t = 0.707 \times s = 0.707 \times 8 = 5.66 \text{ mm}$$

(b) Allowable stress (f)

$$f = \frac{\text{Ult stress of electrode}}{\text{Safety factor}} = \frac{410}{3} = 136.7 \text{ MPa}$$

(c) Allowable load/mm

= Allowable stress × Throat thickness

= 136.7 × 5.66 = 773 N/mm

[illegible]

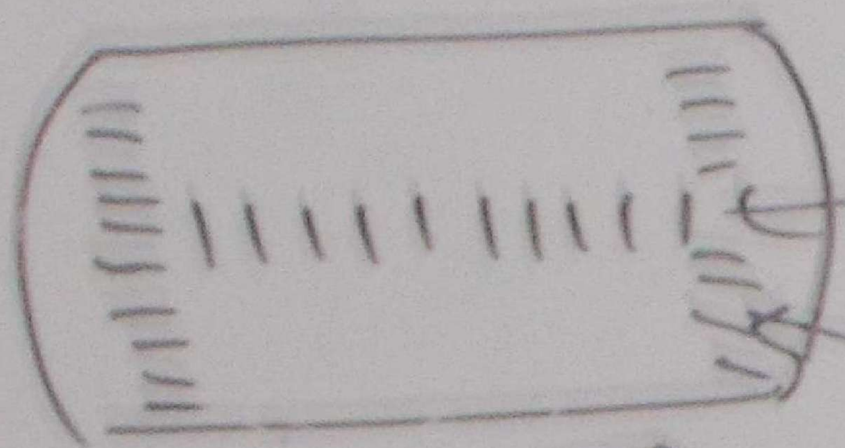
(d)

$$\text{Length of weld} = \frac{F}{\text{allowable load/mm}} = \frac{56.2}{773} = 68$$

PRESSURE VESSELS

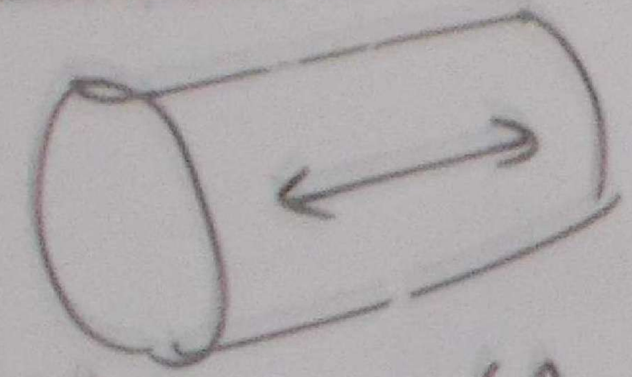
POSSIBLE STRESS → LONGITUDINAL SEAN

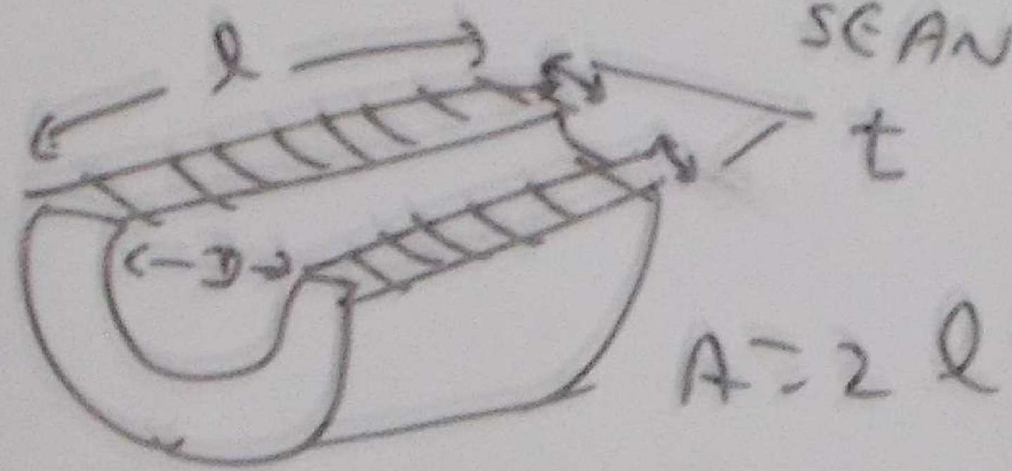
POSSIBLE STRESS → CIRCUMFERENTIAL SEAN



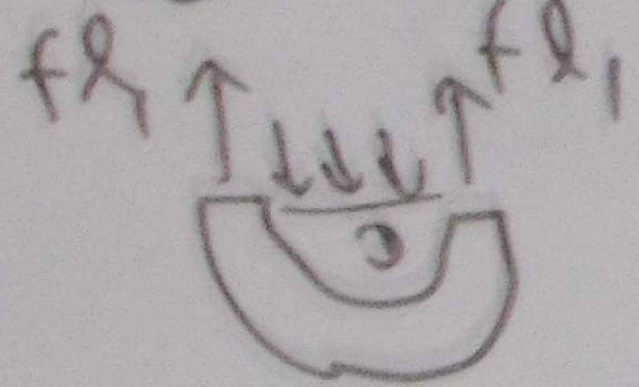
LONGITUDINAL SEAN

CIRCUMFERENTIAL SEAN





$A = 2lt$



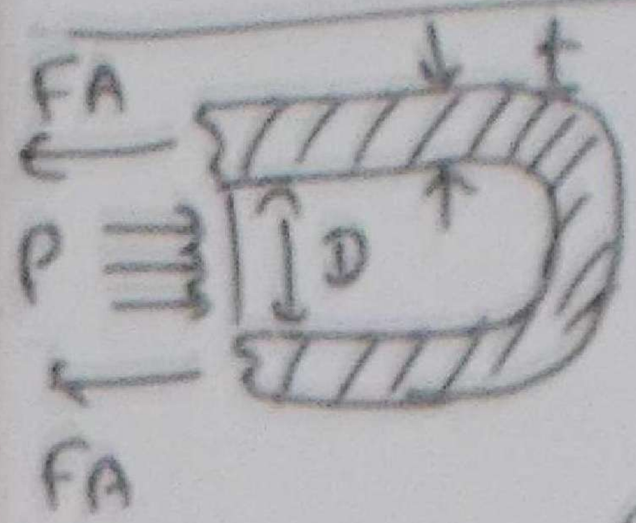
$$F = p\,l\,D$$

- F → FORCE
- p → PRESSURE
- l → LENGTH OF VESSEL
- D → INTERNAL DIAMETER OF VESSEL

$$\text{HOOP STRESS} = \frac{F}{A} = \frac{plD}{2lt} = \frac{pD}{2t}$$

↕ LONGITUDINAL SEAN

AXIAL STRESS



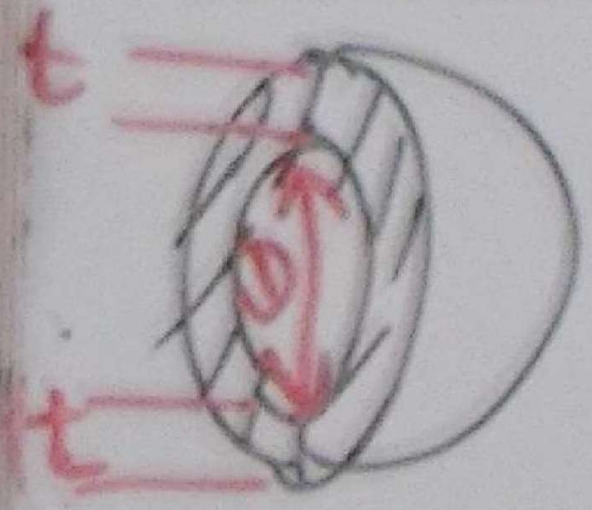
$$F = P \frac{\pi D^2}{4} \leftarrow A$$

$$A = \pi D t$$

$$f = \frac{F}{A} = \frac{P \frac{\pi D^2}{4}}{\pi D t} = \frac{PD}{4t}$$

$$\text{JOINT EFFICIENCY} = \frac{\text{THICKNESS}}{\text{REQUIRED THICKNESS}}$$

STRESSES IN SPHERICAL SHELLS



$$f = \frac{F}{A} = \frac{P \frac{\pi D^2}{4}}{\pi D t} = \frac{PD}{4t}$$

$$f = \frac{\text{ULTIMATE STRENGTH}}{\text{SAFETY FACTOR}}$$

Pa HOOP STRESS = ?

CYLINDERICAL AIR RESERVOIR

1.2 m LONG, DIAMETER = 350 mm

WALL THICKNESS 6 mm, PRESSURE 1 MPa

$$\text{HOOP STRESS} = \frac{PD}{2t} = \frac{1 \times 350}{2 \times 6} = 29.2 \text{ MPa}$$

Pb FIND AXIAL STRESS OF ABOVE PROBLEM

$$f_A = \frac{PD}{4t} = \frac{1 \text{ MPa} \times 350 \text{ mm}}{4 \times 6 \text{ mm}}$$

$$= 14.6 \text{ MPa}$$

Pb STEAM BOILER DRUM

DIAMETER = 1.2 m

MAXIMUM ALLOWABLE STRESS = 75 MPa

PRESSURE = 1.5 MPa

$$f_H = \frac{PD}{2t}$$

$$t = \frac{PD}{2f_H} = \frac{1.5 \times 1200\,mm}{2 \times 73\,MPa} = 12\,mm$$

Q8

JOINT EFFICIENCY = 80%

THICKNESS = 12 mm

REQUIRED THICKNESS

$$\text{REQUIRED THICKNESS} = \frac{\text{THICKNESS}}{\text{JOINT EFFICIENCY}} = \frac{12}{0.8} = 15\,mm$$

Q9 ULTIMATE STRENGTH = 380 MPa

SAFETY FACTOR = 5 , DIAMETER = 1 m

THICKNESS = 10 mm

PRESSURE = ? (a) SPHERICAL (b) CYLINDRICAL

$$f = \frac{380}{5} = 76\,MPa \qquad f = \frac{PD}{4t}$$

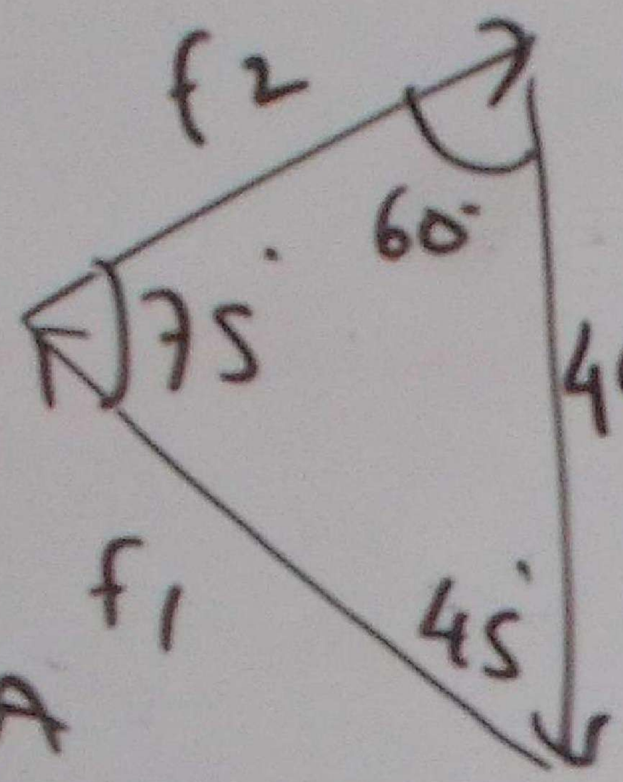
(a) SPHERICAL $\Rightarrow P = \frac{4tf}{D} = \frac{4 \times 10 \times 76}{1000} = 3.04\,MPa$

(b) CYLINDERICAL $\Rightarrow$

$$f = \frac{PD}{2t} \rightarrow P = \frac{2ft}{D} = \frac{2 \times 76 \times 10}{1000} = 1.52\,MPa$$

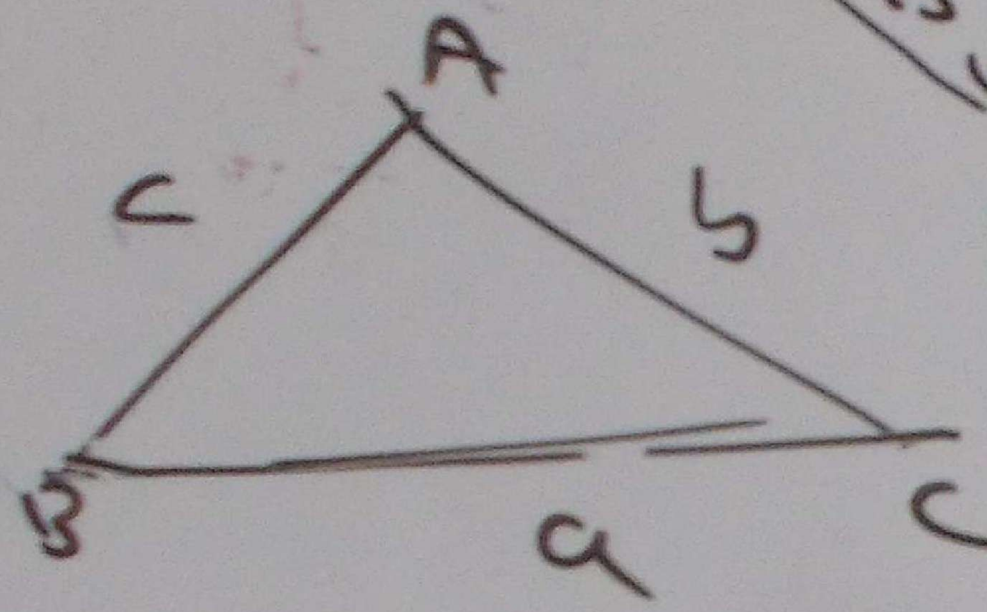
EQUILBRIUM

Ph



SINE RULE

$F_1 = ?$ $F_2 = ?$



$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

$$\frac{F_1}{\sin 60} = \frac{F_2}{\sin 45} = \frac{49.06}{\sin 75}$$

$$\frac{F_1}{\sin 60} = \frac{49.06}{\sin 75}$$

$$F_1 = \frac{49.06 \sin 60}{\sin 75} = 43.92 \text{ KN}$$

Pb HOOP STRESS = ?

CYLINDERICAL AIR RESERVOIR
1.2m LONG, DIAMETER = 350 mm
WALL THICKNESS 6 mm, PRESSURE 1 mpa

$$\text{HOOP STRESS} = \frac{PD}{2t} = \frac{1 \times 350}{2 \times 6} = 29.2 \text{ mpa}$$

Pb FIND AXIAL STRESS OF ABOVE PROBLEM

$$f_A = \frac{PD}{4t} = \frac{1\,\text{mpa} \times 350\,\text{mm}}{4 \times 6\,\text{mm}}$$

$$= 14.6 \text{ mpa}$$

Pb STEAM BOILER DRUM
DIAMETER = 1.2 m
MAXIMUM ALLOWABLE STRESS = 75 mpa
PRESSURE = 1.5 mpa

$$f_H = \frac{PD}{2t}$$

$$t = \frac{PD}{2f_H} = \frac{1.5 \times 1200\ mm}{2 \times 75\ MPa} = 12\ mm$$

Pb

JOINT EFFICIENCY = 80%

THICKNESS = 12 mm REQUIRED THICKNESS

$$\text{REQUIRED THICKNESS} = \frac{\text{THICKNESS}}{\text{JOINT EFFICIENCY}} = \frac{12}{0.8} = 15\ mm$$

Pb ULTIMATE STRENGTH = 380 MPa

SAFETY FACTOR = 5, DIAMETER = 1m

THICKNESS = 10 mm

PRESSURE => (a) SPHERICAL (b) CYLINDRICAL

$$f = \frac{380}{5} = 76\ MPa \qquad f = \frac{PD}{4t}$$

(a) SPHERICAL => $f = \frac{PD}{4t} \Rightarrow P = \frac{4ft}{D} = \frac{4 \times 76 \times 10}{1000} = 3.04\ MPa$

(b) CYLINDRICAL =>

$$f = \frac{PD}{2t} \rightarrow P = \frac{2ft}{D} = \frac{2 \times 76 \times 10}{1000} = 1.52\ MPa$$

Pb HOOP STRESS?

CYLINDERICAL AIR RESERVOIR 1.2m LONG, DIAMETER = 350 mm WALL THICKNESS 6 mm, PRESSURE 1 mpa

$$\text{HOOP STRESS} = \frac{PD}{2t} = \frac{1 \times 350}{2 \times 6} = 29.2 \text{ mpa}$$

Pb FIND AXIAL STRESS OF ABOVE PROBLEM

$$f_A = \frac{PD}{4t} = \frac{1 \text{ mpa} \times 350 \text{ mm}}{4 \times 6 \text{ mm}}$$

$$= 14.6 \text{ mpa}$$

Pb STEAM BOILER DRUM
DIAMETER = 1.2 m
MAXIMUM ALLOWABLE STRESS = 75 mpa
PRESSURE = 1.5 mpa

$$f_H = \frac{PD}{2t}$$

$$t = \frac{PD}{2f_H} = \frac{1.5 \times 1200\,mm}{2 \times 75\,mpa} = 12\,mm$$

Pb

JOINT EFFICIENCY = 80%

THICKNESS = 12 mm

REQUIRED THICKNESS

$$\text{REQUIRED THICKNESS} = \frac{\text{THICKNESS}}{\text{JOINT EFFICIENCY}} = \frac{12}{0.8} = 15\,mm$$

Pb ULTIMATE STRENGTH = 380 MPa

SAFETY FACTOR = 5, DIAMETER = 1m

THICKNESS = 10 mm

PRESSURE = ? (a) SPHERICAL (b) CYLINDRICAL

$$f = \frac{380}{5} = 76\,MPa \qquad f = \frac{PD}{4t}$$

(a) SPHERICAL $\Rightarrow P = \frac{4tf}{D} = \frac{4 \times 10 \times 76}{1000} = 3.04\,MPa$

(b) CYLINDRICAL $\Rightarrow$

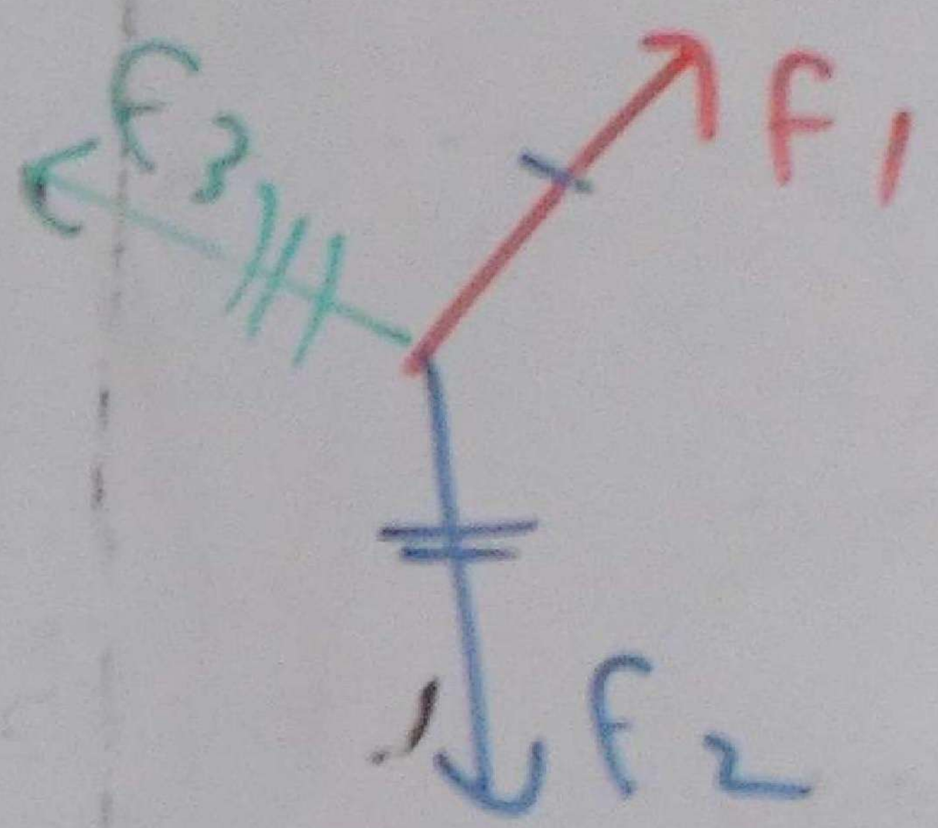
$$f = \frac{PD}{2t} \rightarrow P = \frac{2ft}{D} = \frac{2 \times 76 \times 10}{1000} = 1.52\,MPa$$

$$\frac{F_2}{\sin 45} = \frac{49.06}{\sin 75}$$

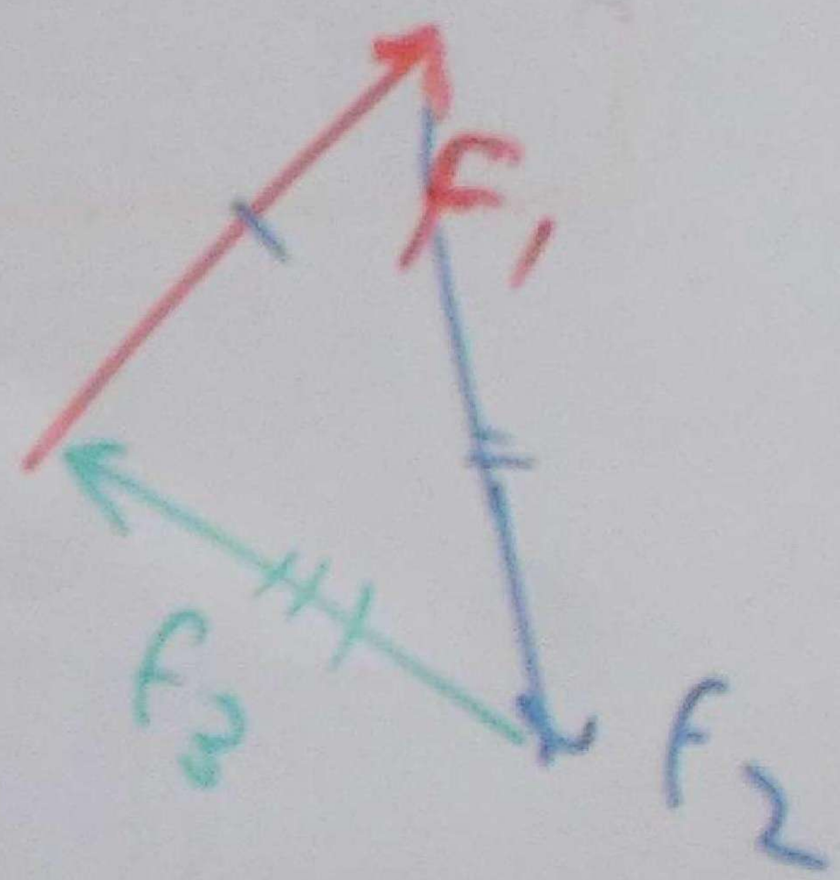
$$f_2 = \frac{49.06 \sin 45}{\sin 75}$$

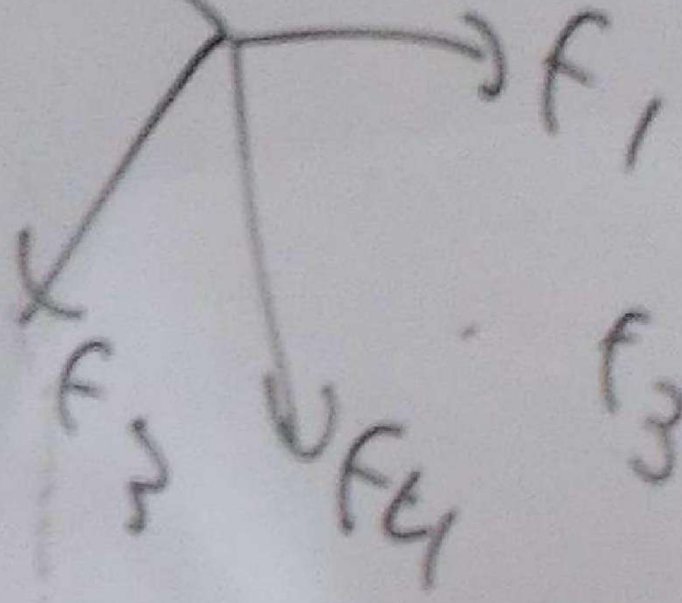
$$= 35.91 \text{ kN}$$

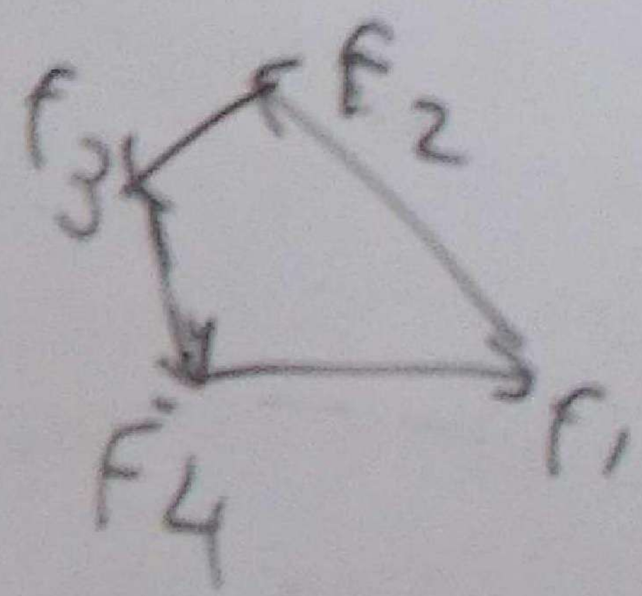
EQUILIBRIUM



F_2 EQUILIBRIUM

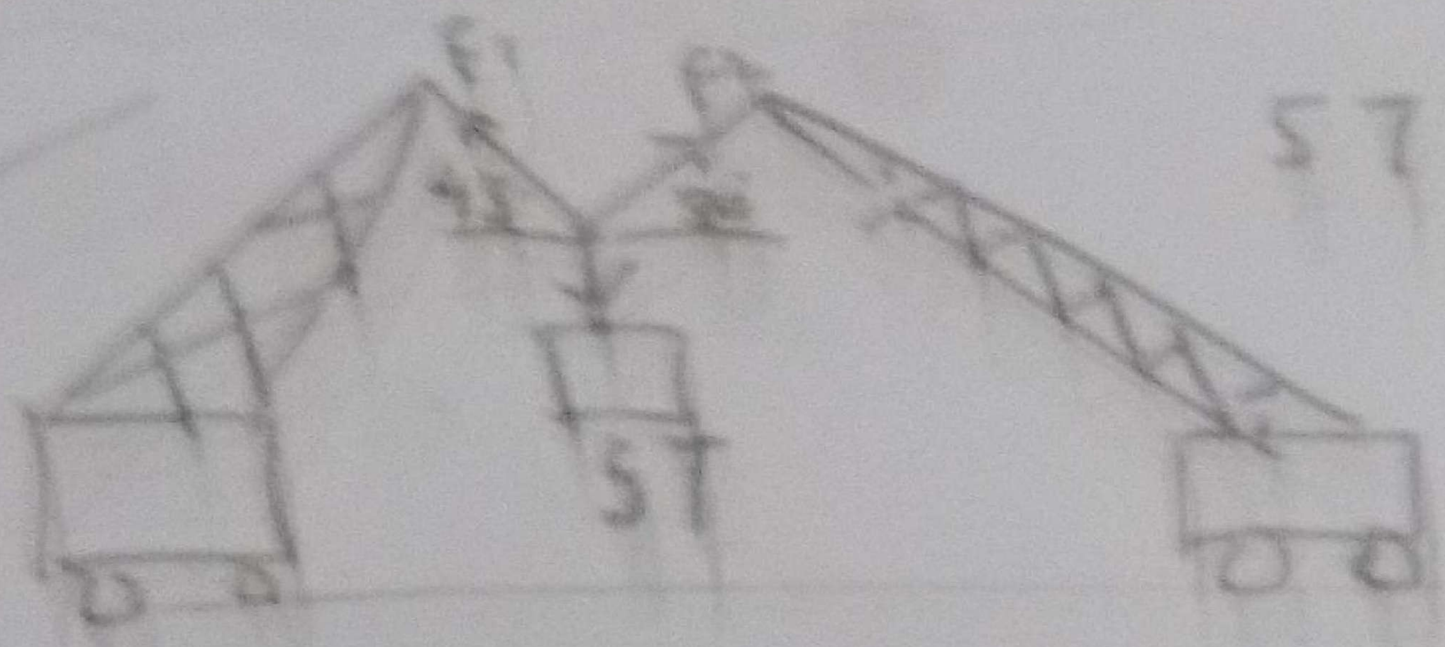




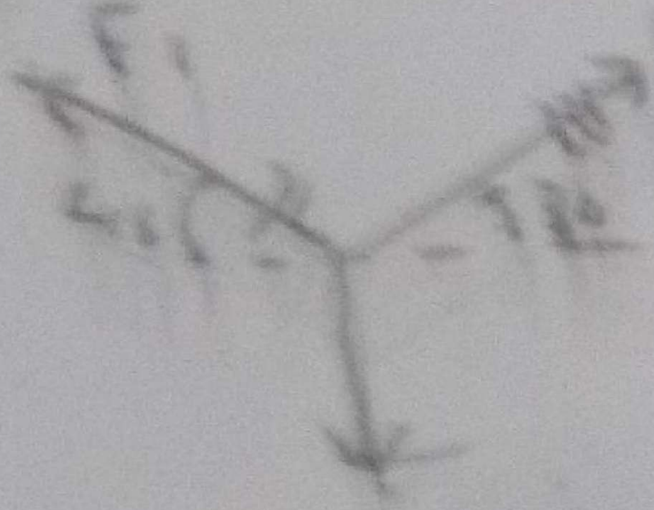


FREE BODY DIAGRAM

Ab



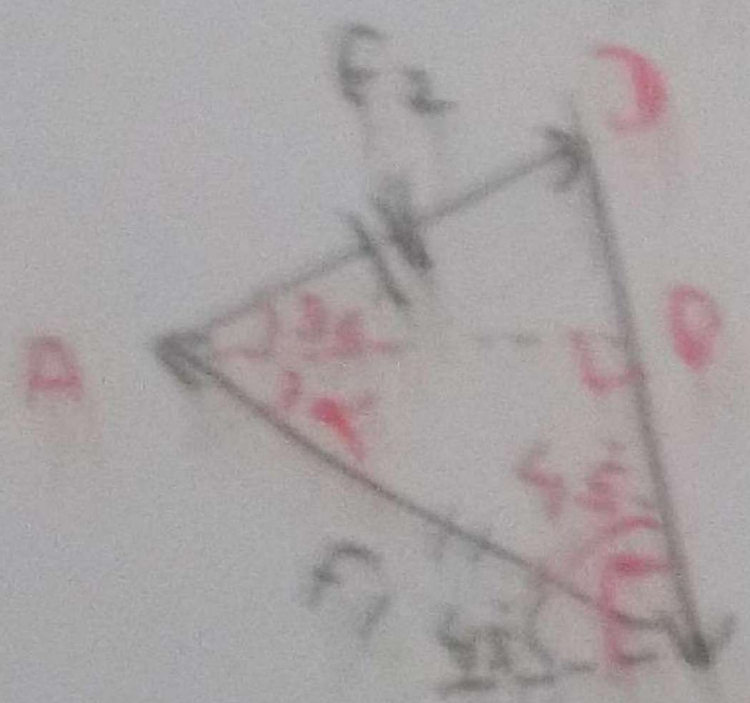
ST = 5000 kg



$= F = mg = 5000 \times 9.81$ kg m/s²

$= 49050$ N

$= 49.05$ kN



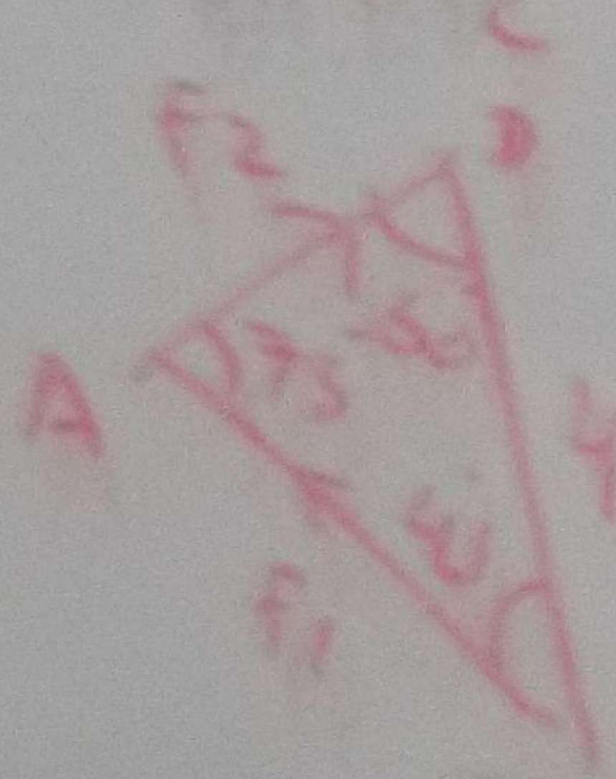
$\alpha + \hat{C} + \hat{ABC} = 180$

$\alpha + 45 + 90 = 180$

$\alpha + 135 = 180$

$\alpha = 180 - 135$

$= 45$



$\hat{A} = 30 + 45$

$= 75$

$\hat{B} = 180 - (45 + 75)$

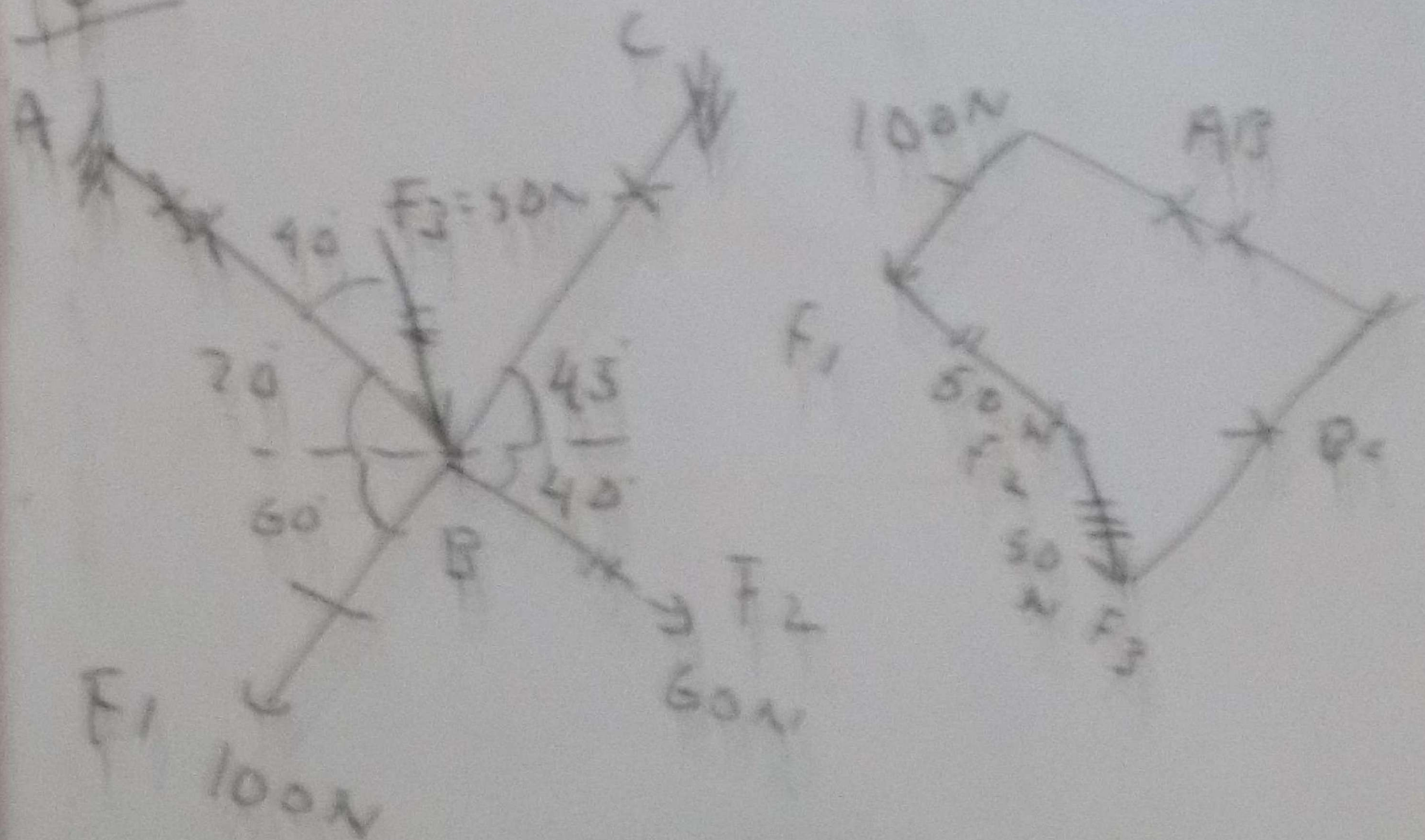
$\hat{D} = 60$

$$\frac{44.05\,kN}{\sin 75} = \frac{F_1}{\sin 60} = \frac{F_2}{\sin 45}$$

$$F_1 = \frac{44.05 \times \sin 60}{\sin 75} = 43.98\ kN$$

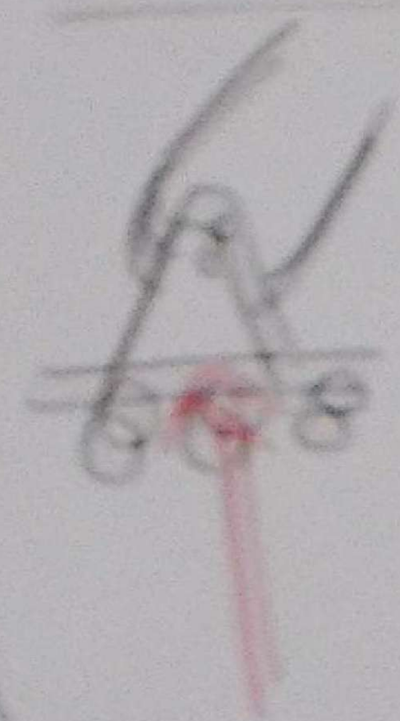
$$F_2 = \frac{44.05 \times \sin 45}{\sin 75} = 35.61\ kN$$

Pb

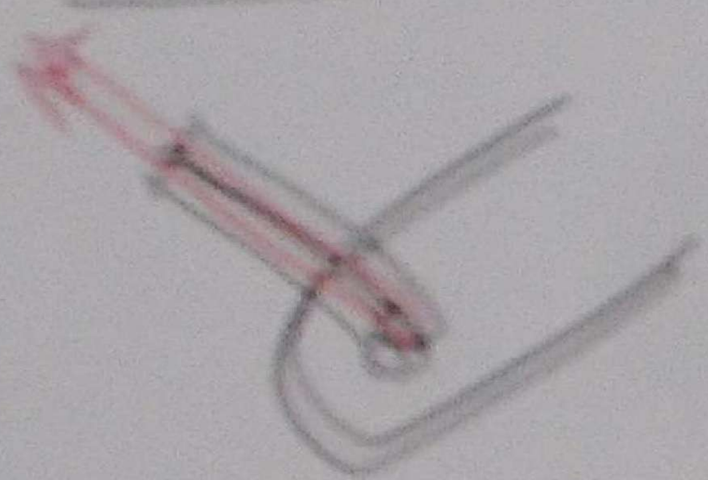


NORMAL REACTION

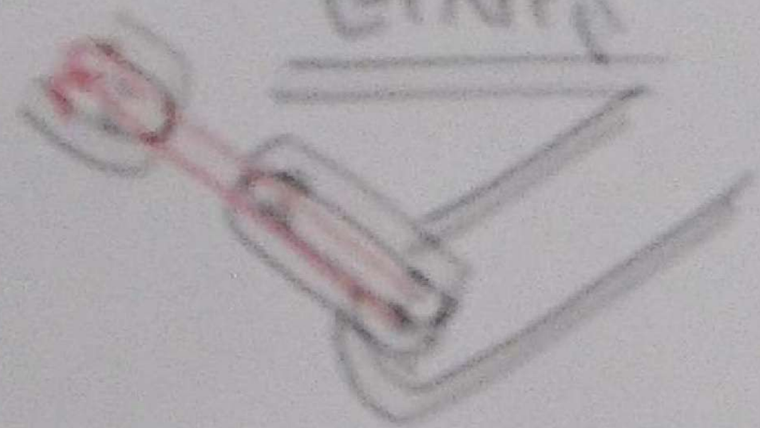
BALL ROLLER SUPPORT



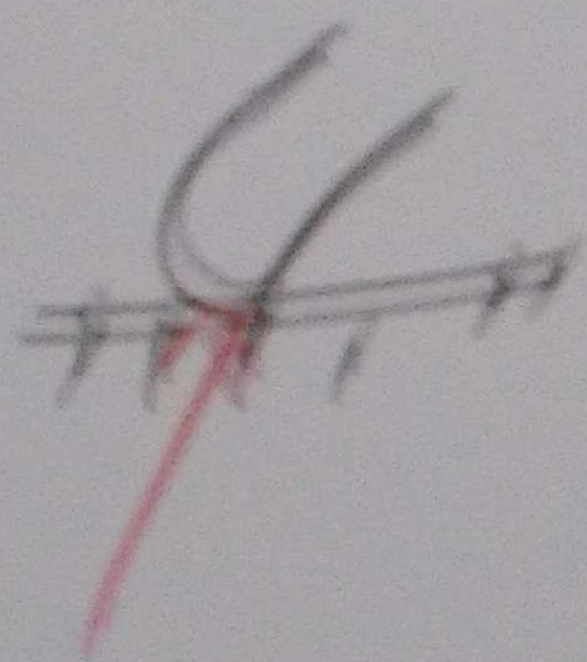
CABLE



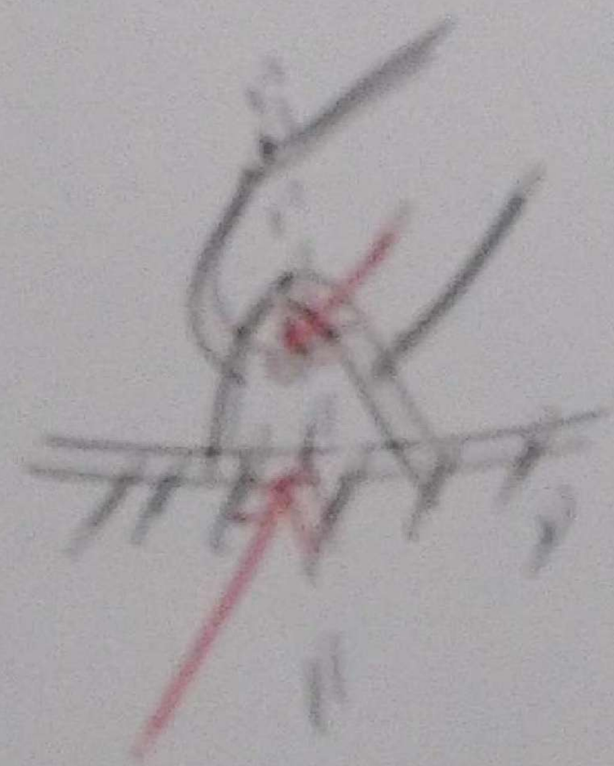
LINK



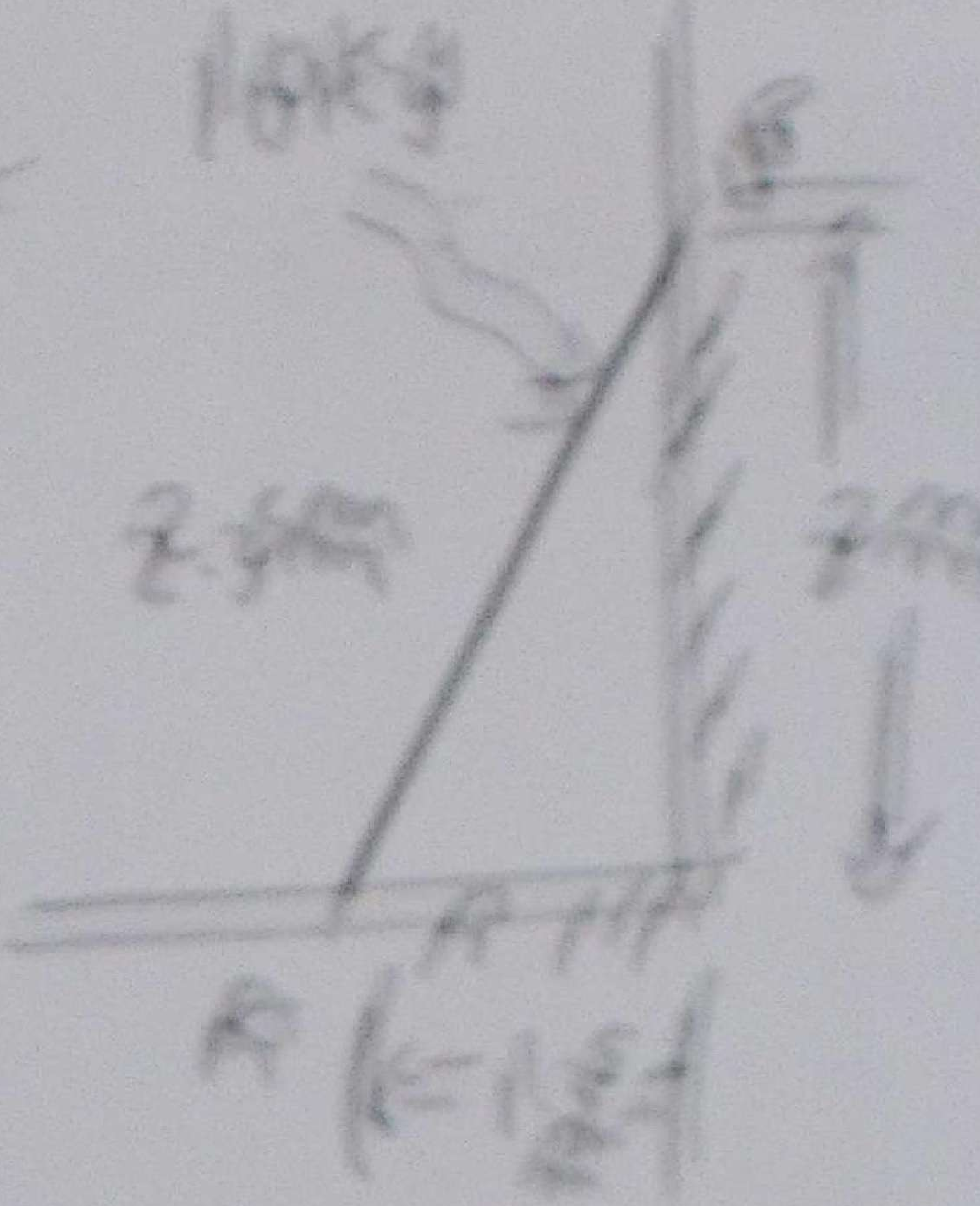
REACTION AT ANY ANGLE



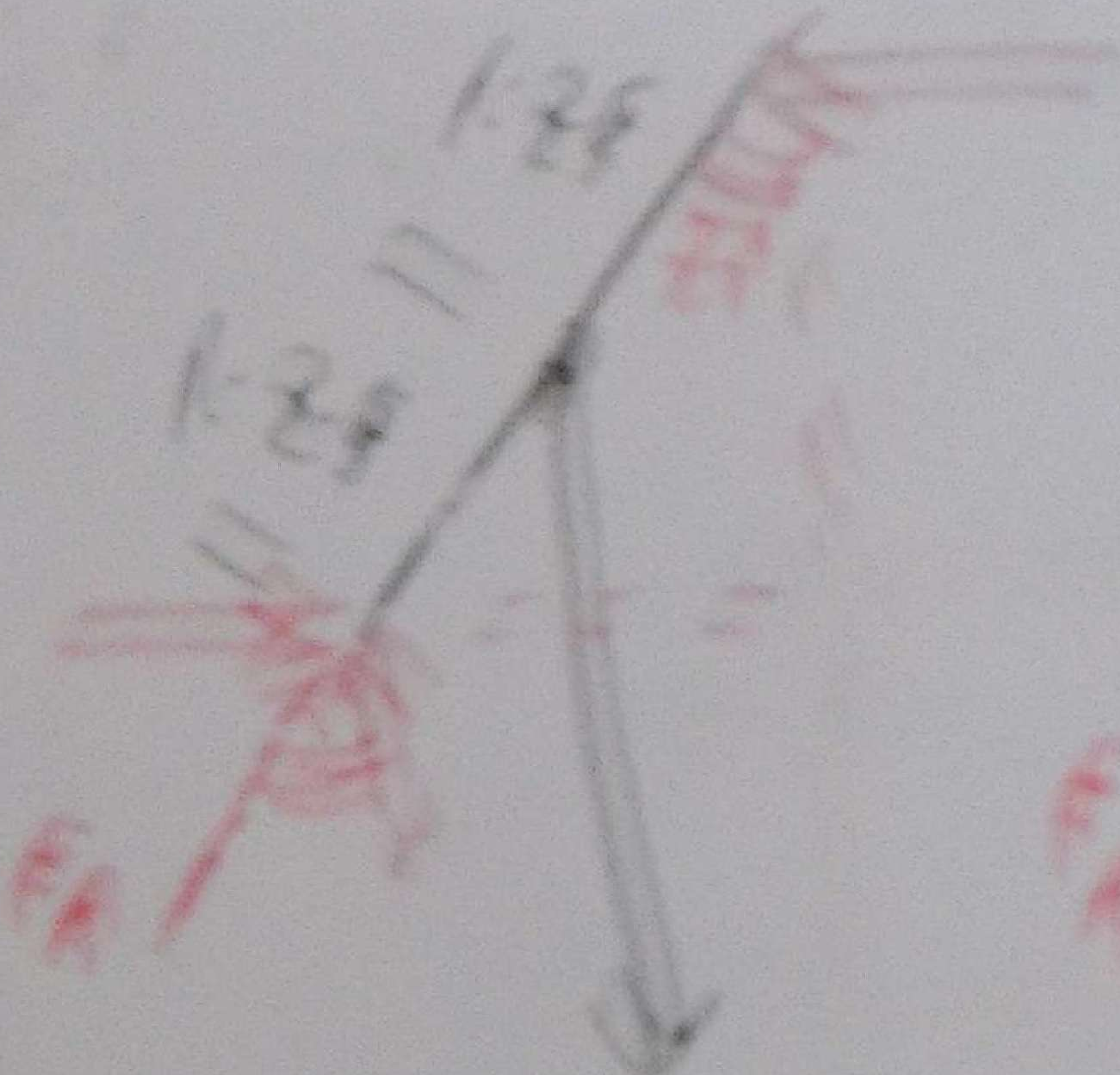
PIN OR HINGED



Q6

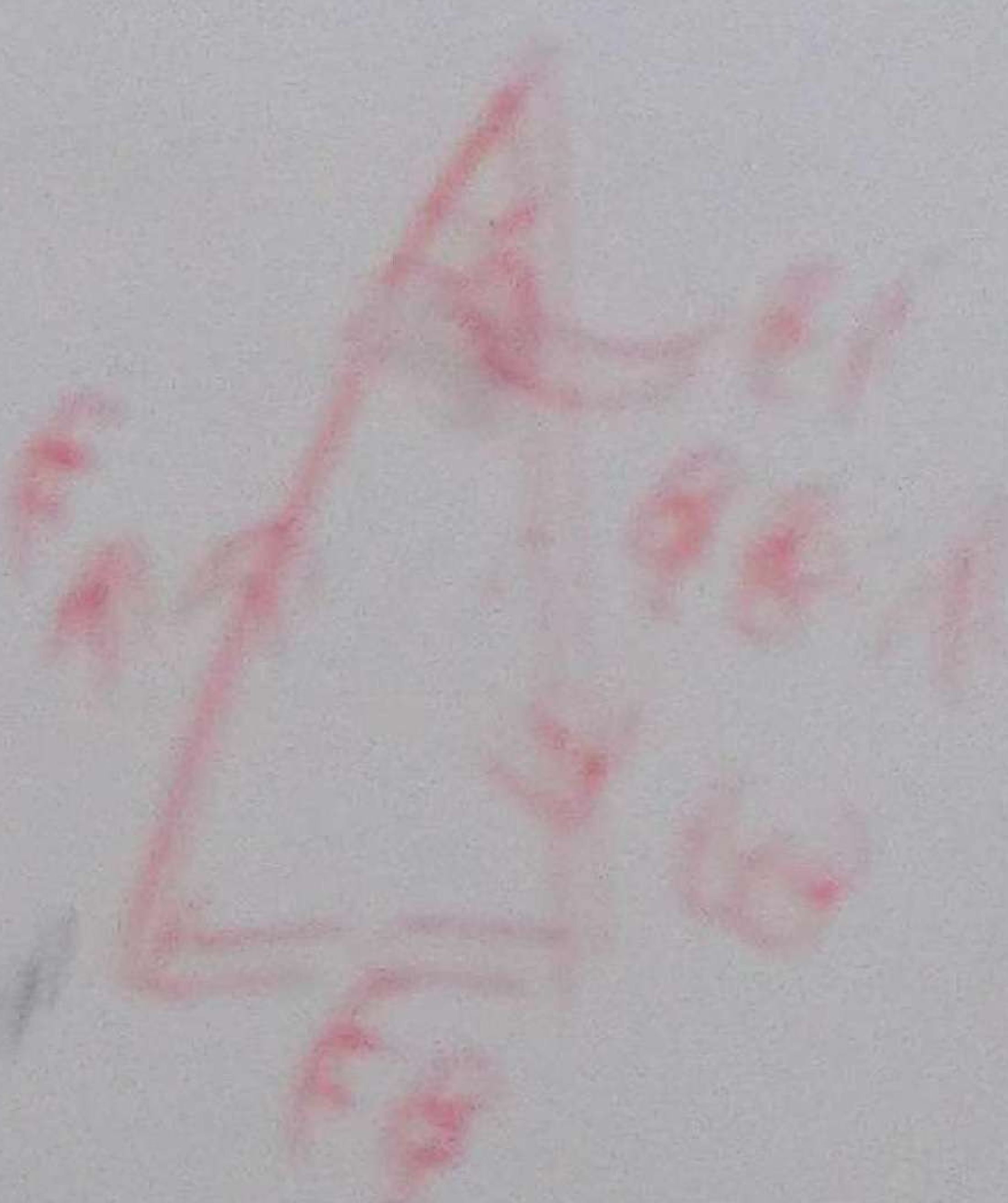


Find the reactions at A and B



$\theta = \tan^{-1} \frac{1.6}{?}$

$= 61$



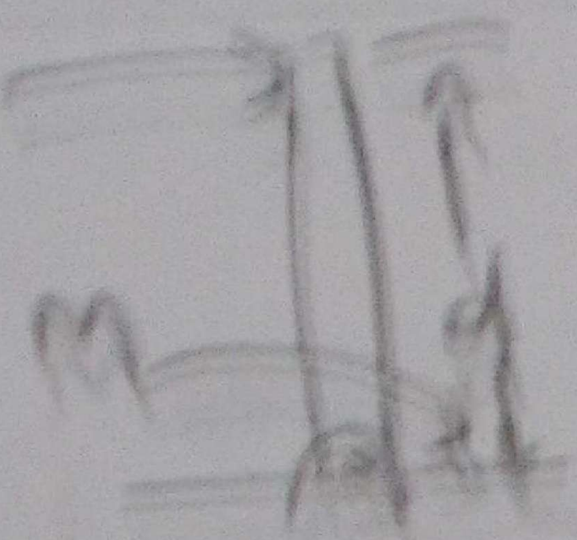
$F = mg = 10 \times 9.81$

$= 98.1$ N

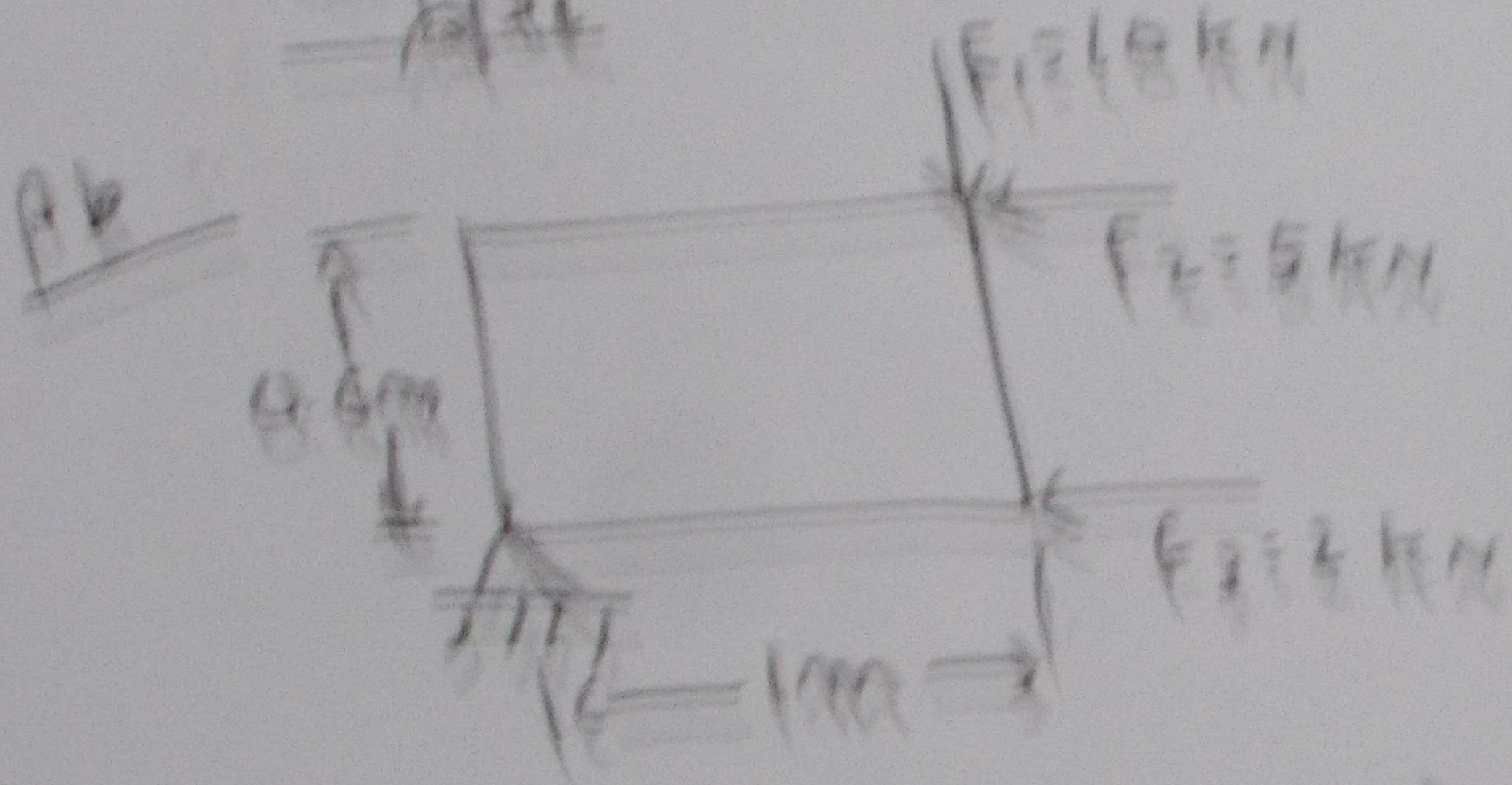
$F_A = 105$ N

$F_B = 36.8$ N

MOMENT OF THE FORCE



$M = F \times d$ $\quad M = Nm$



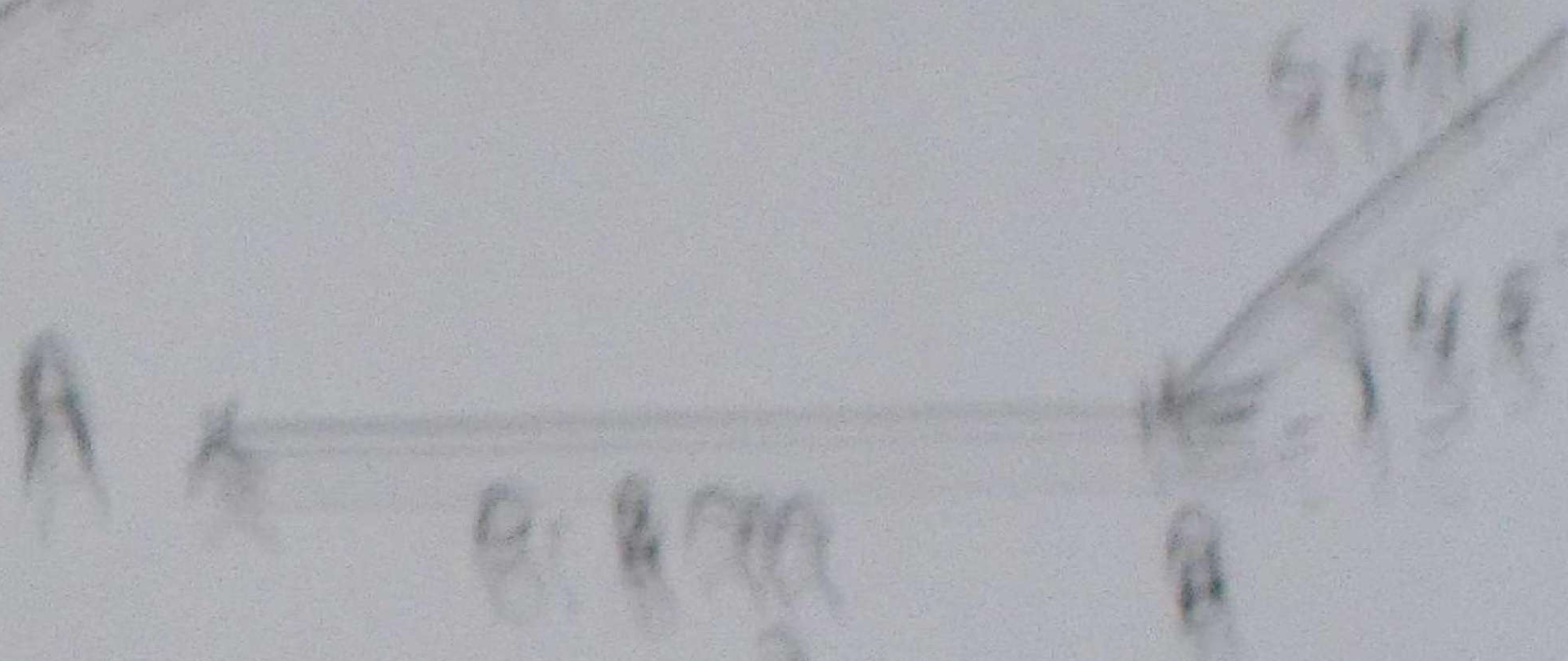
F_1 $M_1 = 10\,kN \times 1\,m = 10\,kN{-}m$

F_2 $M_2 = 5\,kN \times 0.6\,m = 3\,kN{-}m$

F_3 $M_3 = 2\,kN \times 0 = 0\,kN{-}m$

$M_T = 10 - 3 = 7\,kN{-}m$

1b



moment = ?

$\theta = 180 - (90 + 45) = 45$

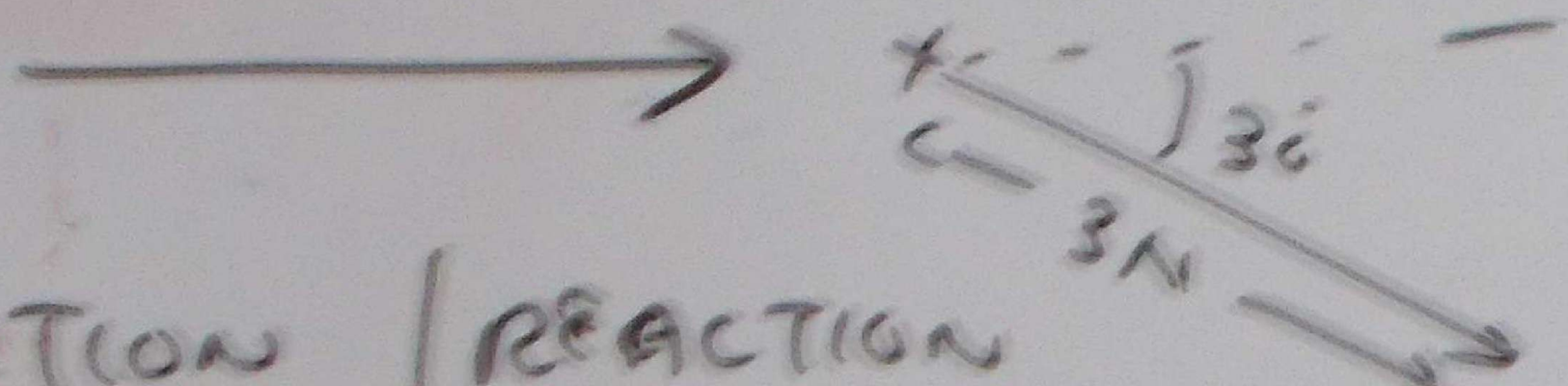
$AB = ? = 0.8 (\cos 45) = 0.566\,m$

$Moment = 50 \times 0.566 = 28.3$

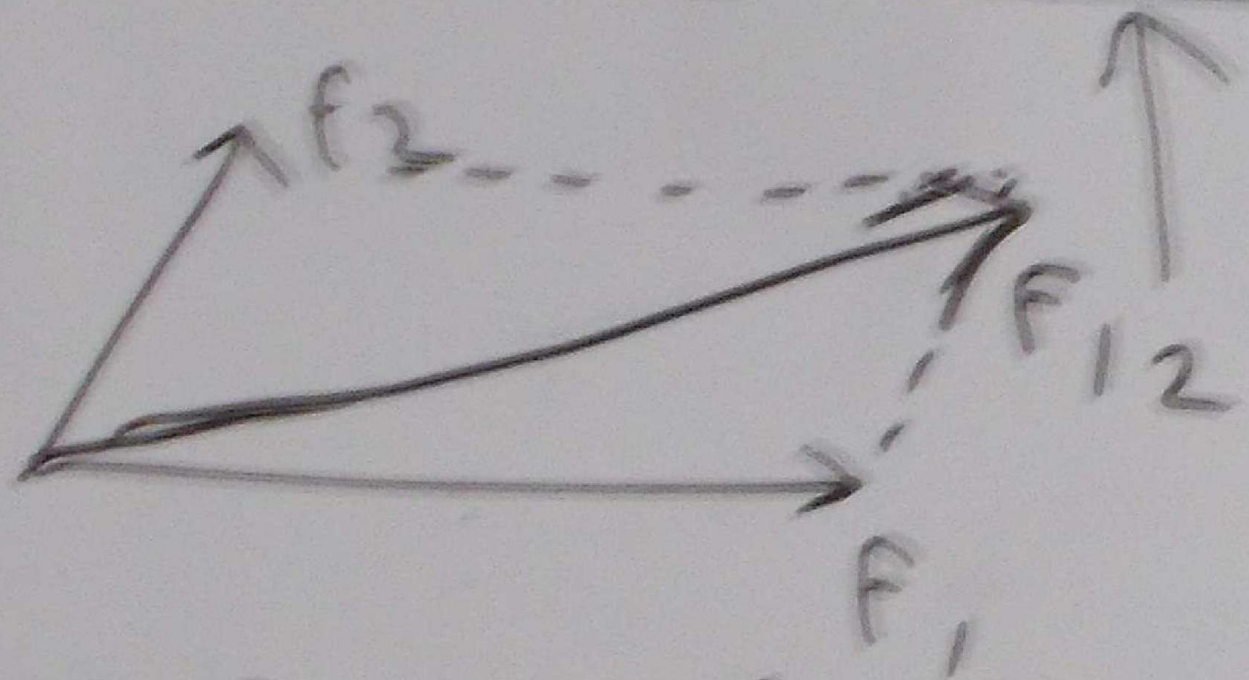
clockwise

FORCE — VECTOR

DIRECTION / MAGNITUDE

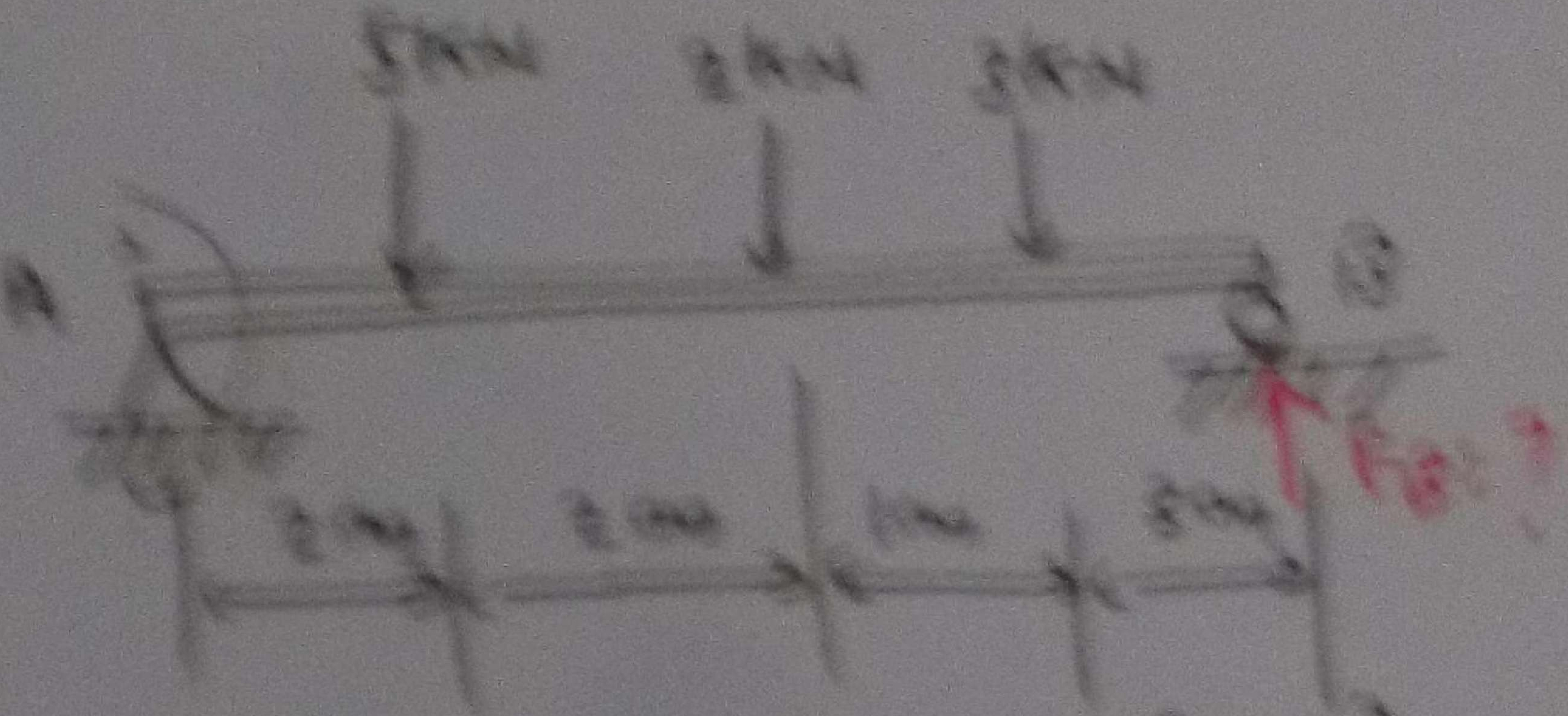


ACTION / REACTION



<u>EX</u> A FORCE OF 5KN ACTING UP AND TO THE RIGHT AT 30° TO THE HORIZONTAL, CALCULATE HORIZONTAL AND VERTICAL COMPONENTS.

ADDITION OF MOMENTS



moment of B = ?

$M_B = ?$ $F_B = ?$

$M_A = \sum F d$

$= 5 \times 2 + 2(2+2) + 3(2+2+1)$

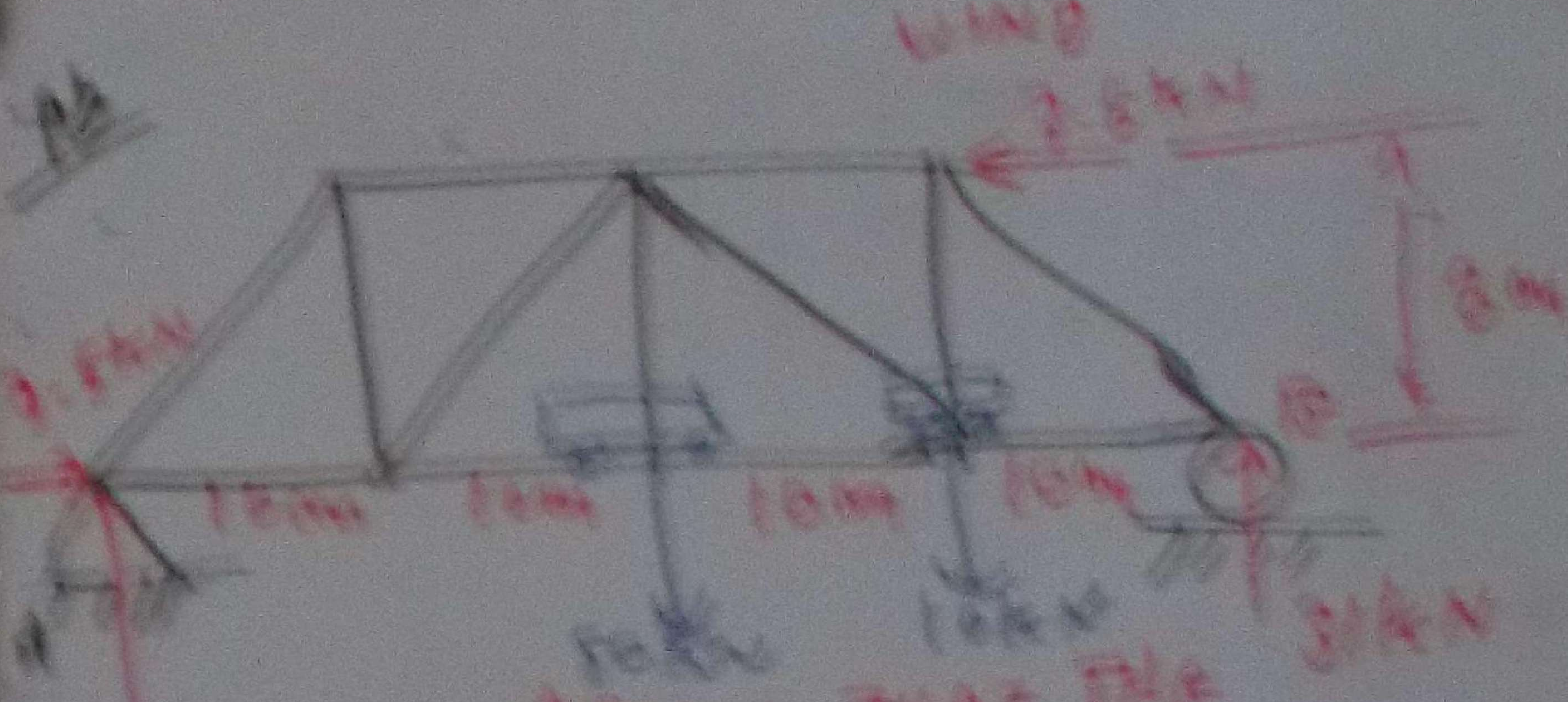
$= 5 \times 2 + 2 \times 4 + 3 \times 5 = 33$ kN-m

EQUILIBRIUM $M_A = M_B$

$M_B = 33$ kN-m

$M_B = F_B \times d = F_B \times (2+2+1+3)$

$33 = F_B \times 8$, $F_B = 33/8 = 4.1$ kN



PROVE THAT THE STRUCTURE IS ROTATIONAL EQUILIBRIUM

+↻ −↺

$\Sigma M_A = (10 \times 20) + (10 \times 30) + (-7.5 \times 8) + (-31 \times 40)$

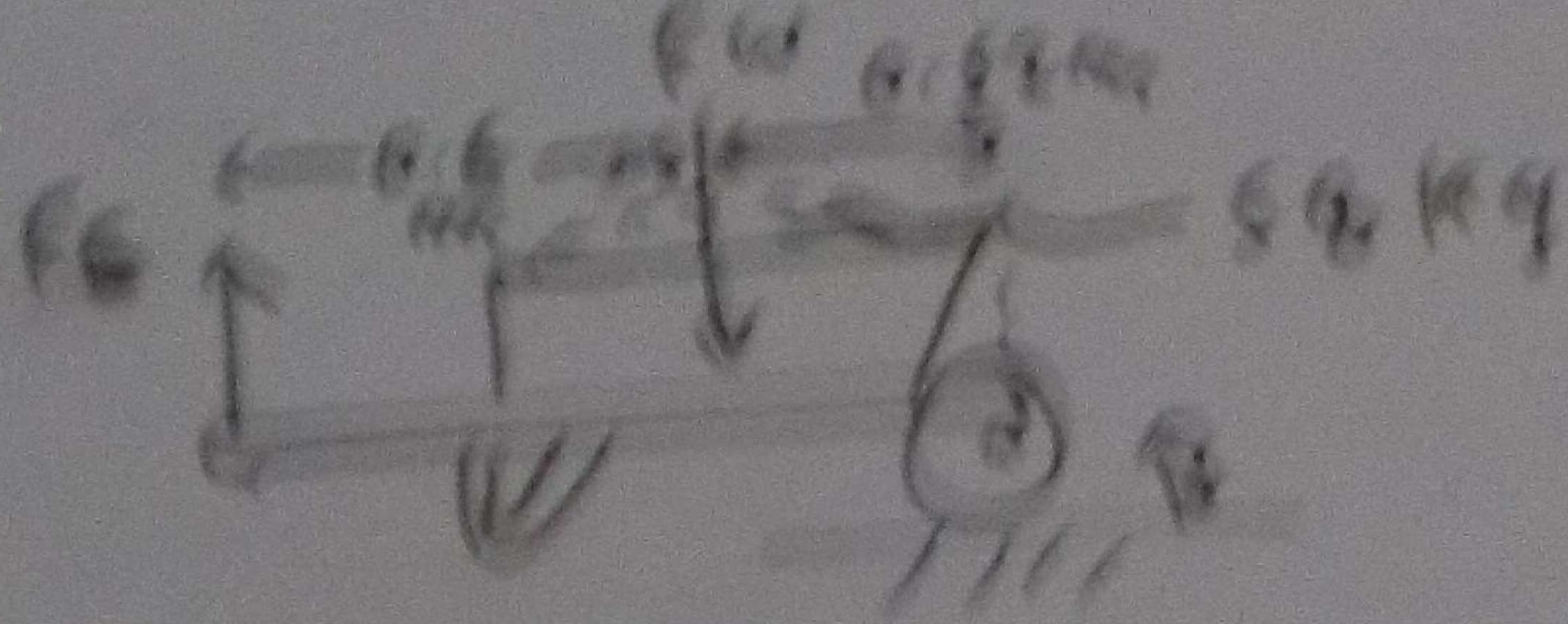
$\Sigma M_A = 0$

+↻ −↺

$\Sigma M_B = (29 \times 40) + (-10 \times 20) + (-10 \times 10) + (-7.5 \times 8)$

$\Sigma M_B = 0$

(b)



$F_E = ?$

$F_W = mg$

$= 58 \times 9.81$

$= 569\ N$

Moment at (B)

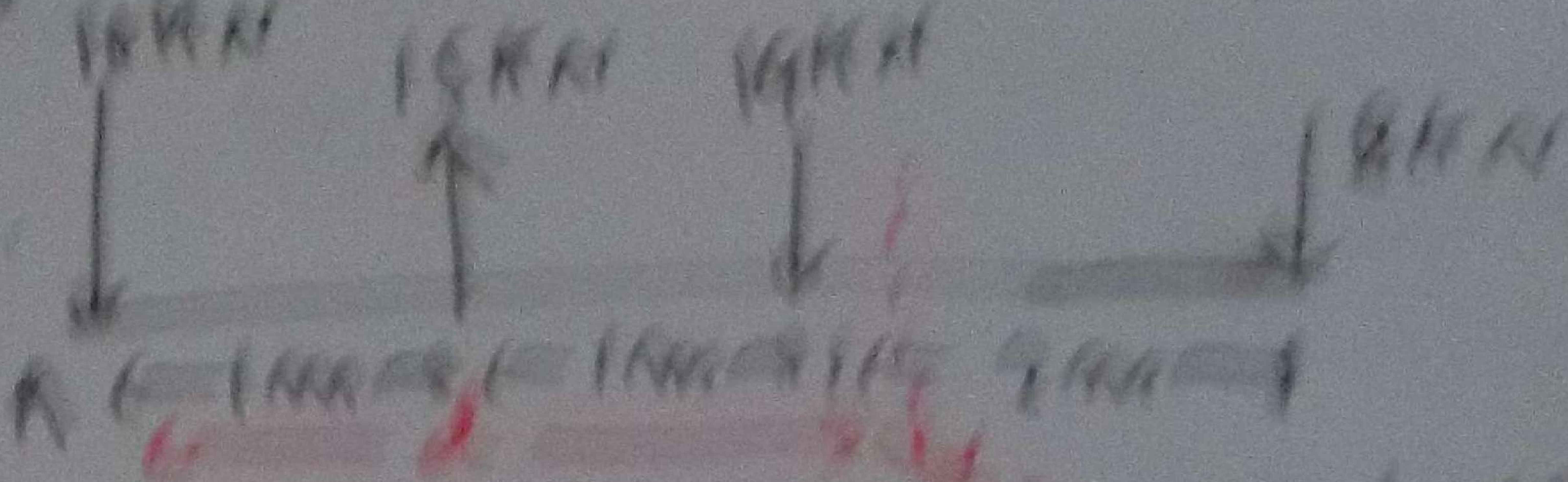
$F_E \times (0.6 + 0.52) = F_E \times 1.12\ N\cdot m$

$F_W \times 0.52 = 569 \times 0.52$

$F_E \times 1.12 = 569 \times 0.52$

$$F_E = \frac{569 \times 0.52}{1.12} = 264\ N$$

Ex.



RESULTANT FORCE AND LOCATION?

$\downarrow + \quad \uparrow -$

$\Sigma F = (+16) + (-15) + (+19) + (18)$

$= 16 - 15 + 19 + 18 = 22 \text{ kN} \downarrow$

ΣM_A @ $\curvearrowright$

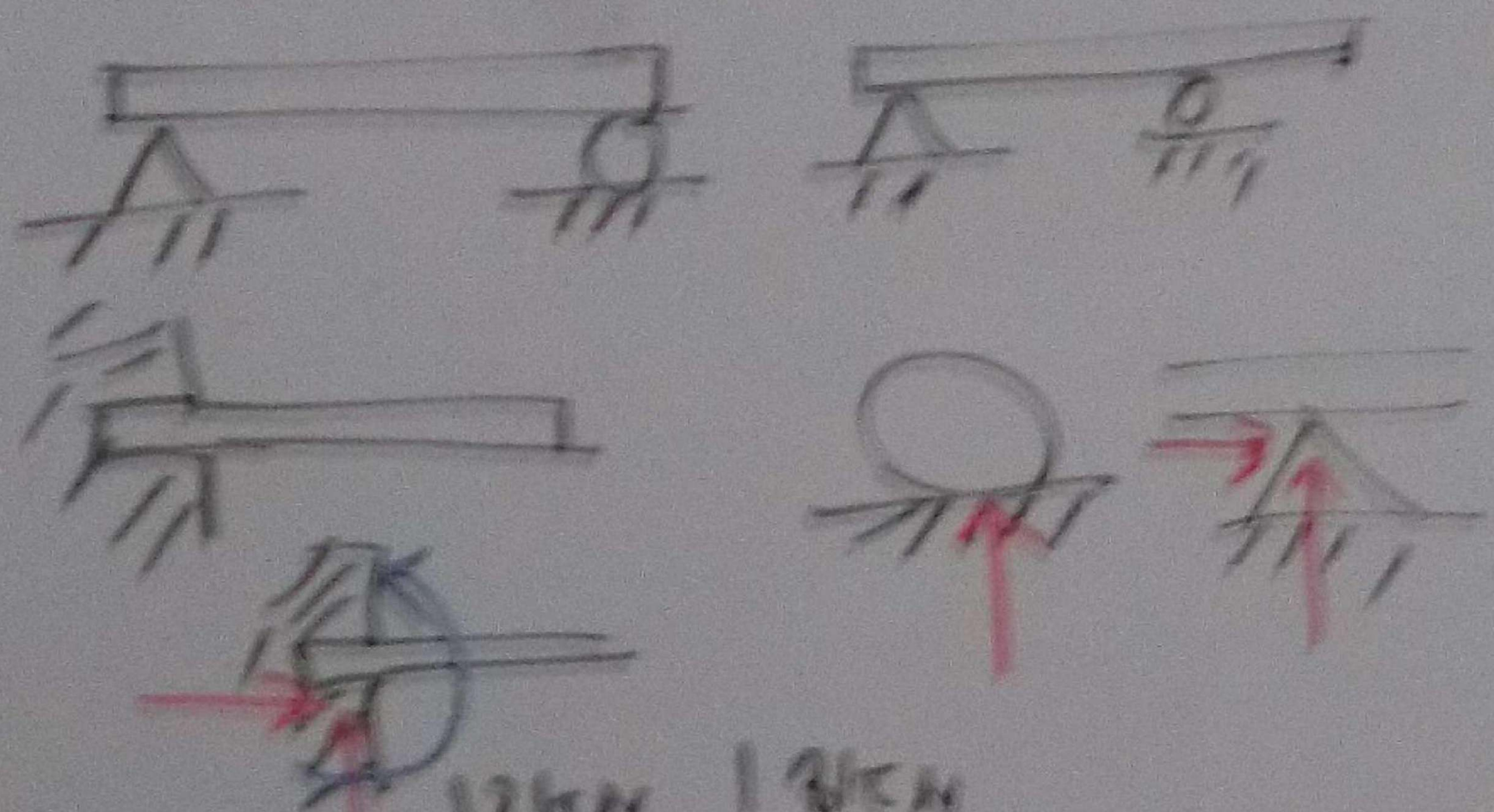
$22 d = (-15 \times 1) + (19 \times 2) + (18 \times 4)$

$= -15 + 38 + 76 = 55 \text{ kN·m}$

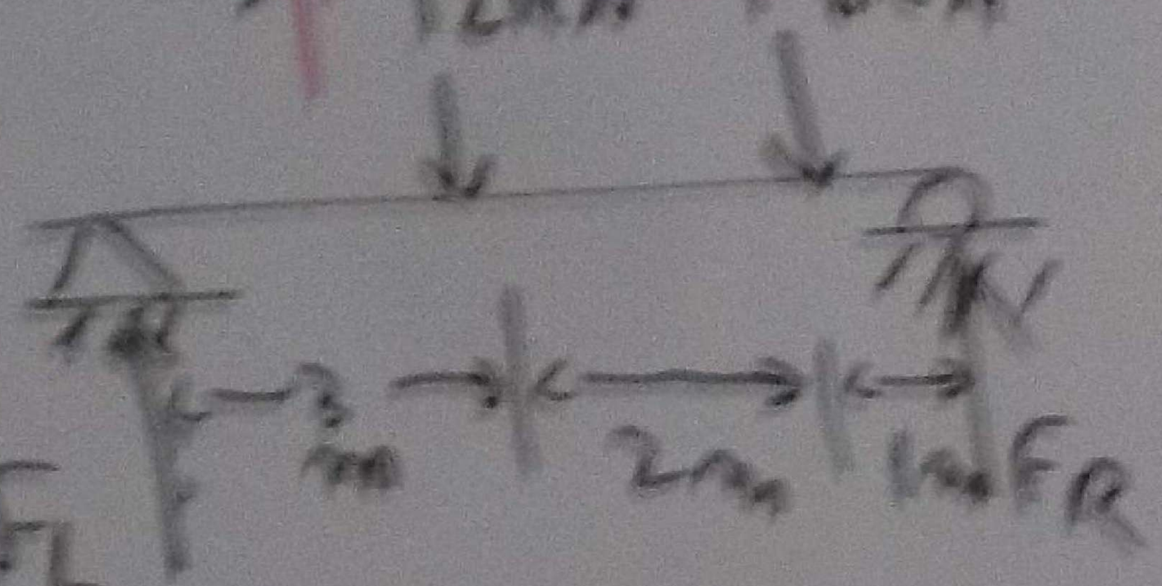
$d = \frac{55}{22} = 2.5 \text{ m}$

STRUCTURAL ANALYSIS

SIMPLY SUPPORTED BEAM



Pb



$\circlearrowleft + \quad \sum M_L = 12\times3 + 18\times5 + (-F_R\times6) = 0$

$36 + 90 - 6F_R = 0$

$\downarrow + \uparrow -$

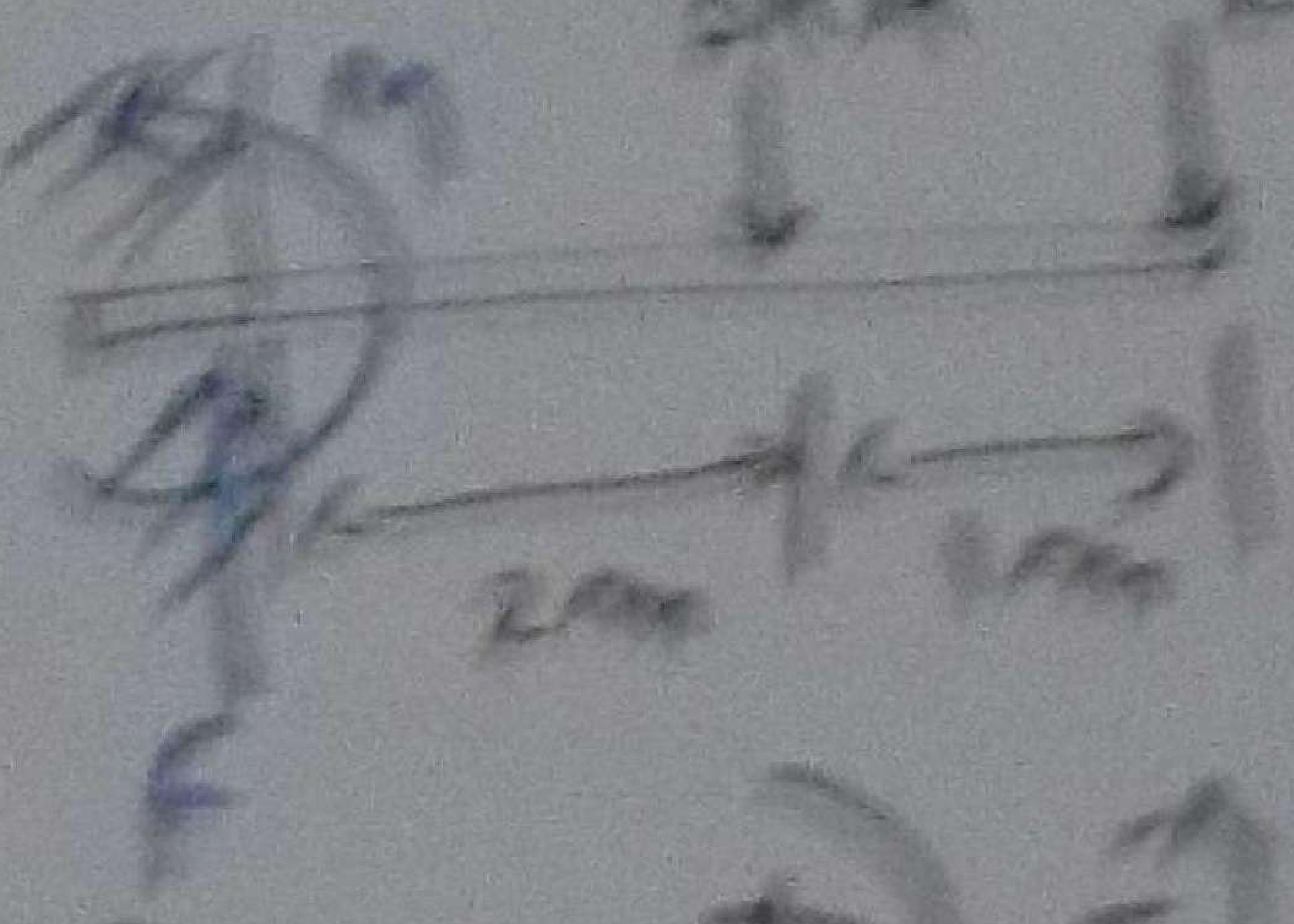
$-6F_R = -126 \Rightarrow F_R = 21\,kN$

$\sum F_y = 0 \quad (+12) + (+18) + (-F_L) + (-F_R) = 0$

$30 - F_L - 21 = 0$

$9 - F_L = 0 \Rightarrow F_L = 9\,kN$

CANTILEVER BEAM



M = ?

F = ?

$\Sigma M = 0$ + ↻ − ↺

$3 \times 2 + 2 \times 3 - M = 0$

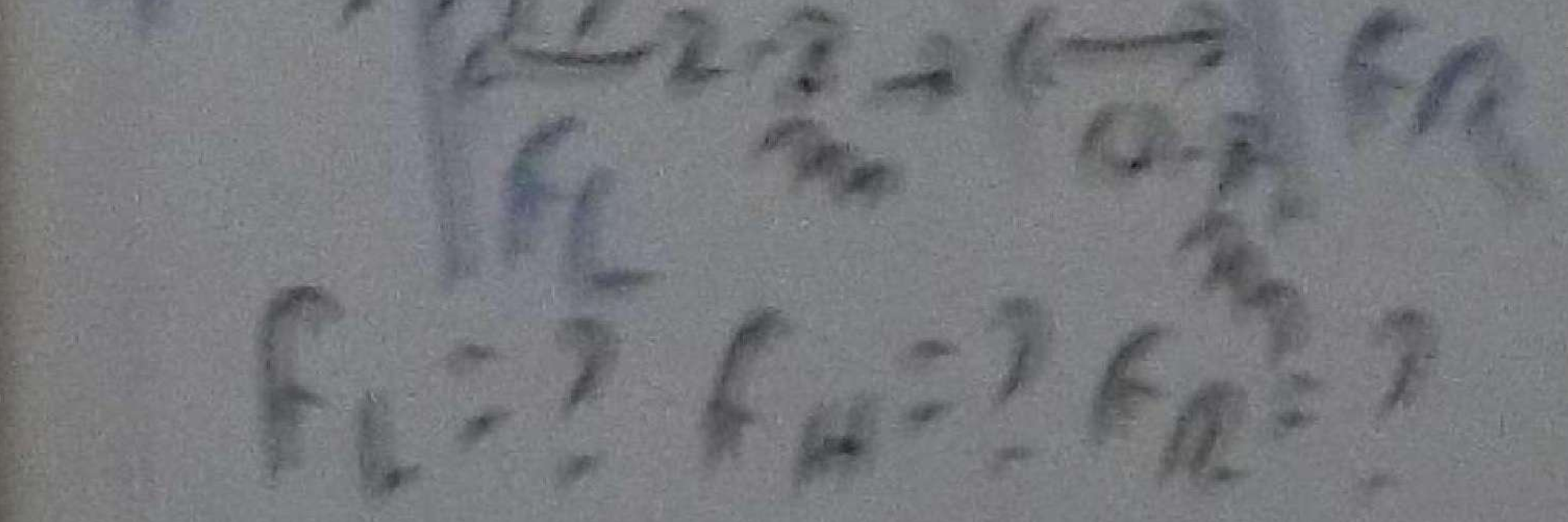
$6 + 6 - M = 0 \rightarrow M = 12$ kN-m

$\Sigma F_y = 0$ ↓+ ↑−

$(+3) + (+2) + (-F) = 0$

$5 - F = 0 \rightarrow F = 5$ kN

Pb



$5 \sin 60 = 4.3$

$5 \cos 60 = 2.5$ kN

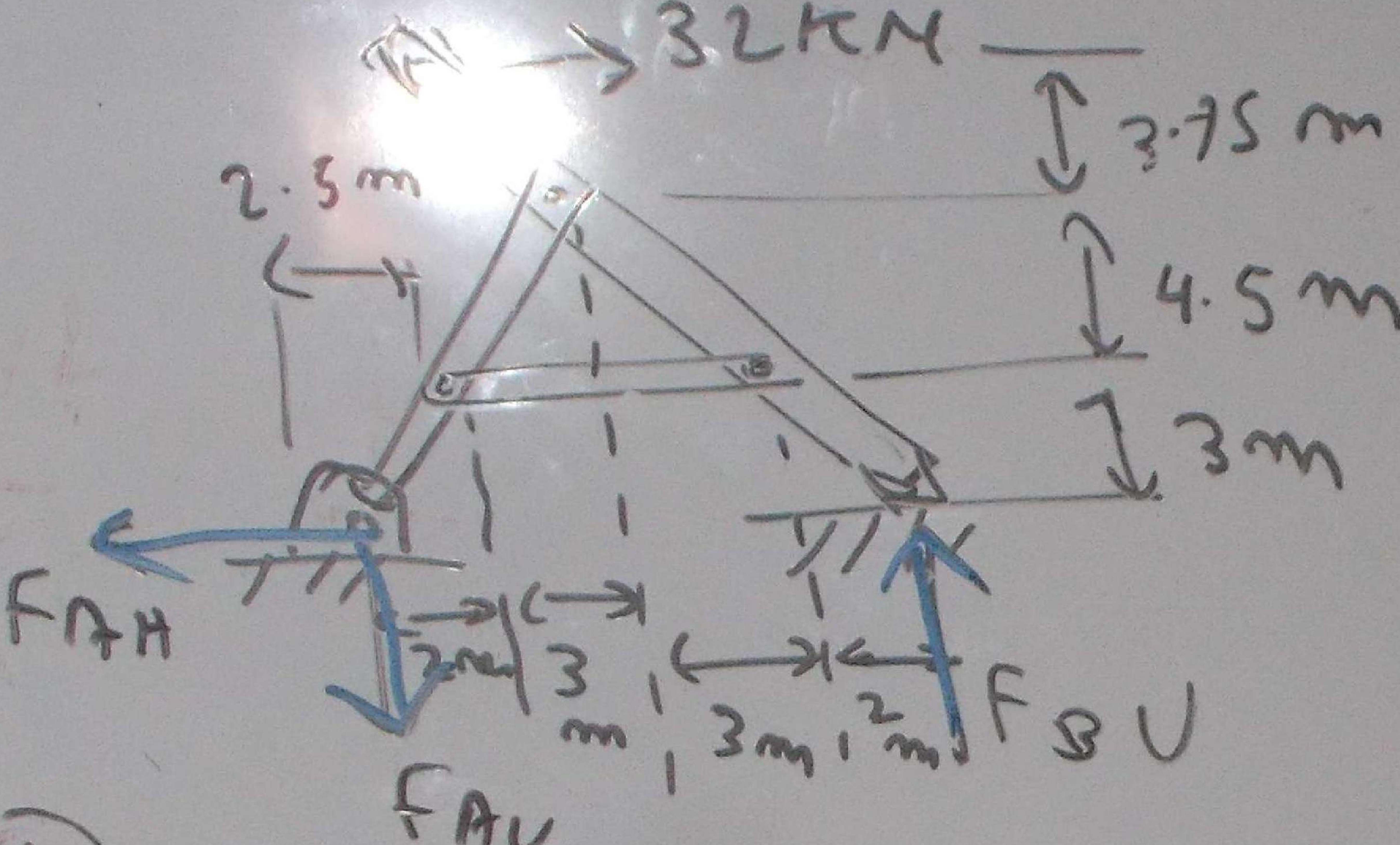
$\Sigma F_x = 0$

$F_H = 2.5$ kN

$F_L = ?$ $F_H = ?$ $F_R = ?$

$\Sigma M_L = 0 \rightarrow 4.3 \times 2.3 + (-F_R \times 3) = 0 \rightarrow F_R = 3.32$ kN

$\Sigma F_y = 0$ $F_L + F_R = 4.3$ | $F_L = 1.01$ kN



F_{AH}?

F_{AV} = ?

F_{BV} = ?

(+ clockwise)

$$\sum M = 0 \rightarrow 32 \times (3.75 + 4.5 + 3) - F_{BV}(2 + 3 + 3 + 2)$$

$$32 \times 11.25 - F_{BV} \times 10 = 0$$

$$F_{BV} = \frac{32 \times 11.25}{10} = 36\,KN \uparrow$$

($\rightarrow$ +, $\leftarrow$ −)

$$\sum F_x = 0 \quad 32 + (-F_{AH}) = 0$$

$$F_{AH} = 32\,KN$$

$$\sum F_y = 0$$

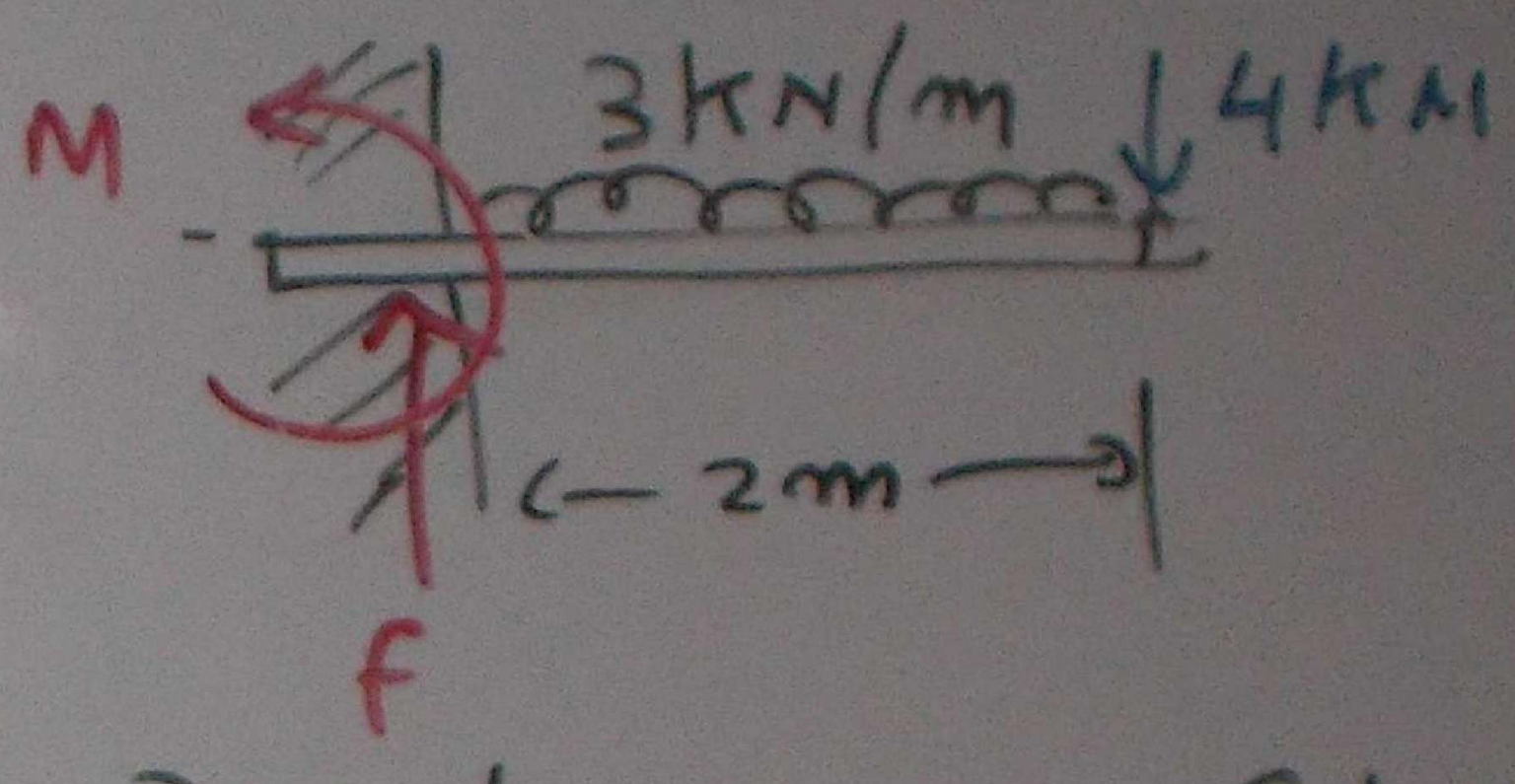
($\uparrow$ +, $\downarrow$ −)

$$F_B + (-F_{AV}) = 0$$

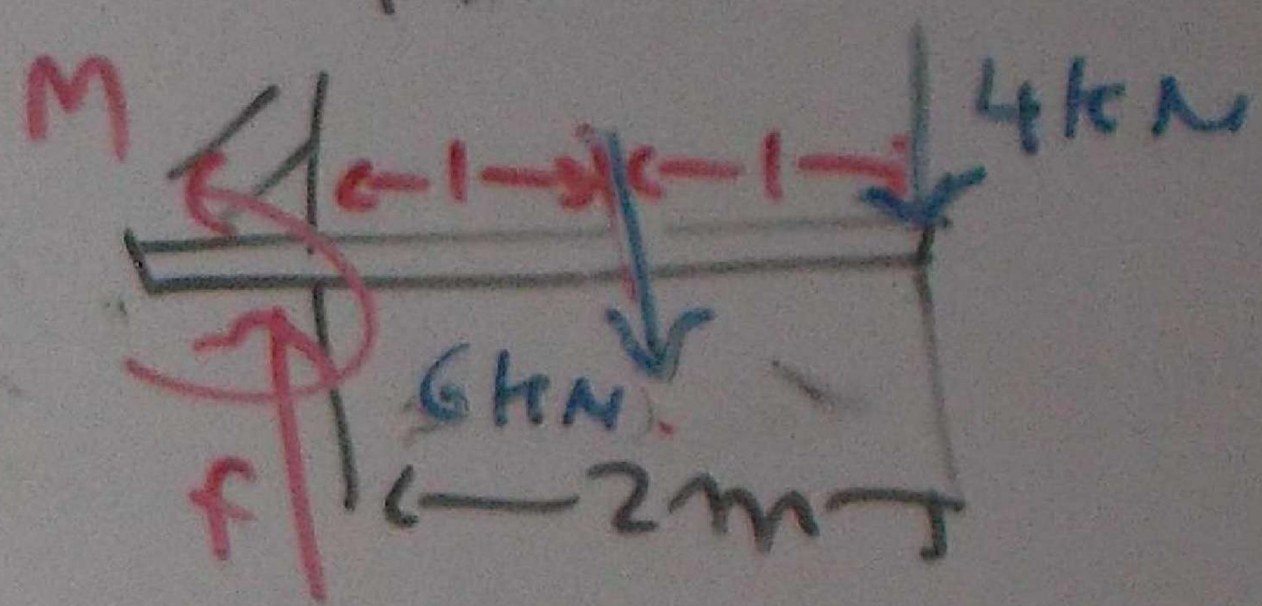
$$36 - F_{AV} = 0$$

$$F_{AV} = 36\,KN \downarrow$$

UNIFORMLY DISTRIBUTED LOAD



$3kN/m \times 2m = 6kN$



$\Sigma F_y = 0 \quad \downarrow + \quad \uparrow -$

$(+6) + (+4) + (-F) = 0$

$10 - F = 0 \rightarrow F = 10kN$

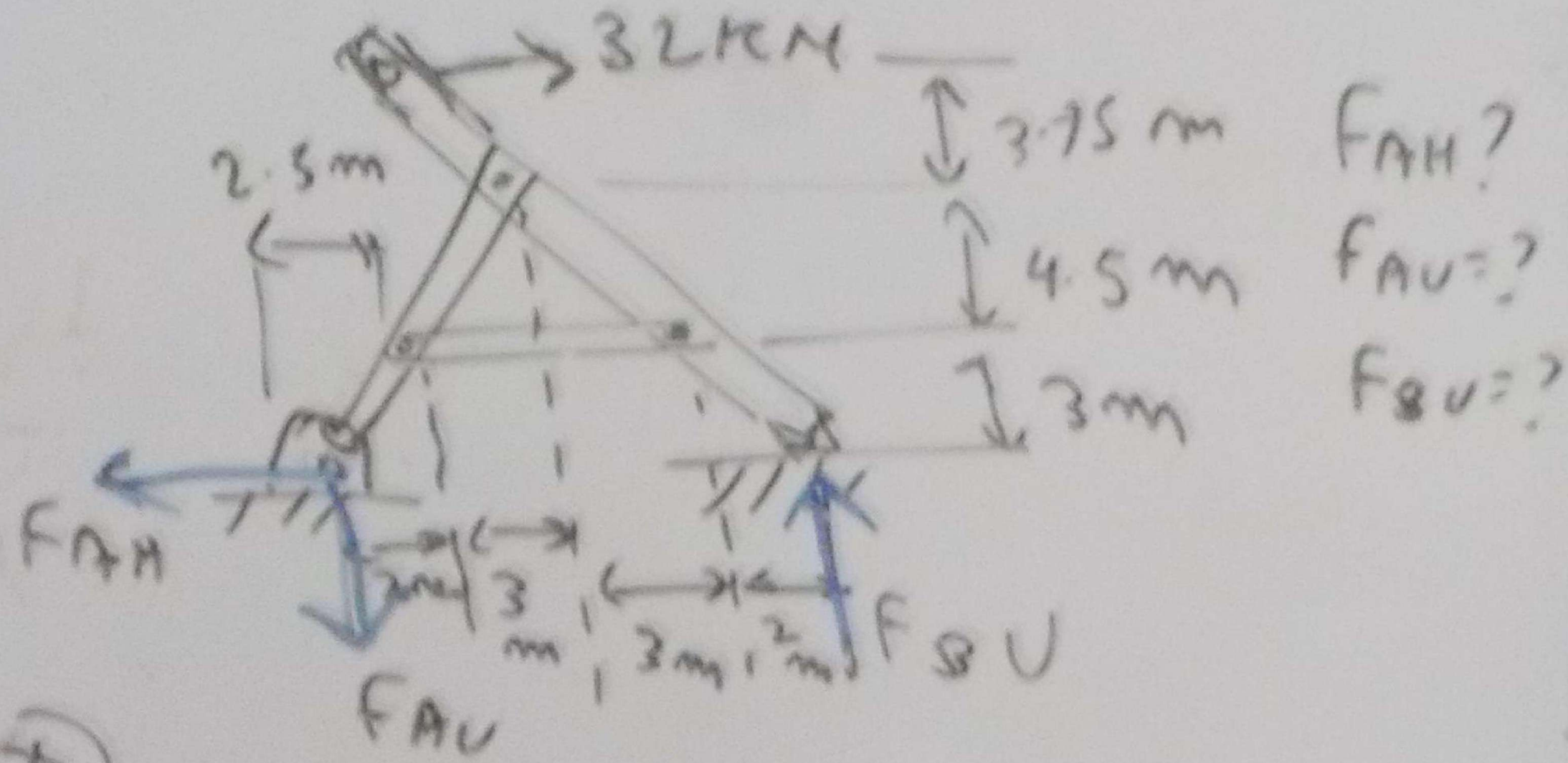
$\Sigma M = 0$

$\circlearrowleft + \quad (6 \times 1) + (4 \times 2) + (-M) = 0$

$6 + 8 - M = 0$

$M = 14kN\text{-}m$

REACTION IN PM



$$\sum M = 0 \rightarrow 32 \times (3.75 + 4.5 + 3) - F_{BV}(2+3+3+2)$$

$$32 \times 11.25 - F_{BV} \times 10 = 0$$

$$F_{BV} = \frac{32 \times 11.25}{10} = 36\,kN \uparrow$$

$$\sum F_x = 0 \qquad 32 + (-F_{AH}) = 0$$

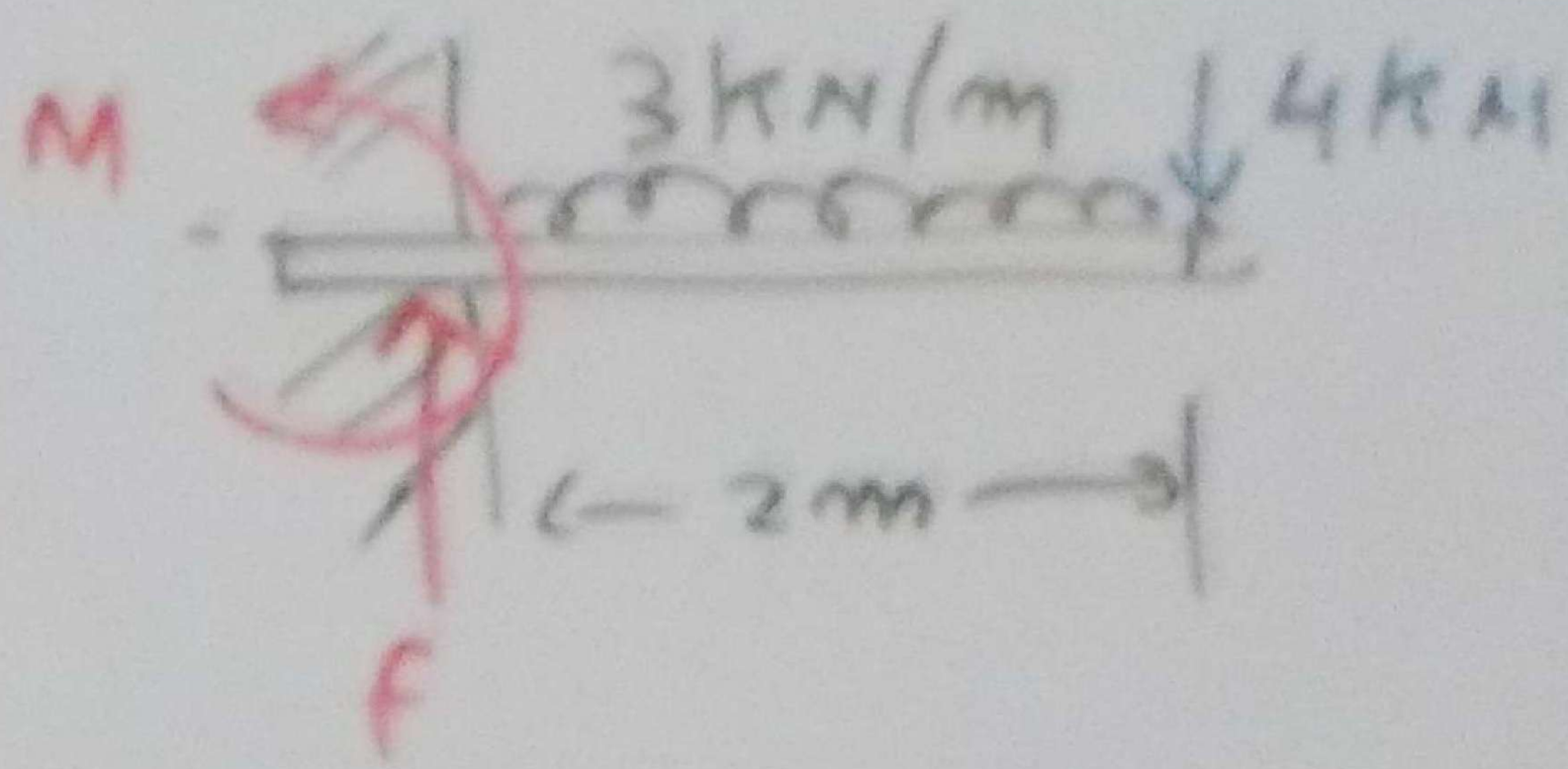
$$F_{AH} = 32\,kN$$

$$\sum F_y = 0 \qquad F_B + (-F_{AV}) = 0$$

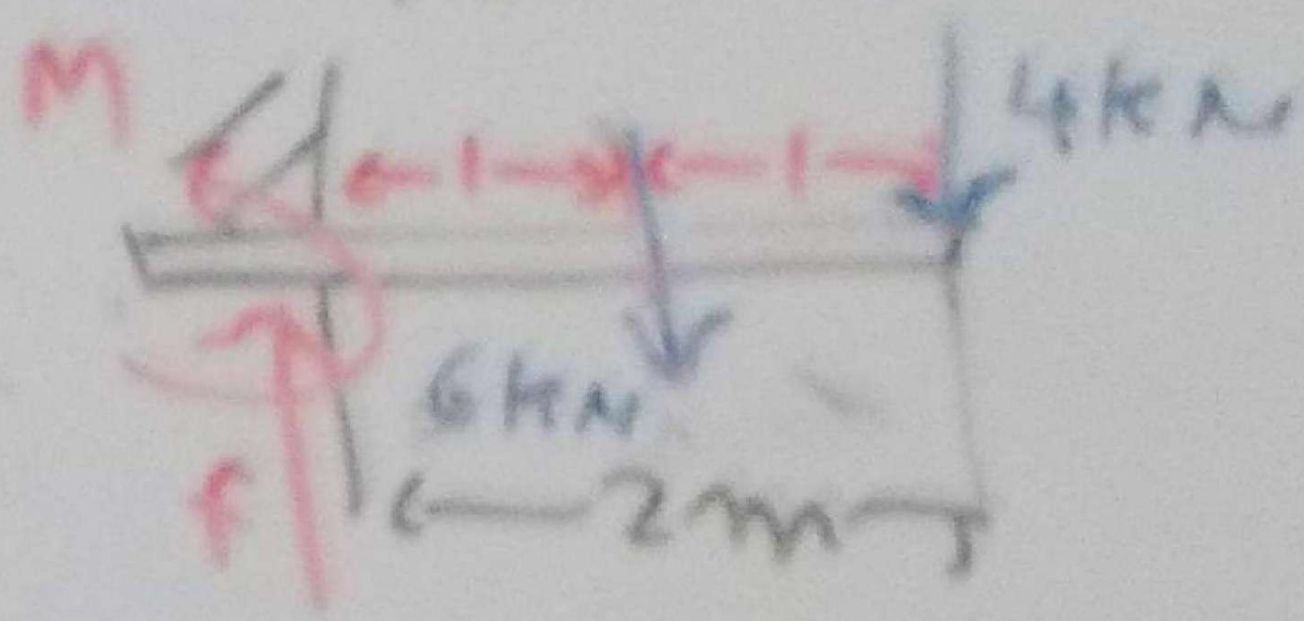
$$36 - F_{AV} = 0$$

$$F_{AV} = 36\,kN \downarrow$$

UNIFORMLY DISTRIBUTED LOAD



$3kN/m \times 2m = 6kN$



$\Sigma F_y = 0 \quad \downarrow + \quad \uparrow -$

$(+6) + (+4) + (-F) = 0$

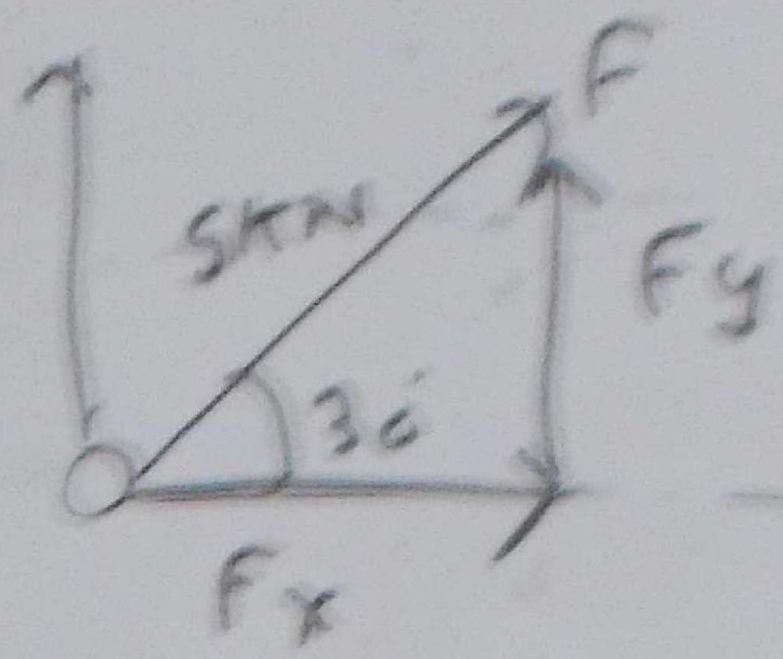
$10 - F = 0 \rightarrow F = 10kN$

$\Sigma M = 0$

$\circlearrowleft + \quad (6 \times 1) + (4 \times 2) + (-M) = 0$

$6 + 8 - M = 0$

$M = 14kN\text{-}m$



$F_x = F\cos 30°$

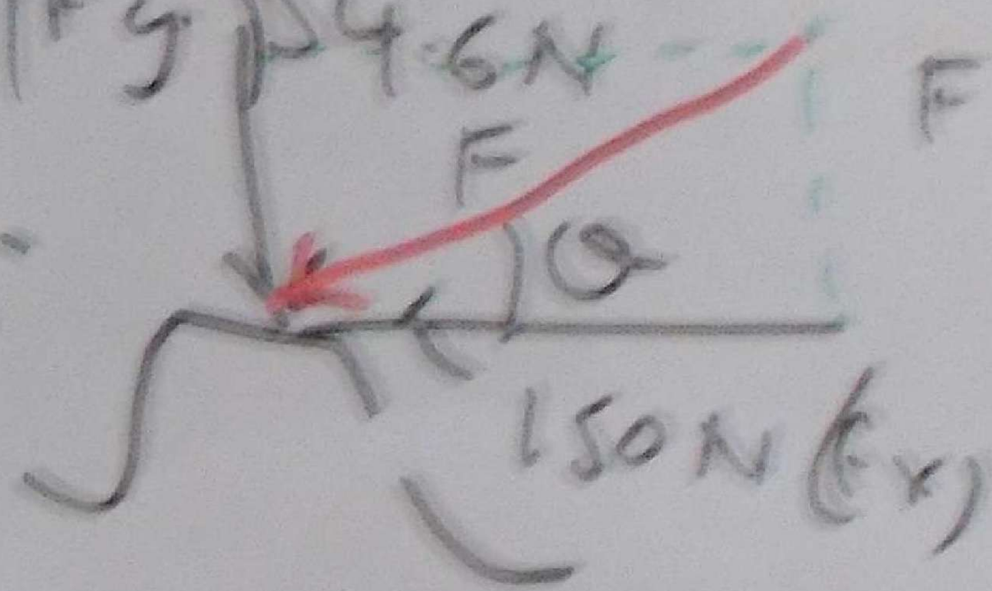
$= 5 \times 0.866$

$= 4.33\ KN$

$F_y = F\sin 30$

$= 5 \times 0.5 = 2.5\ KN$

Ex IF THE COMPONENTS OF FORCE ACTING ON A GEAR TOOTH ARE 546 N IN RADIAL DIRECTION AND 150 N IN TANGENTIAL DIRECTION, DETERMINE TOTAL FORCE AND THE ANGLE BETWEEN FORCE AND TANGENTIAL DIRECTION.

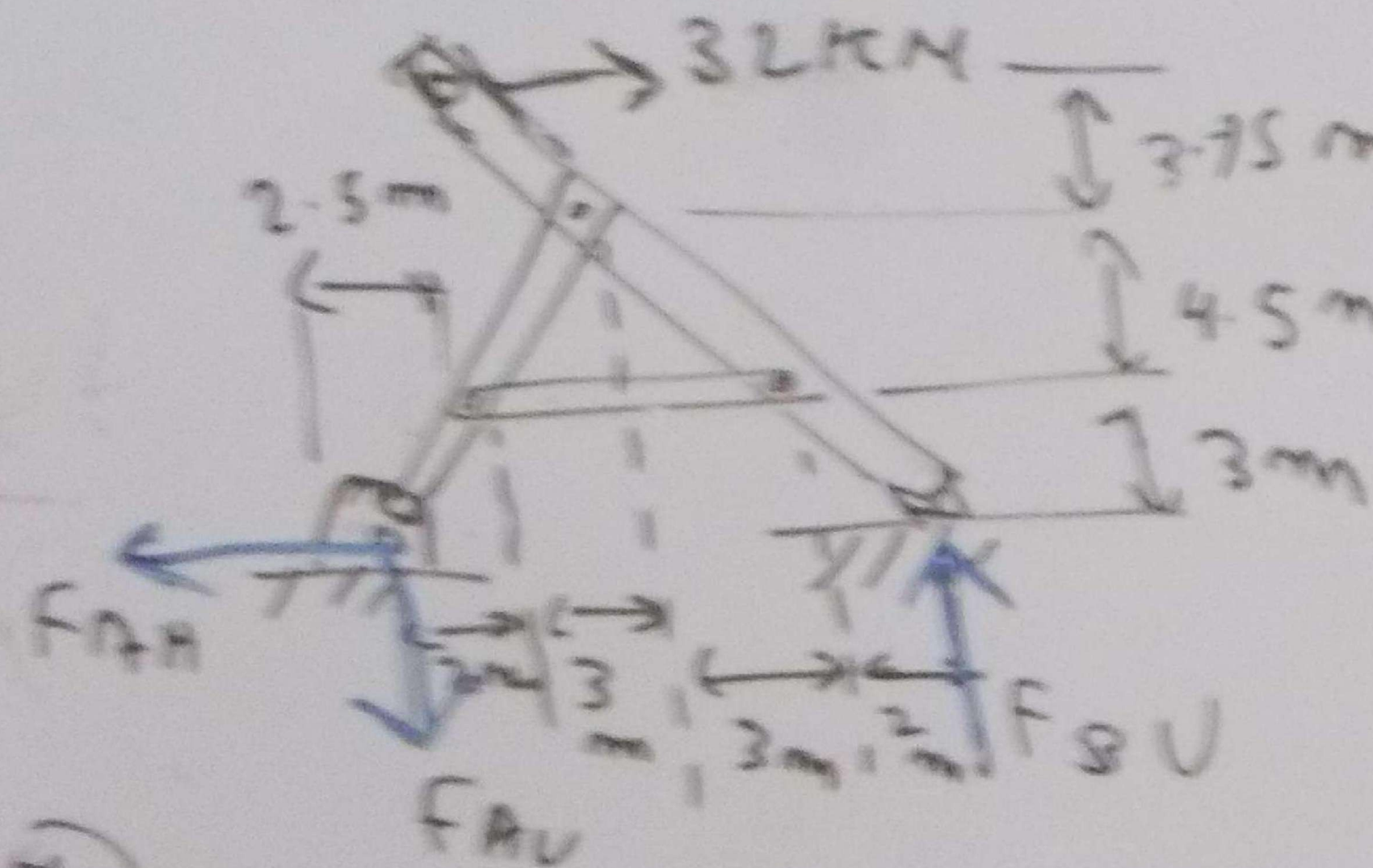


$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{150^2 + 546^2} = 199.6\ N$

$\theta = \tan^{-1}\frac{546}{150} = 20°$

REACTION IN PIN



F_{AH}?

F_{AU}=?

F_{BU}=?

$\circlearrowleft + \quad \sum M = 0 \rightarrow 32 \times (3.75 + 4.5 + 3) - F_{BU}(2 + 3 + 2) = 0$

$32 \times 11.25 - F_{BU} \times 10 = 0$

$F_{BU} = \frac{32 \times 11.25}{10} = 36\,KN \uparrow$

$\sum F_x = 0 \quad 32 + (-F_{AH}) = 0$

$F_{AH} = 32\,KN$

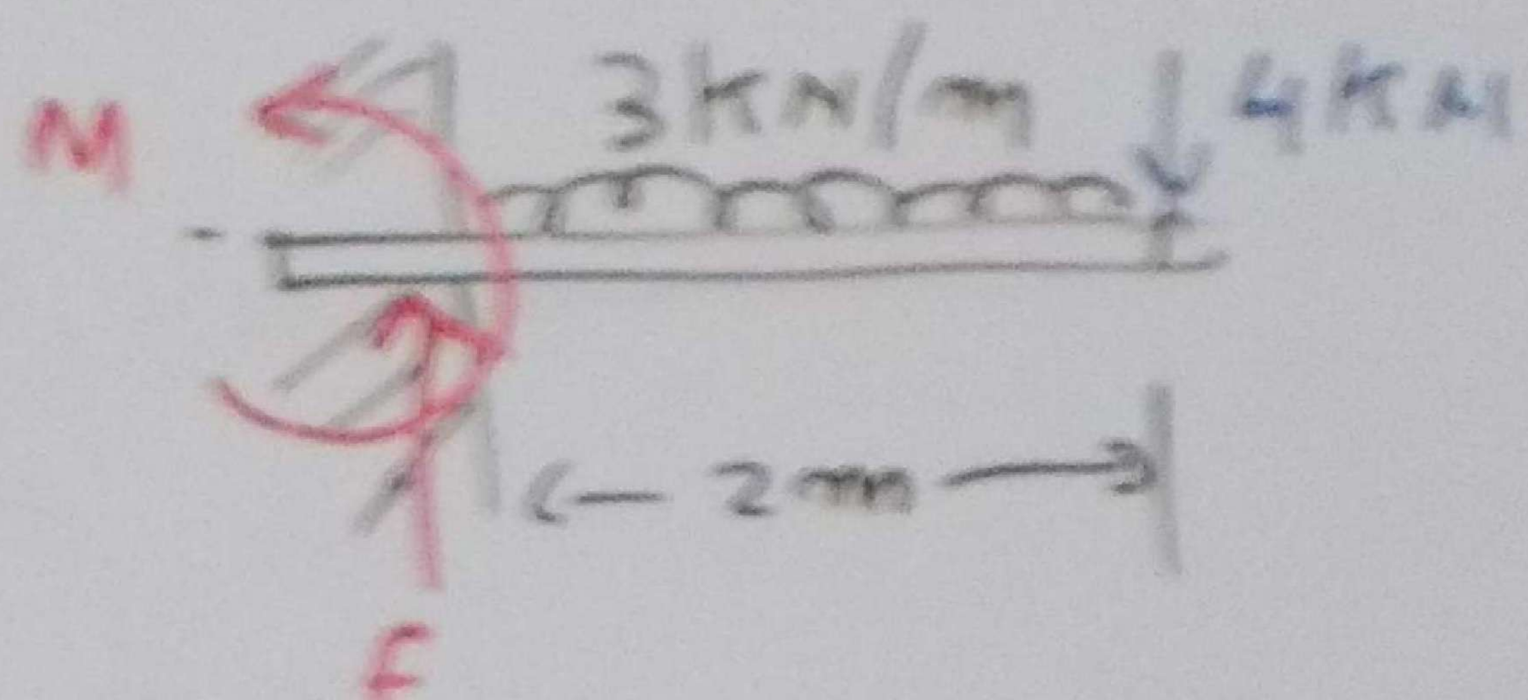
$\sum F_y = 0 \quad \uparrow + \downarrow -$

$F_B + (-F_{AU}) = 0$

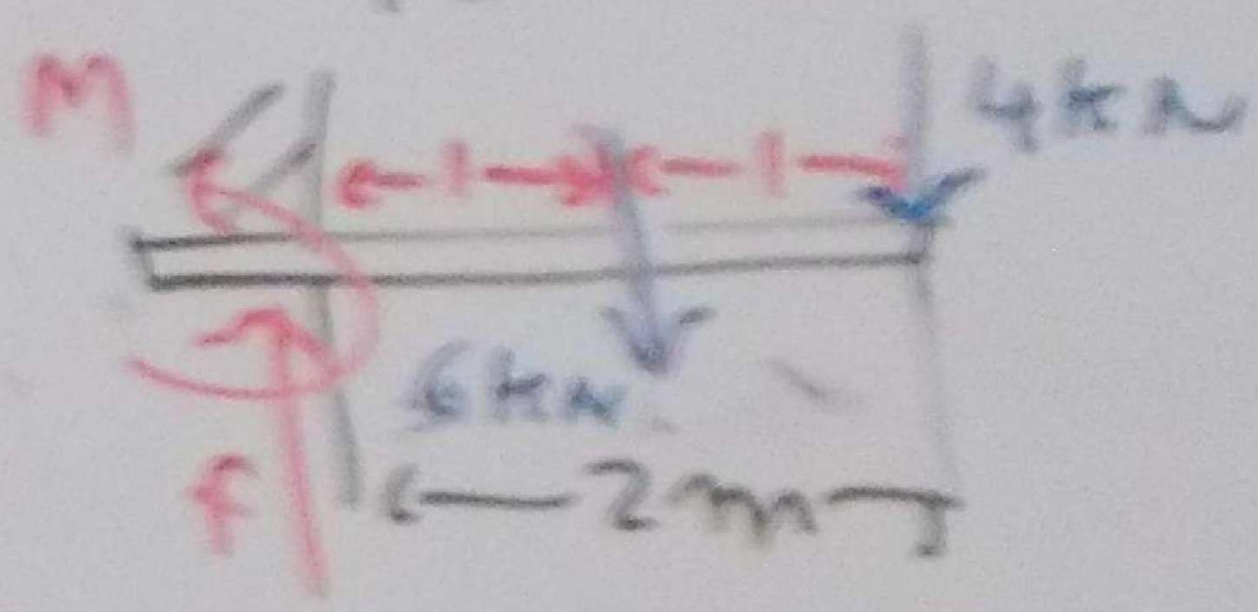
$36 - F_{AU} = 0$

$F_{AU} = 36\,KN \downarrow$

UNIFORMLY DISTRIBUTED LOAD



$3kN/m \times 2m = 6kN$



$\Sigma F_y = 0 \quad \downarrow + \quad \uparrow -$

$(+6) + (+4) + (-f) = 0$

$10 - f = 0 \rightarrow f = 10\,kN$

$\Sigma M = 0$

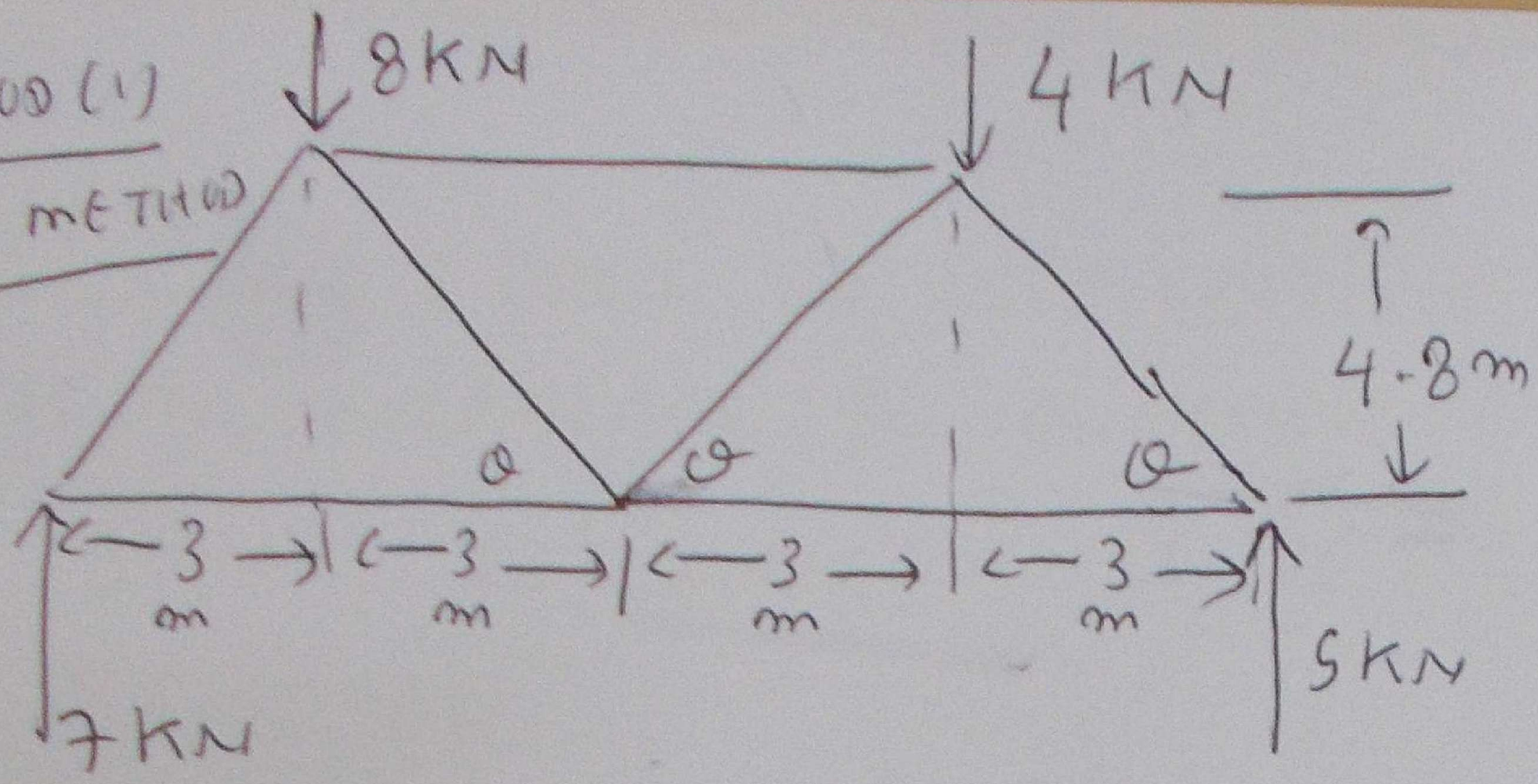
$\circlearrowleft + \quad -J \quad (6 \times 1) + (4 \times 2) + (-M) = 0$

$6 + 8 - M = 0$

$M = 14\,kN\text{-}m$

METHOD (1)

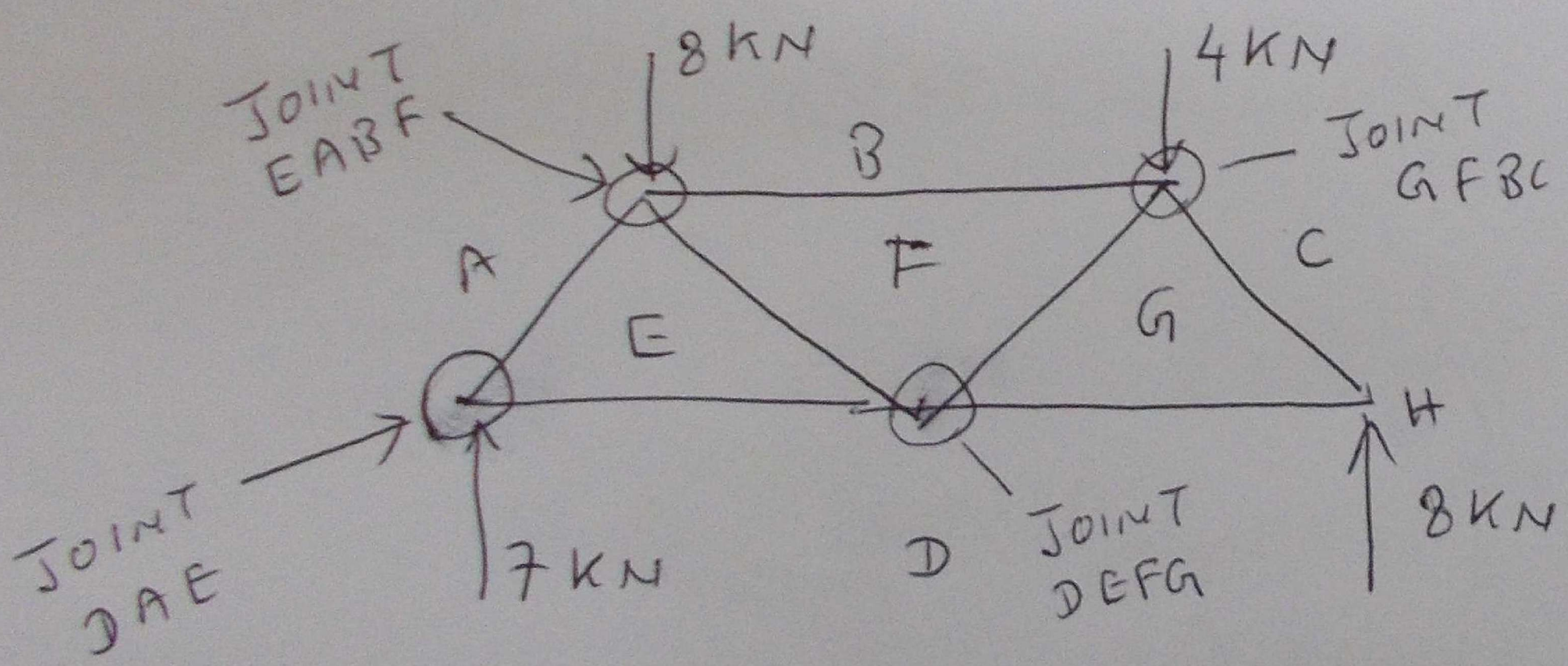
JOINT METHOD



Find $\theta \rightarrow \tan\theta = \frac{4.8}{3}$

$\theta = \tan^{-1}\frac{4.8}{3} = 58^\circ$

PUT THE REGION, NAME THE JOINTS

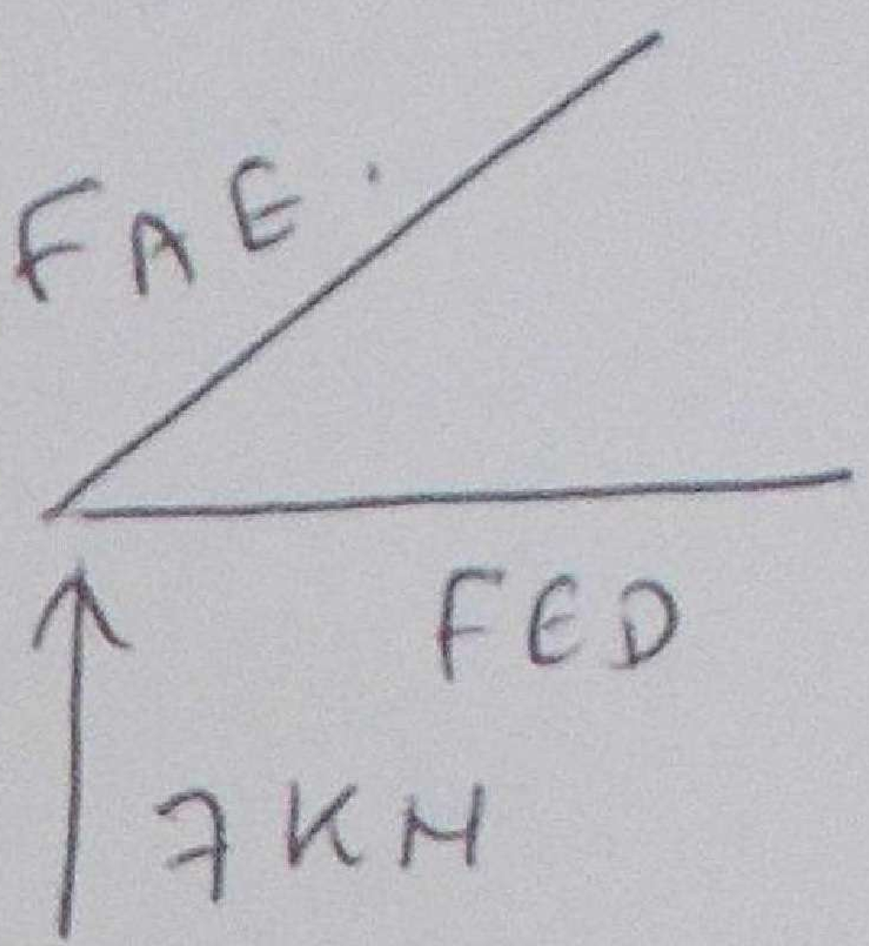


NAME BY ↺ ANTICLOCKWISE

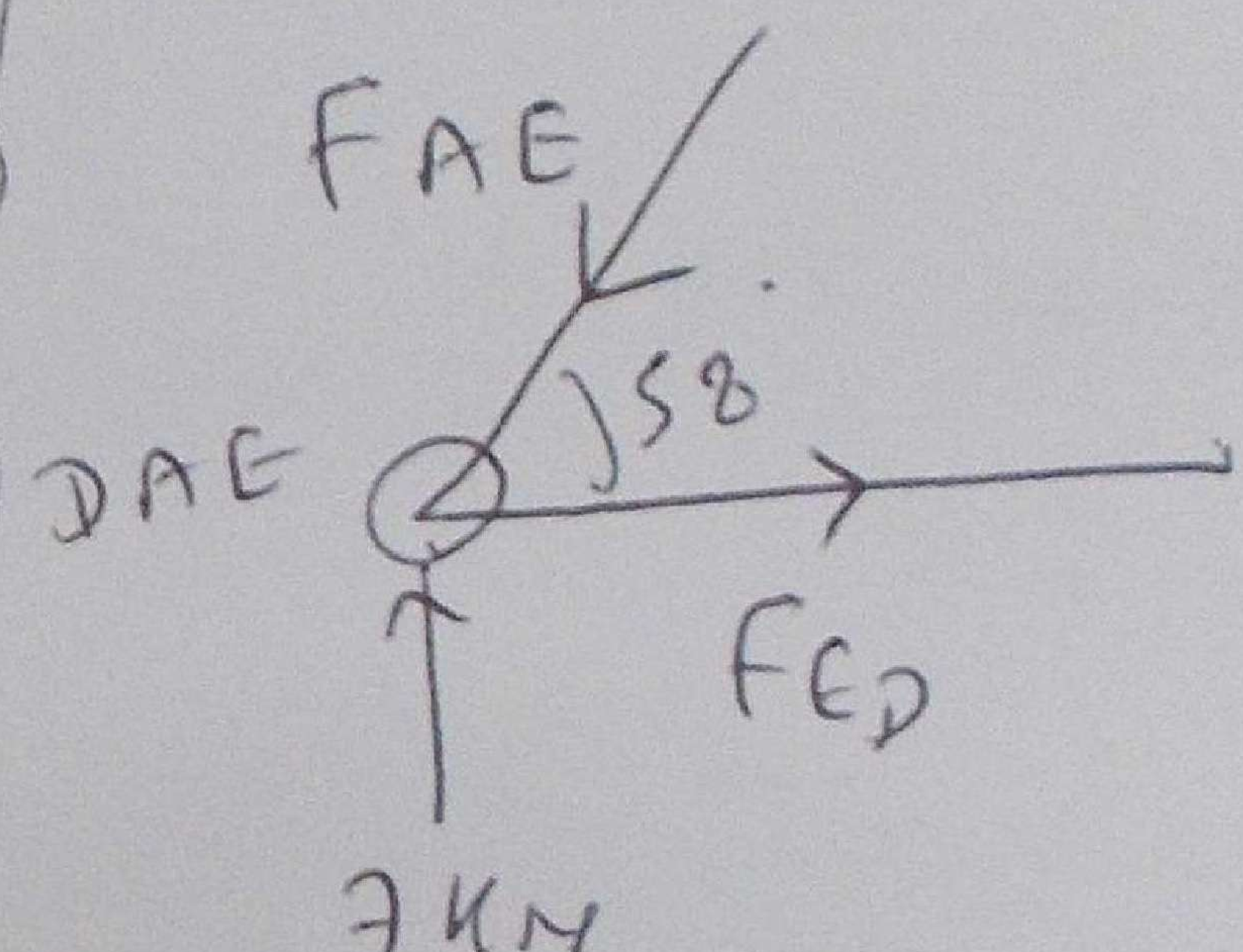
DAE, EABF, DEFG, GFBC etc

JOINT DAE

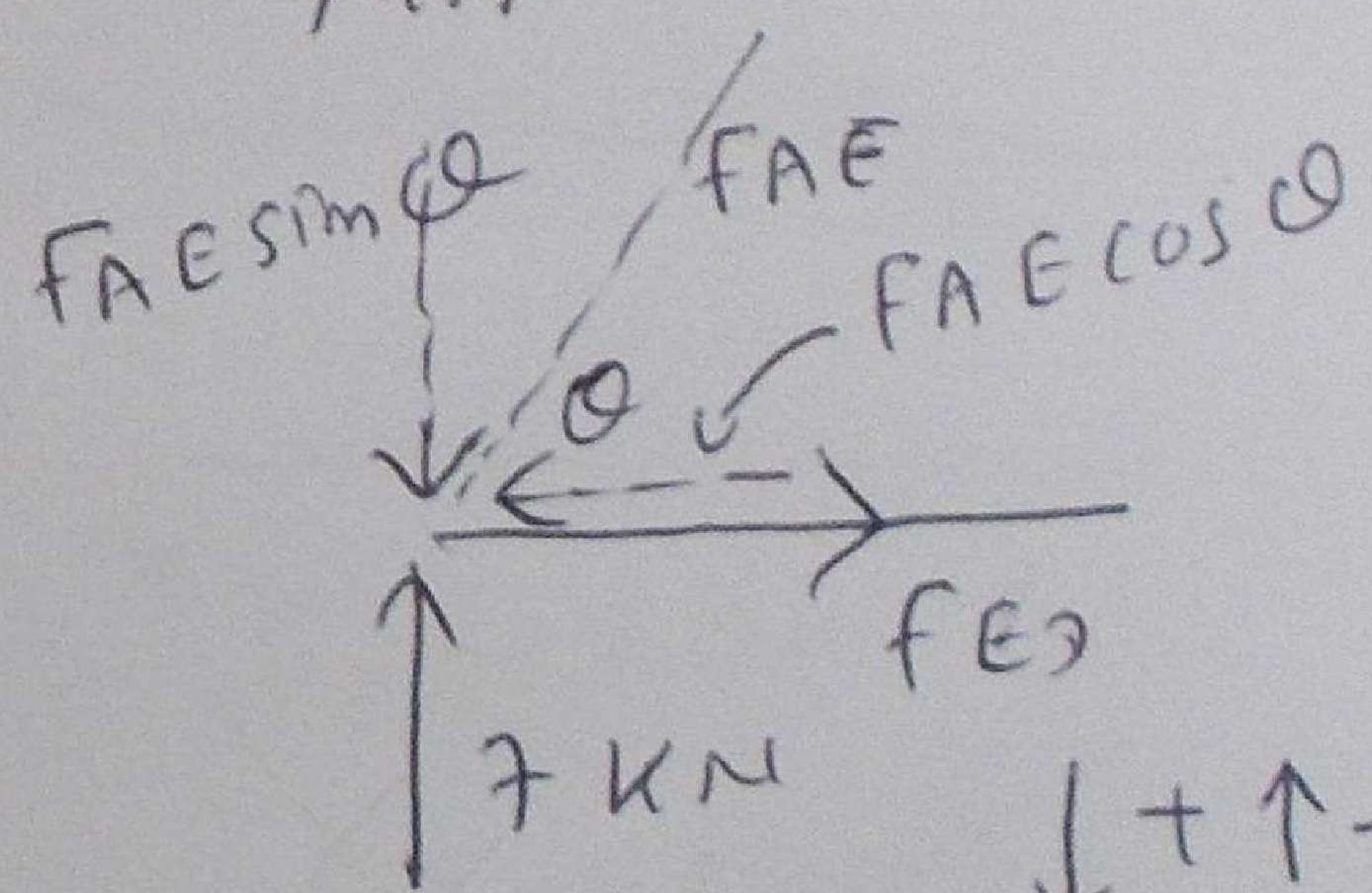
FORCE → NAME ①



ASSUME THE DIRECTION
IF THE ANSWER IS POSITIVE, IT'S DIRECTION IS CONFIRMED
IF NEGATIVE, REVERSE DIRECTION



* Toward Joint — compression
* Away from joint - Tension



↓+ ↑−

$\Sigma F_y = 0 \rightarrow$ $F_{AE} \sin\theta + (-7) = 0$

$F_{AE} \sin 58 = 7$

$F_{AE} = 7/\sin 58 = 8.25$ ↙

←+ Toward Joint - compression

$\Sigma F_x = 0$ →−

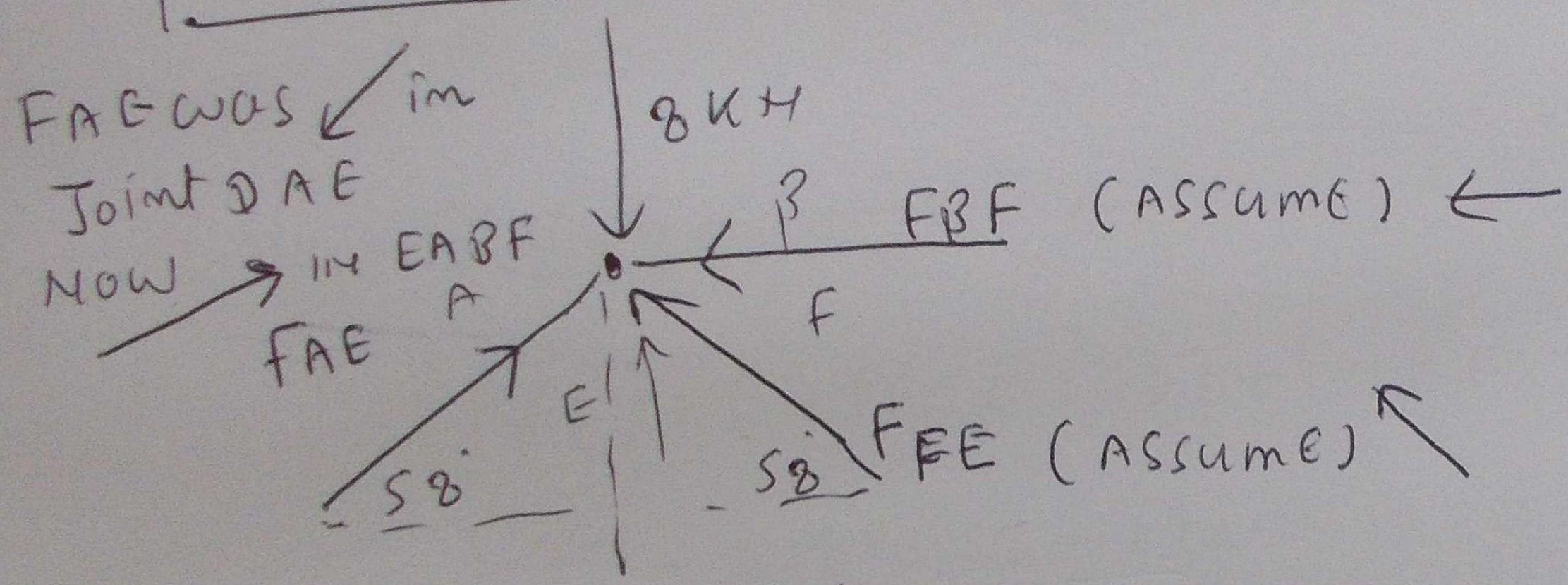
$(+F_{AE} \cos\theta) + (-F_{ED}) = 0$

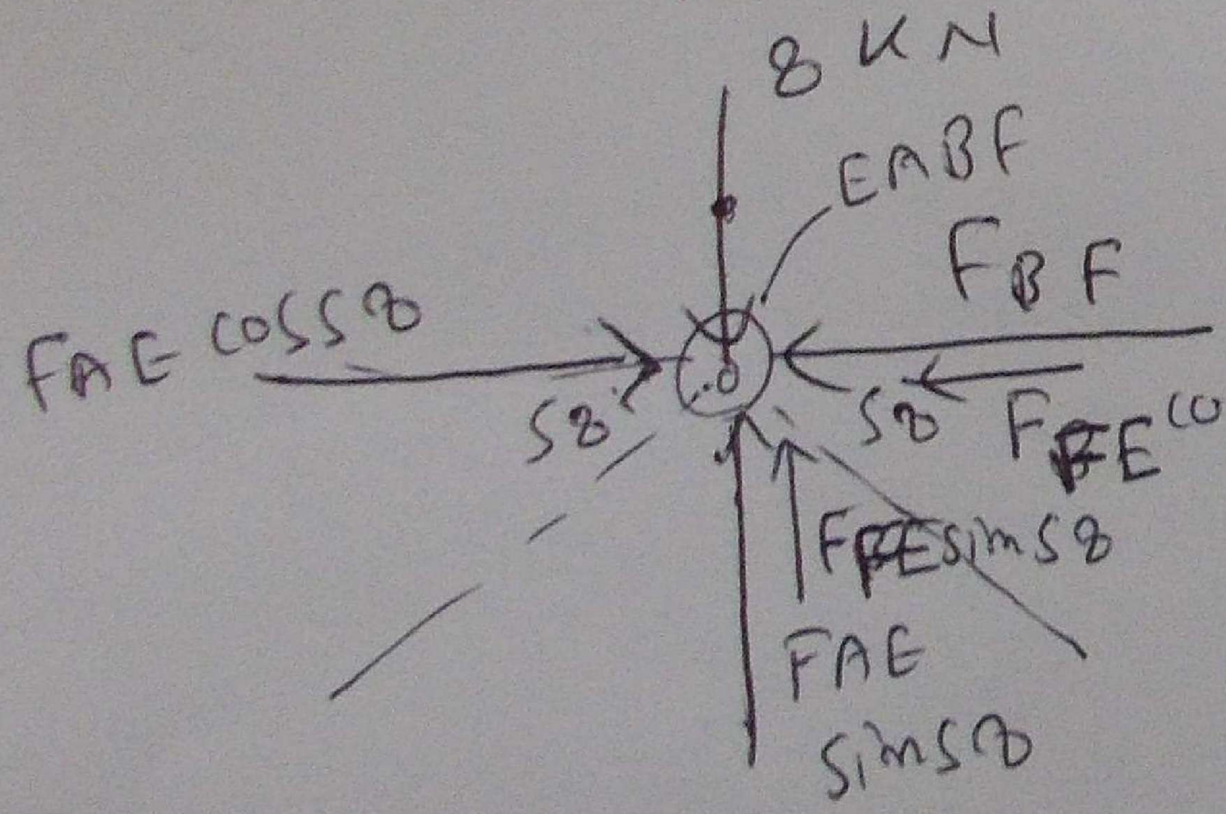
~~FAE~~ $= F_{ED} = 8.25 \cos 58$

TENSION $= 4.37$ kN →

NOTE — IF FORCE IN ONE SECTION IS TRANSFERRED TO ANOTHER JOINT, IT'S DIRECTION IS TO BE REVERSED.

JOINT EABF





$$\Sigma F_y = 0 \quad \uparrow + \quad \downarrow -$$

$$(+F_{FE}\sin 58) + (F_{AE}\sin 58) - 8 = 0$$

$$F_{FE}\sin 58 + 8.25\sin 58 - 8 = 0$$

$$F_{FE} = \frac{8 - 8.25\sin 58}{\sin 58} = 1.12\ \text{kN}$$

Toward The joint – compression

$\sum F_x = 0$ $\xrightarrow{+}$ $\xleftarrow{-}$

$(F_{AE} \cos 58) + (-F_{BF}) + (-F_{FE} \cos 58) = 0$

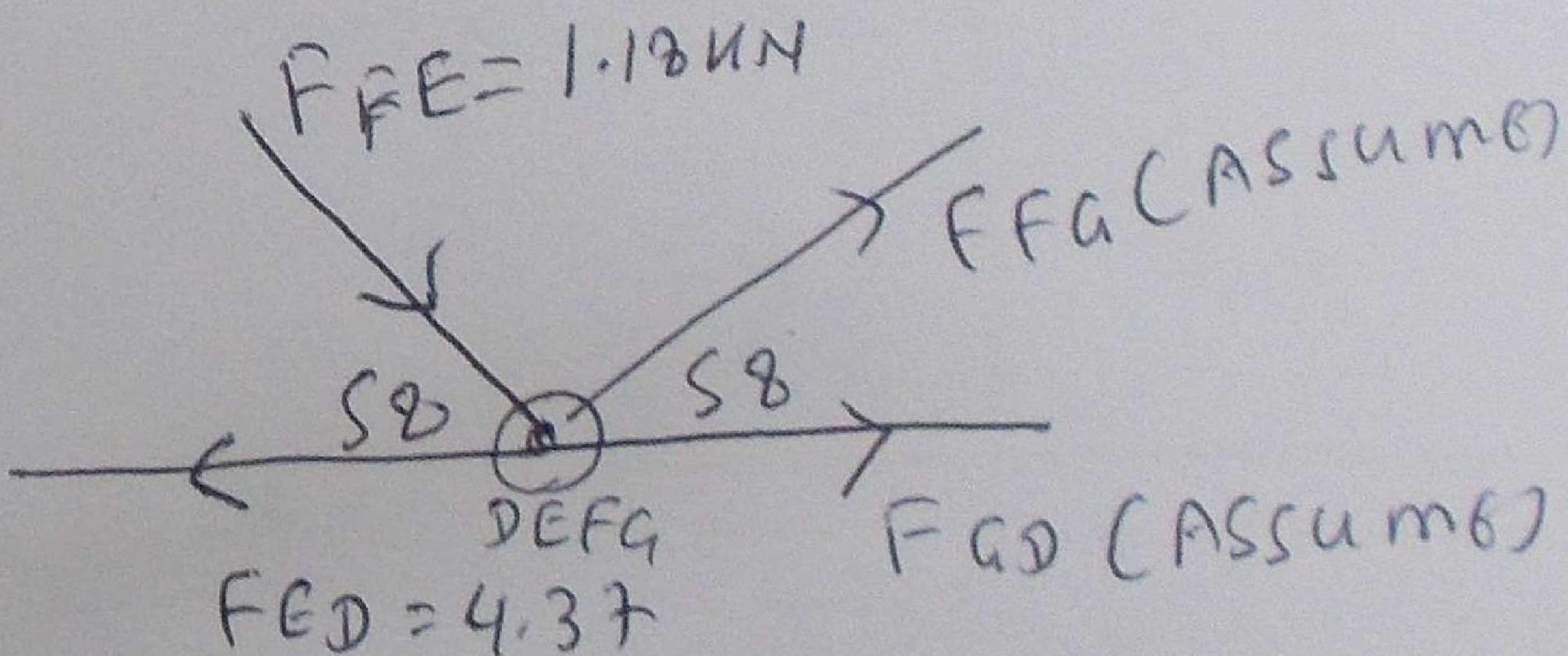
$8.25 \cos 58 - F_{BF} - 1.18 \cos 58 = 0$

$F_{BF} = 8.25 \cos 58 - 1.18 \cos 58$

$= 3.75 \text{ kN} \leftarrow$

Toward joint – compression

JOINT DEFG



$\sum F_y = 0$ $\downarrow + \uparrow -$

$(F_{FE} \sin 58) + (-F_{FG} \sin 58) = 0$

$1.18 \sin 58 = F_{FG} \sin 58$

$F_{FG} = 1.18 \text{ kN}$ (Tension) ↗

$\sum F_x = 0$ $\xleftarrow{+}$ $\xrightarrow{-}$

$(+F_{ED}) + (-F_{FE} \cos 58) + (-F_{FG} \cos 58) + (F_{GD}) = 0$

$= 4.37 - 1.18 \cos 58 - 1.18 \cos 58 - F_{GD} = 0$

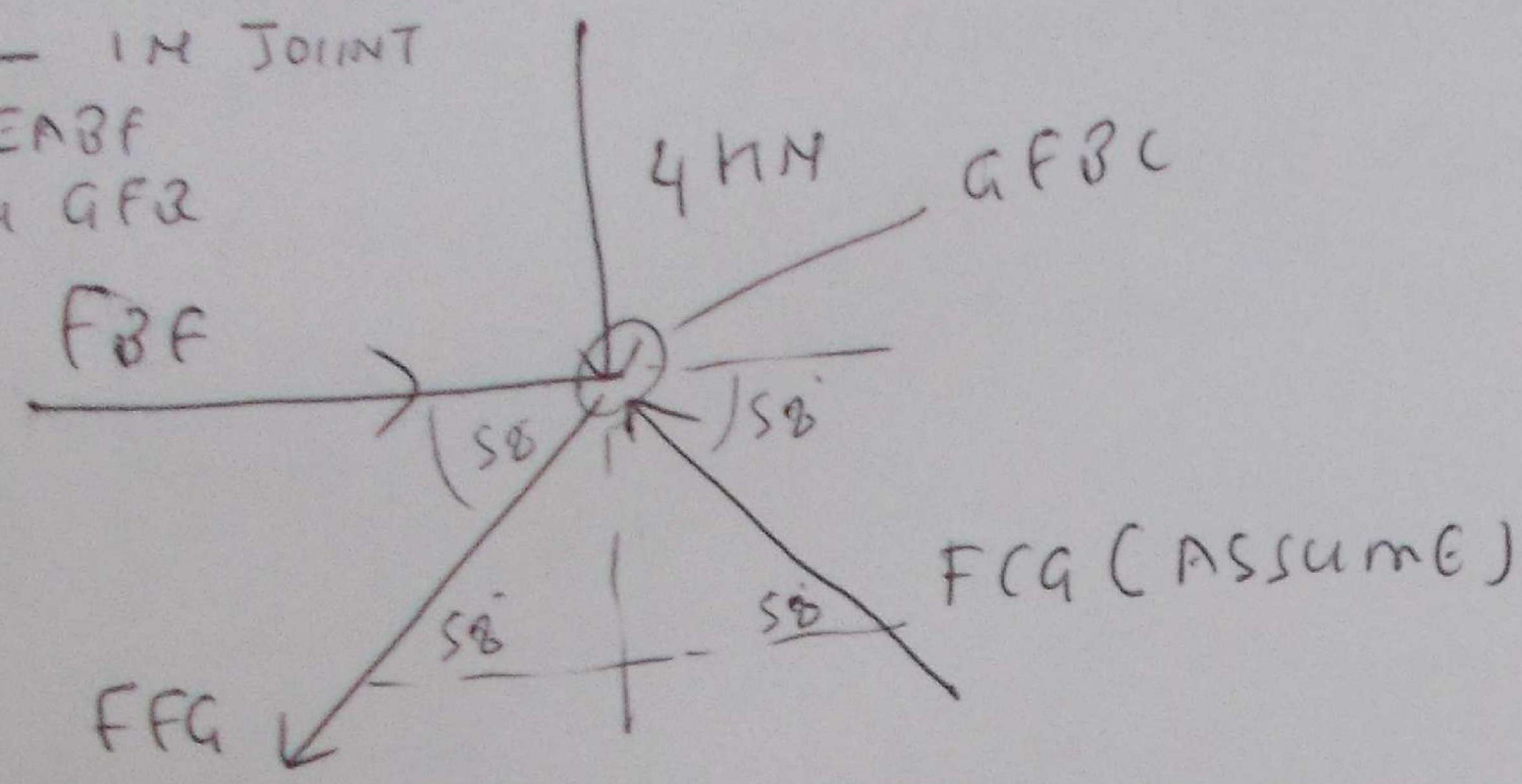
$F_{GD} = 3.12 \text{ kN} \longrightarrow$ (Away from joint)

Tension

JOINT GFBC

FBF was ← IN JOINT EABF

NOW → IN GFB



$\Sigma F_x = 0$ $\xrightarrow{+}$ $\xleftarrow{-}$

$(F_{BF}) + (-F_{FG}\cos 58) + (-F_{CG}\cos 58) = 0$

$3.75 - 1.18\cos 58 - F_{CG}\cos 58 = 0$

$$F_{CG} = \frac{3.75 - 1.18\cos 58}{\cos 58}$$

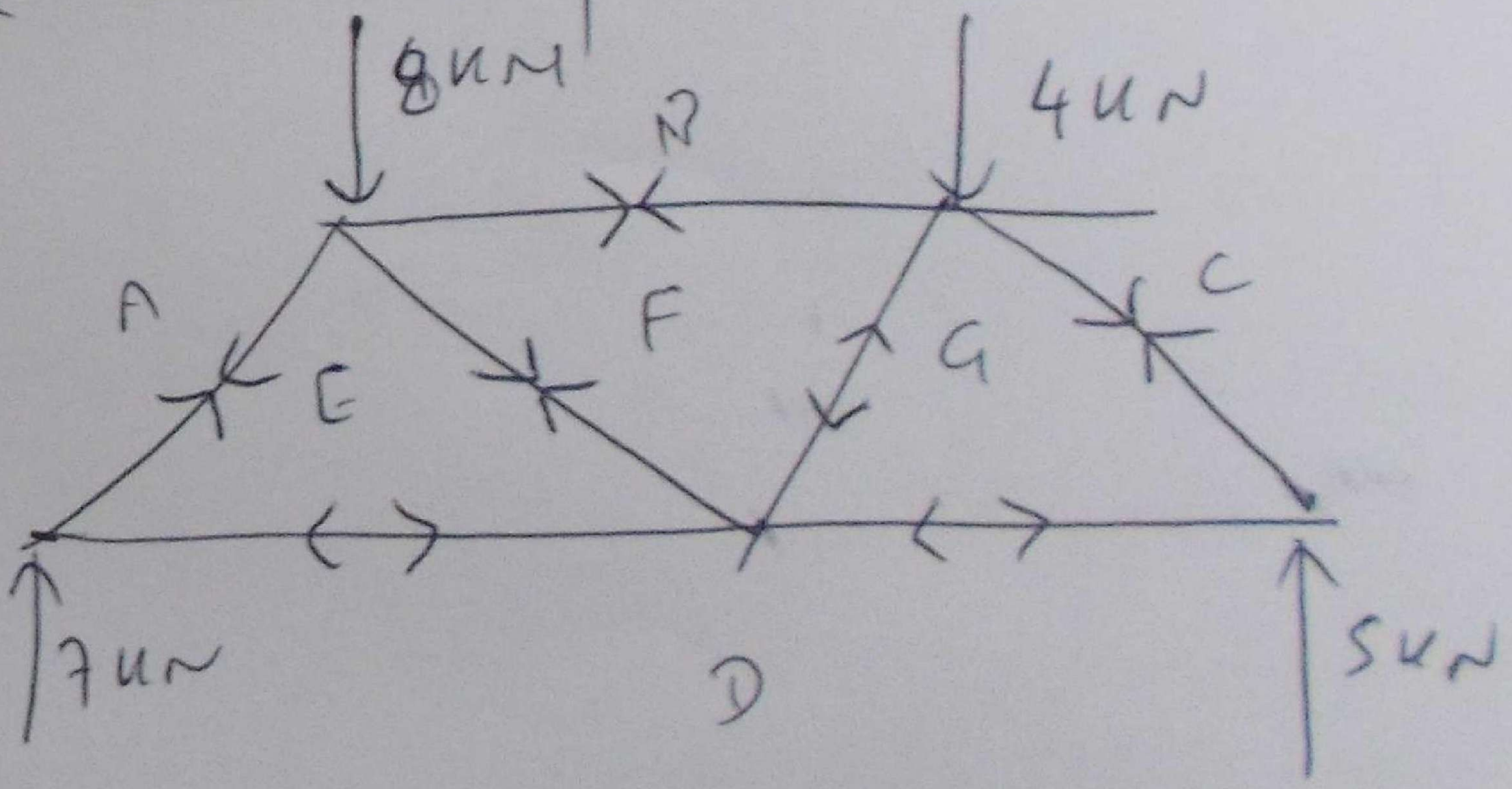
$= 5.9$ KN Toward The joint

Compression

COMPRESSION ✗

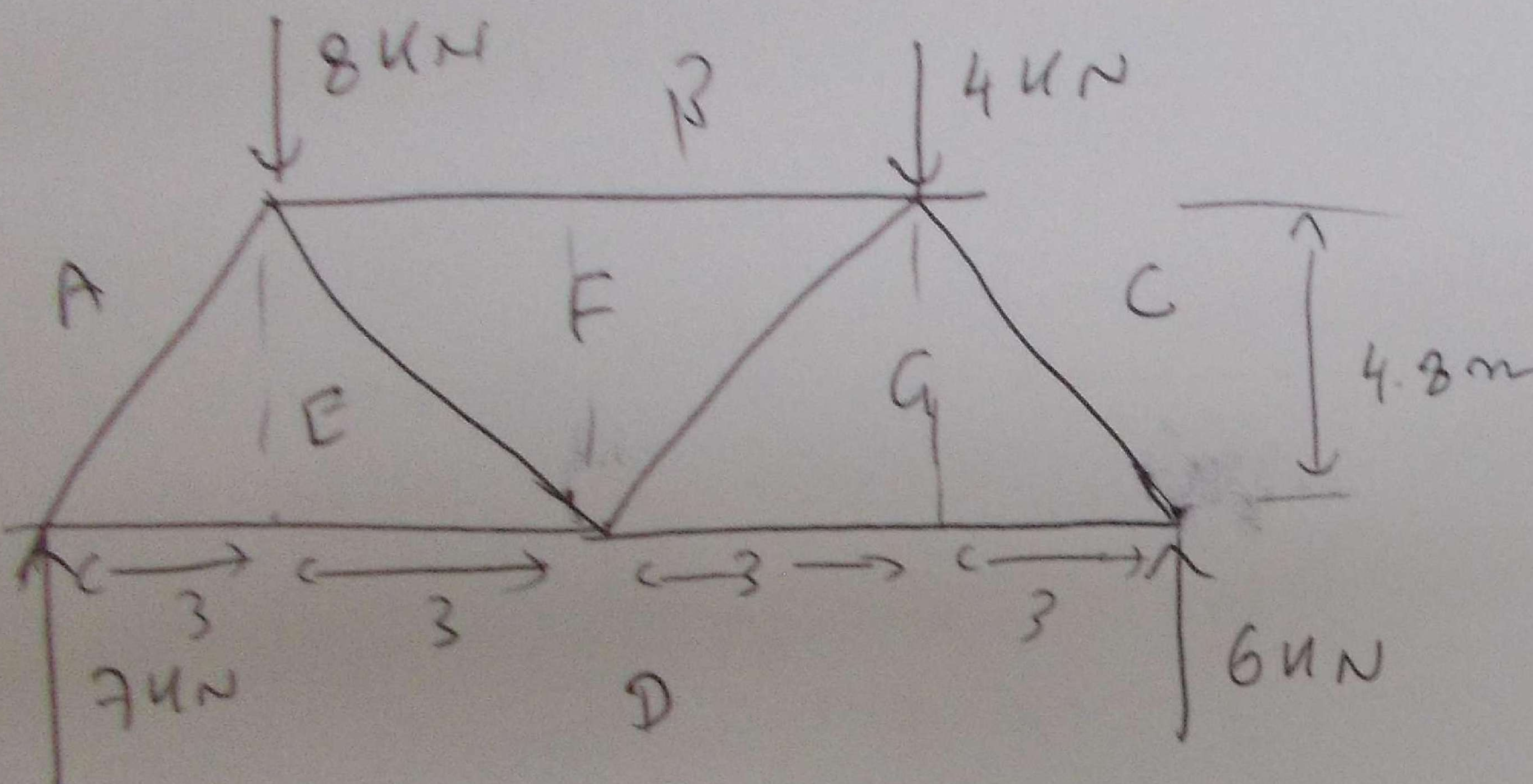
TENSION ✗

TRUSS	FORCE KN	NATURE
FAE	8.25	compression
ED	4.37	TENSION
FE	1.12	compression
BF	3.75	compression
FG	1.12	TENSION
GD	3.12	TENSION
CG	5.9	compression



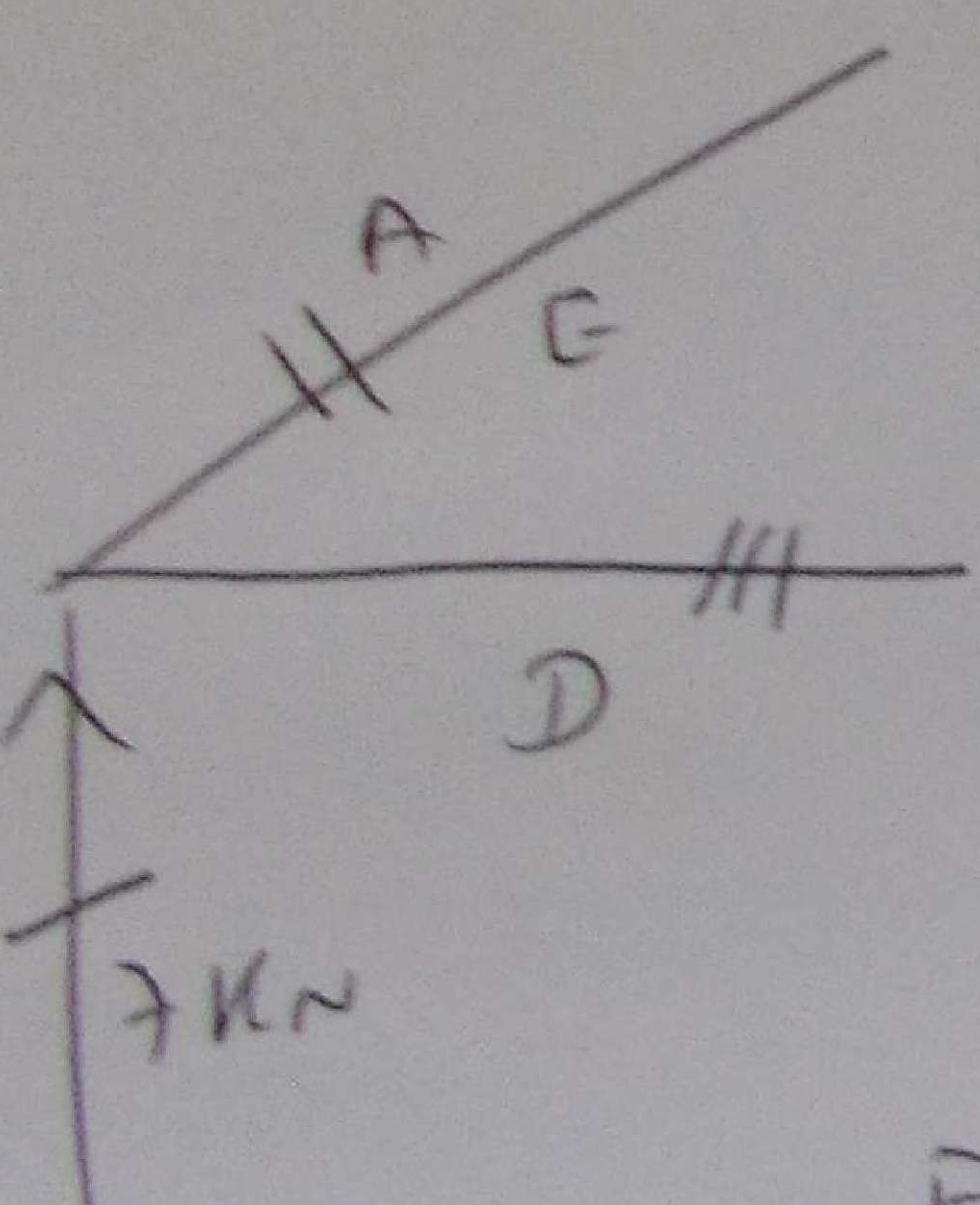
METHOD 2 POLYGON METHOD

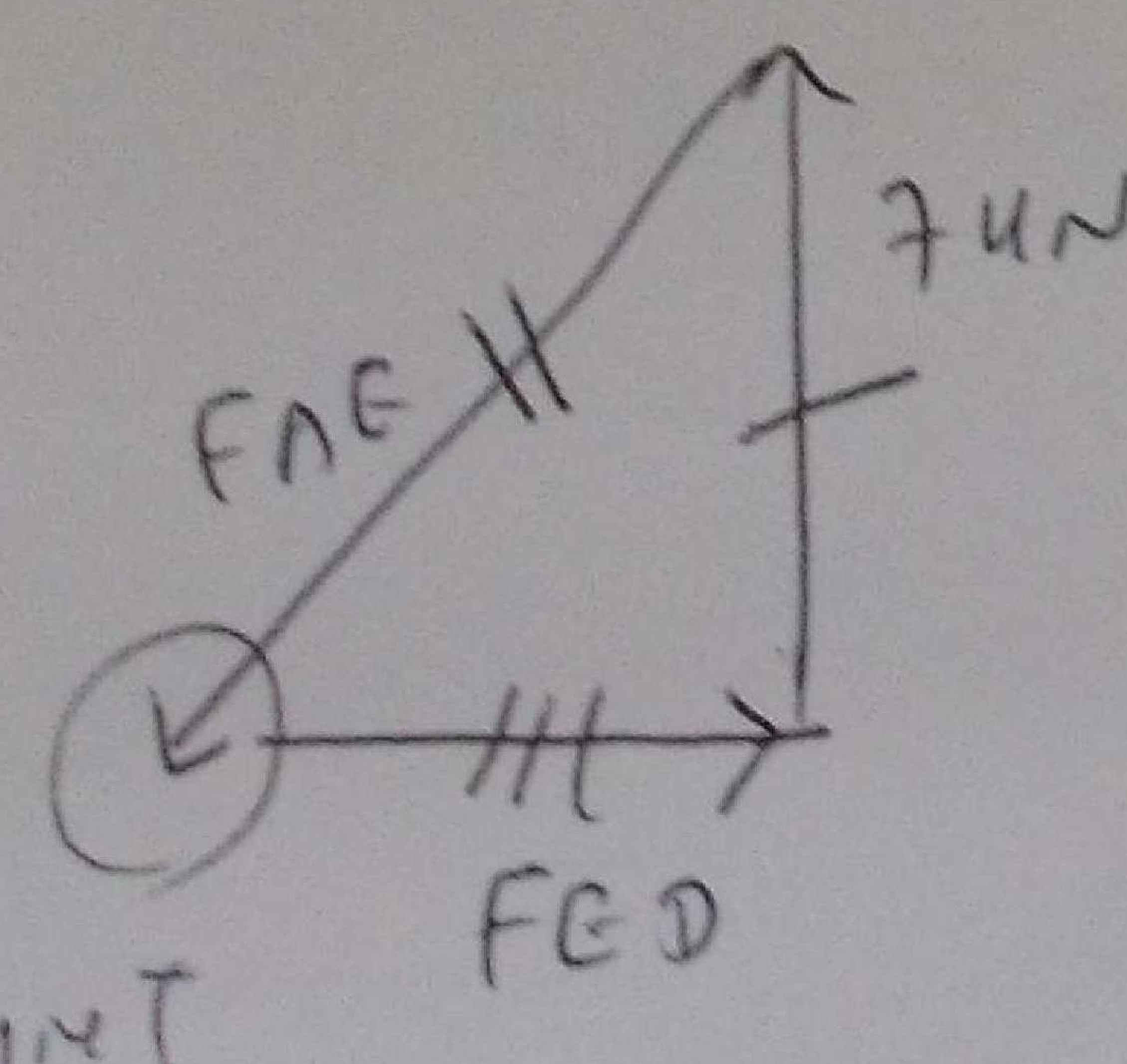
* LOCATE THE JOINTS
* DRAW THE PARALLEL LINE ~~OF~~ TO THE TRUSSES IN THE STRUCTURE
* ARROW HEAD OF ONE FORCE TO BE CONNECTED TO THE ARROW TAIL OF ANOTHER FORCE (ALL IN SCALES)



$\theta = \tan^{-1} \frac{4.8}{3} = 53$

DAE





BY SCALE

FAE = 8.25kN

FED = 4.37kN

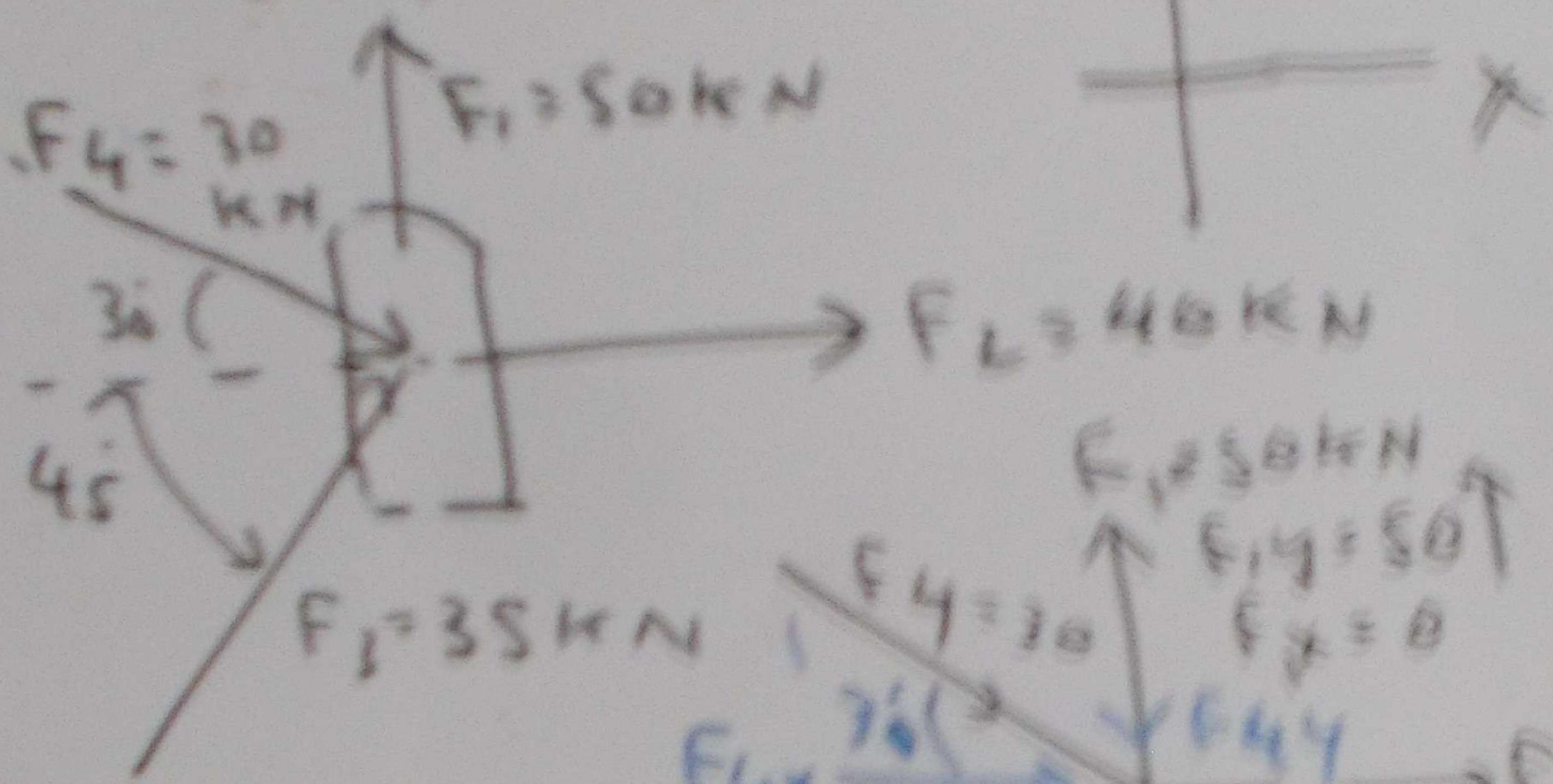
FAE = Toward Joint – compression

FED = award from joint – Tension

E

FAE IS NOW

Pb DETERMINE THE RESULTANT FORCE ACTING ON THE BARGE DUE TO THE COMBINED EFFORT OF FOUR TUG BOATS IN THE FIGURE



$F_{3x} = F_3 \cos 45$

$= 35 \times 0.707$

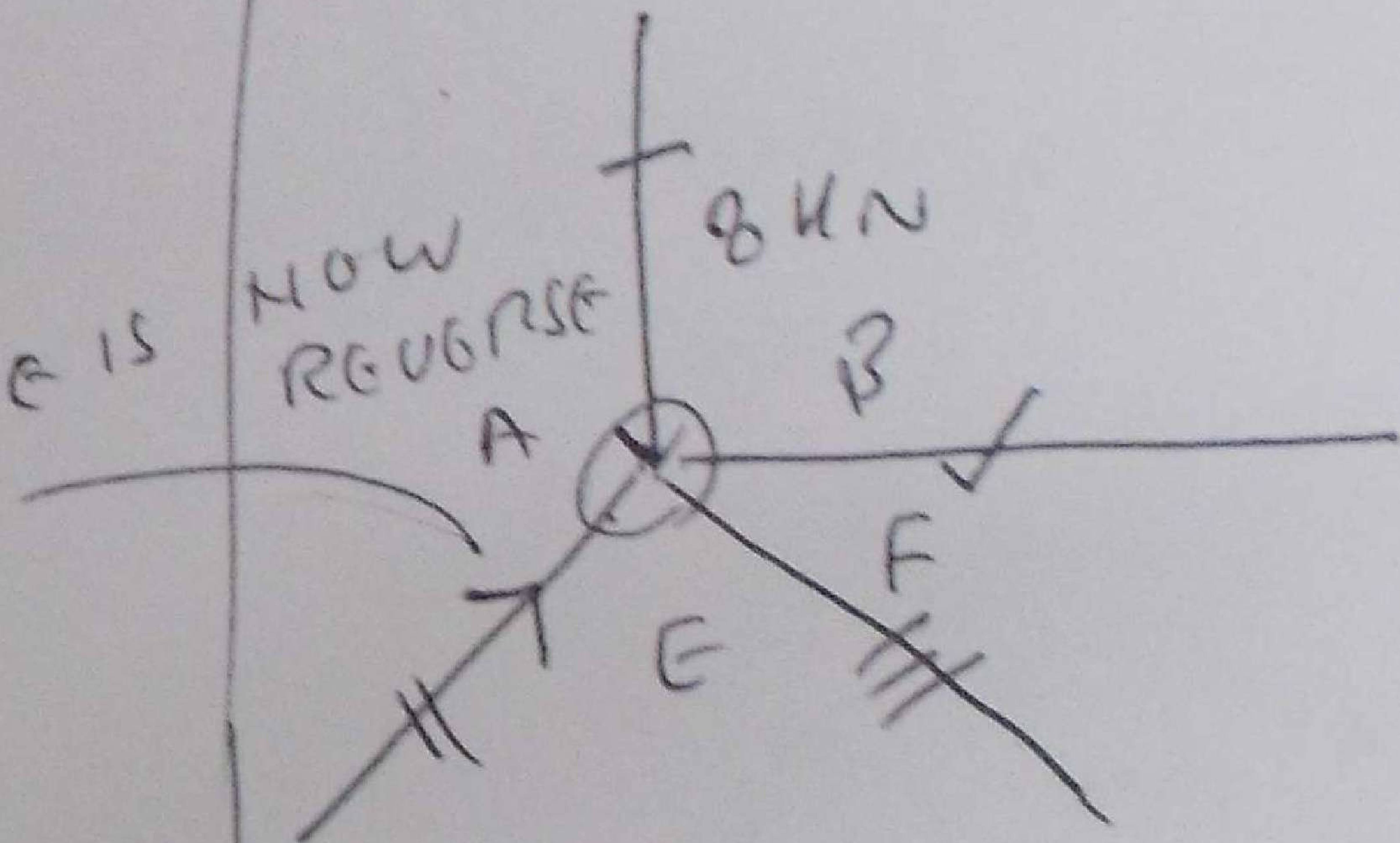
$= 24.7\,N \rightarrow$

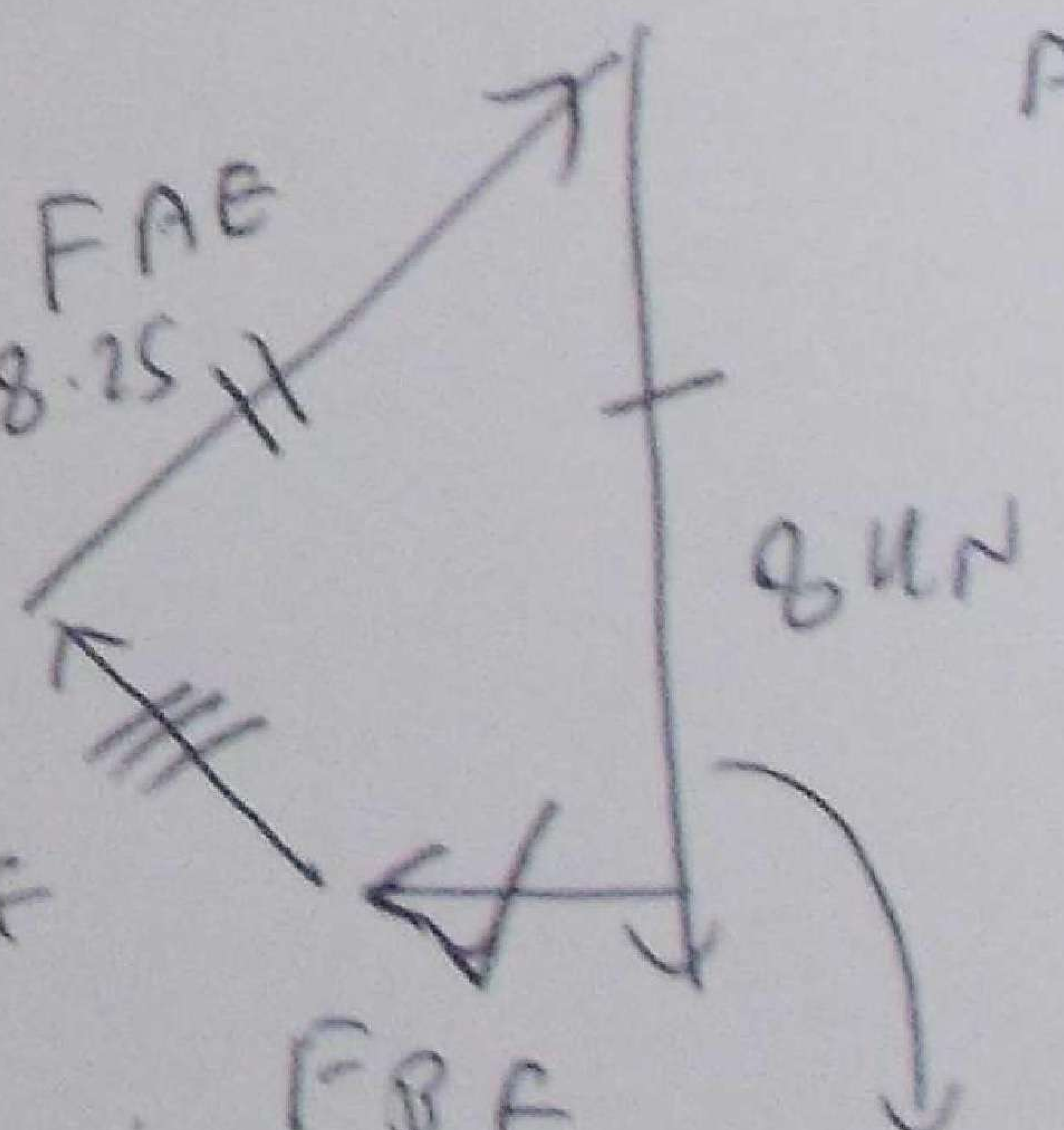
$F_{3y} = F_3 \sin 45$

$= 35 \times 0.707$

$= 24.7\,N \uparrow$

EABF



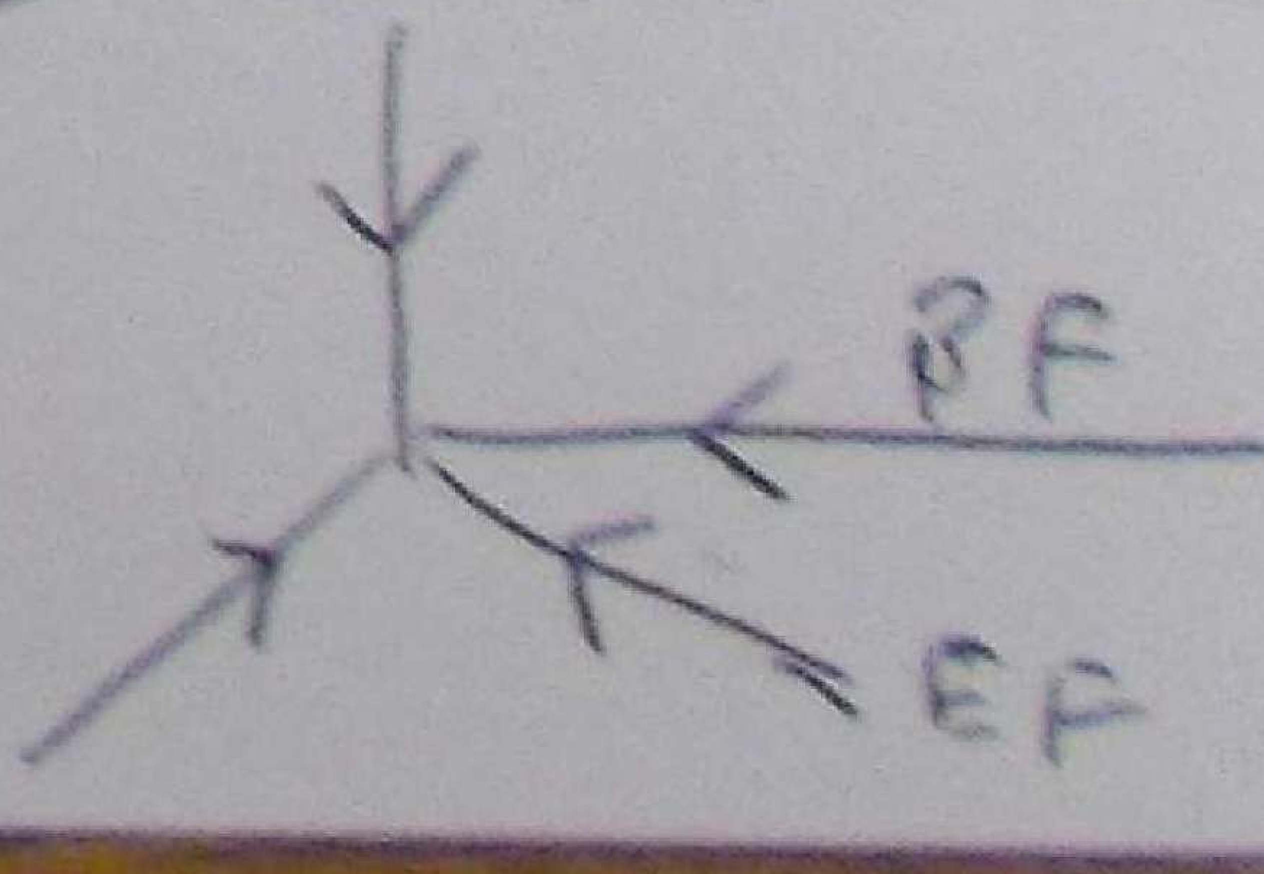


ACCORDING TO SCALE

$F_{BF} = 3.75$ KN

$F_{FE} = 1.13$KN

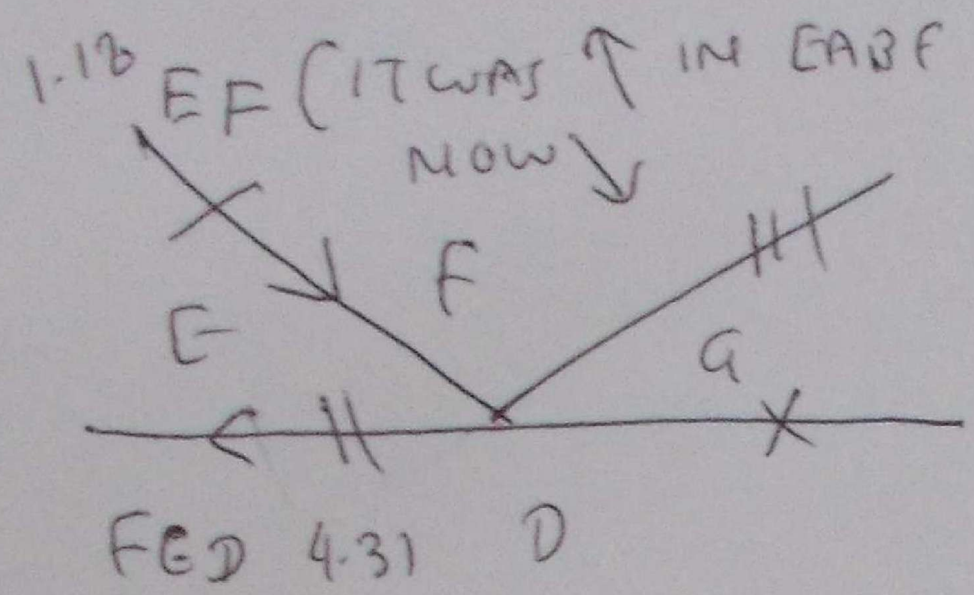
DIRECTION TRANSFER



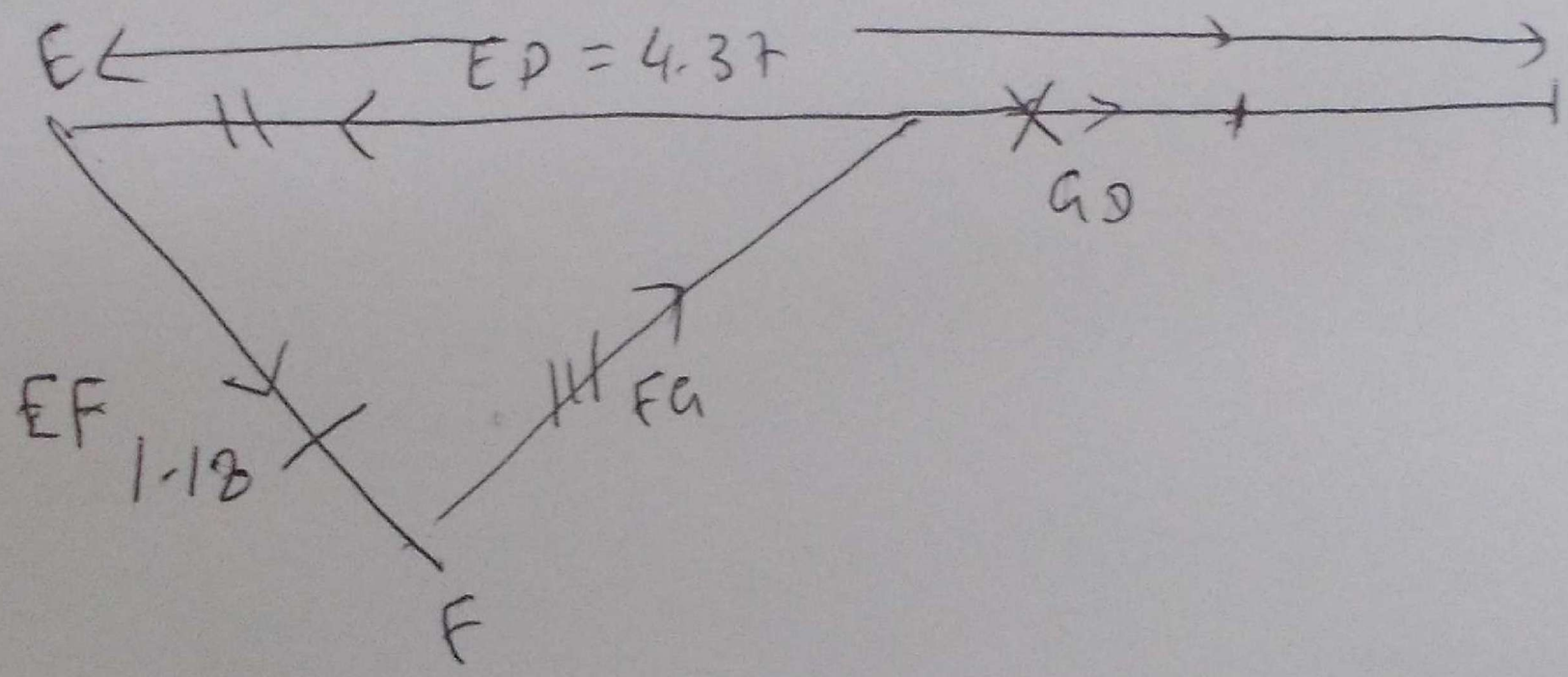
3.75 KN
F_{BF} = COMPRESSION

1.13 KN
F_{EF} - COMPRESSION

JOINT DEFG

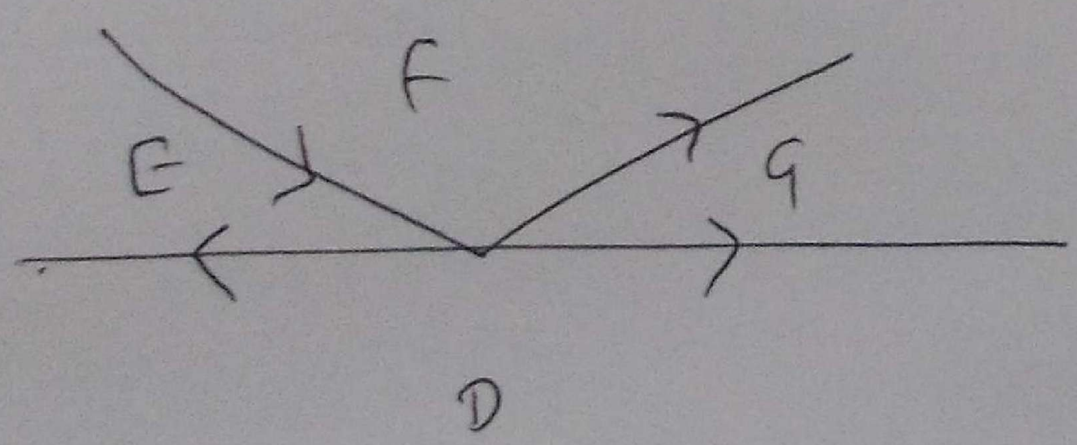


IT WAS → IN DAE NOW REVERSED



BY SCALE FG = 1.13 kN

GD = 3.12 kN ~~FE~~



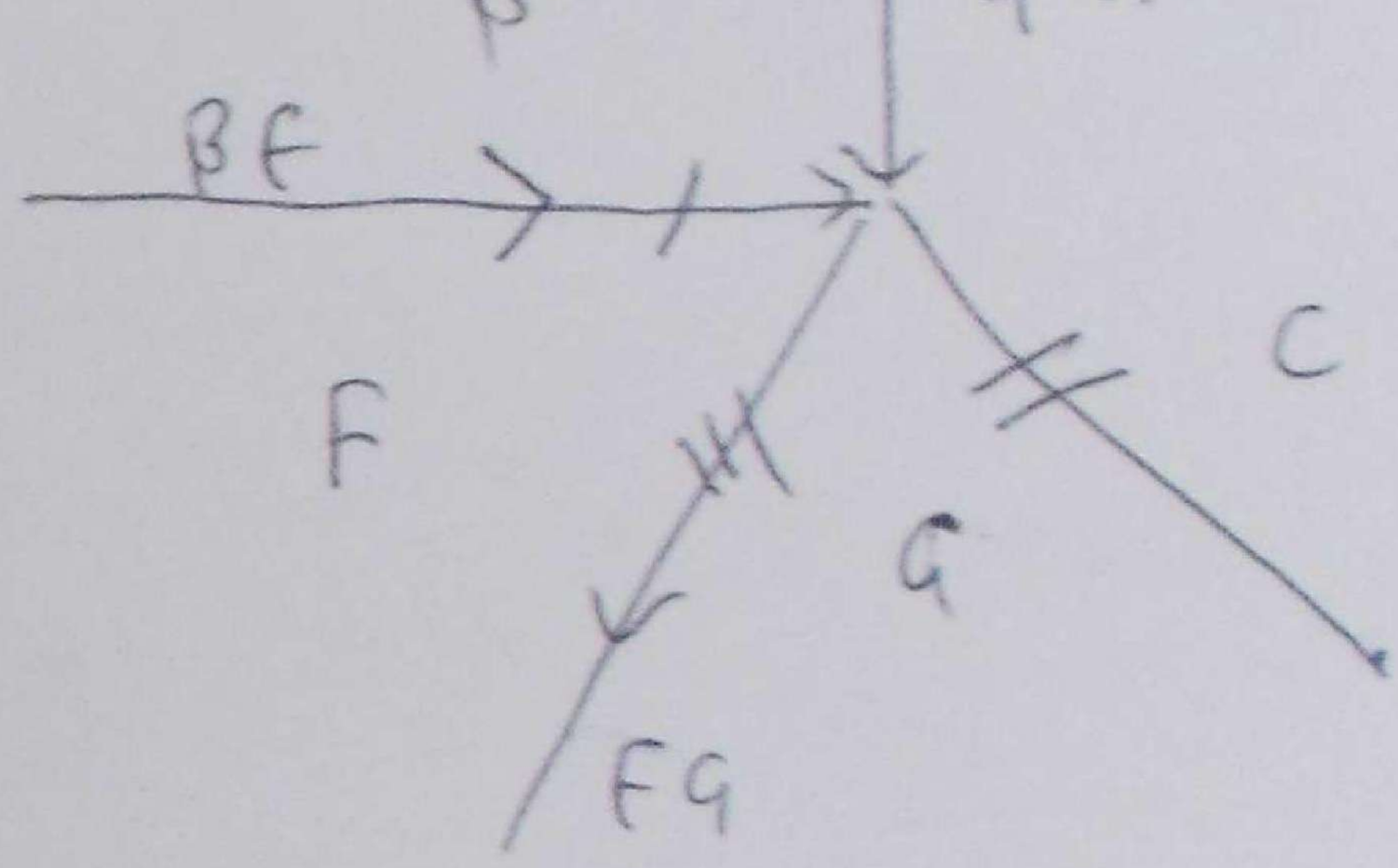
TRANSFER THE DIRECTION

FG = 1.13 kN TENSION

GD = 3.12 kN TENSION

JOINT 9 FBC

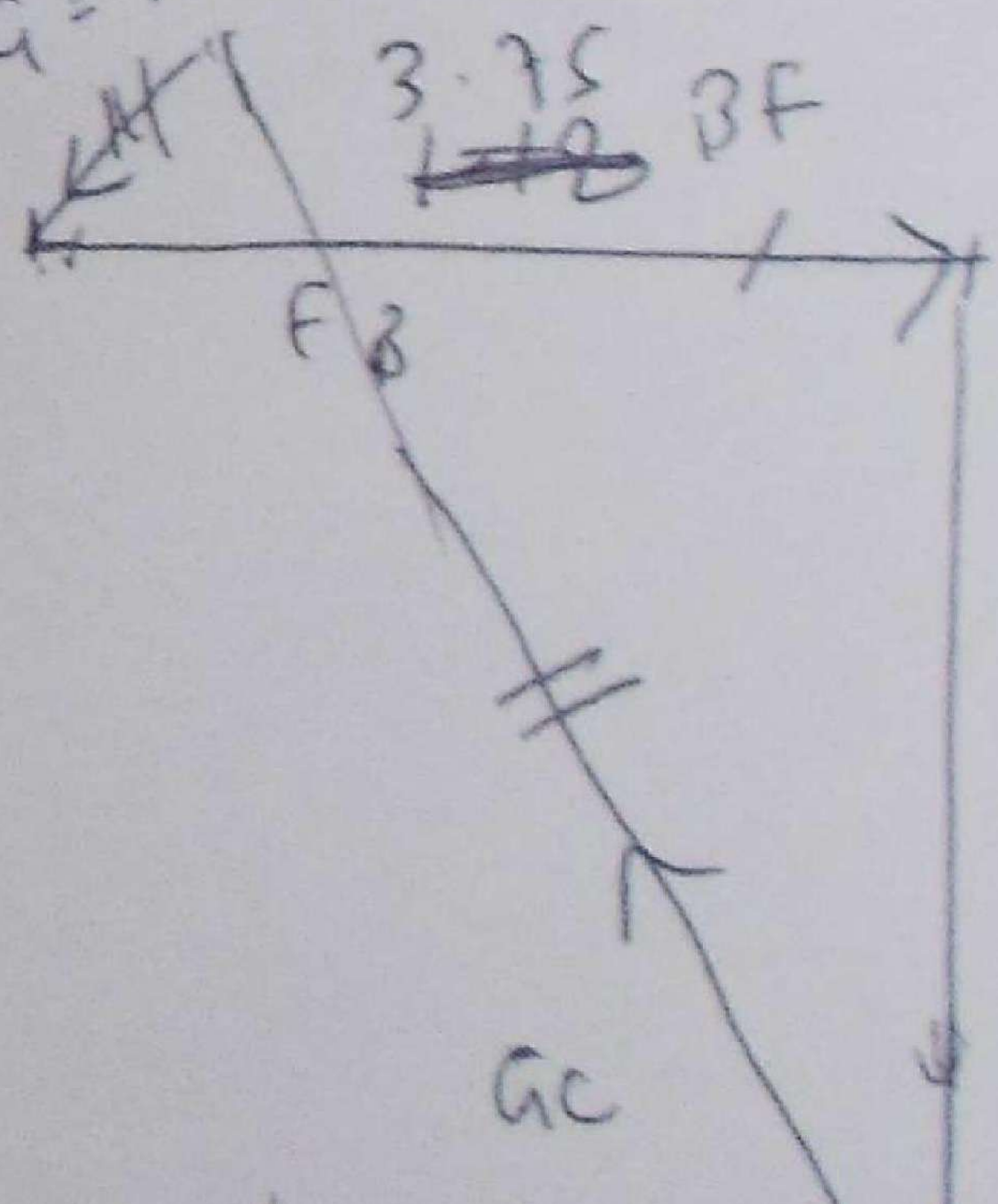
BF was IN ←
EABF, NOW →



4 UN

FG was →
IN DEFG
NOW ←

FG = 1.12

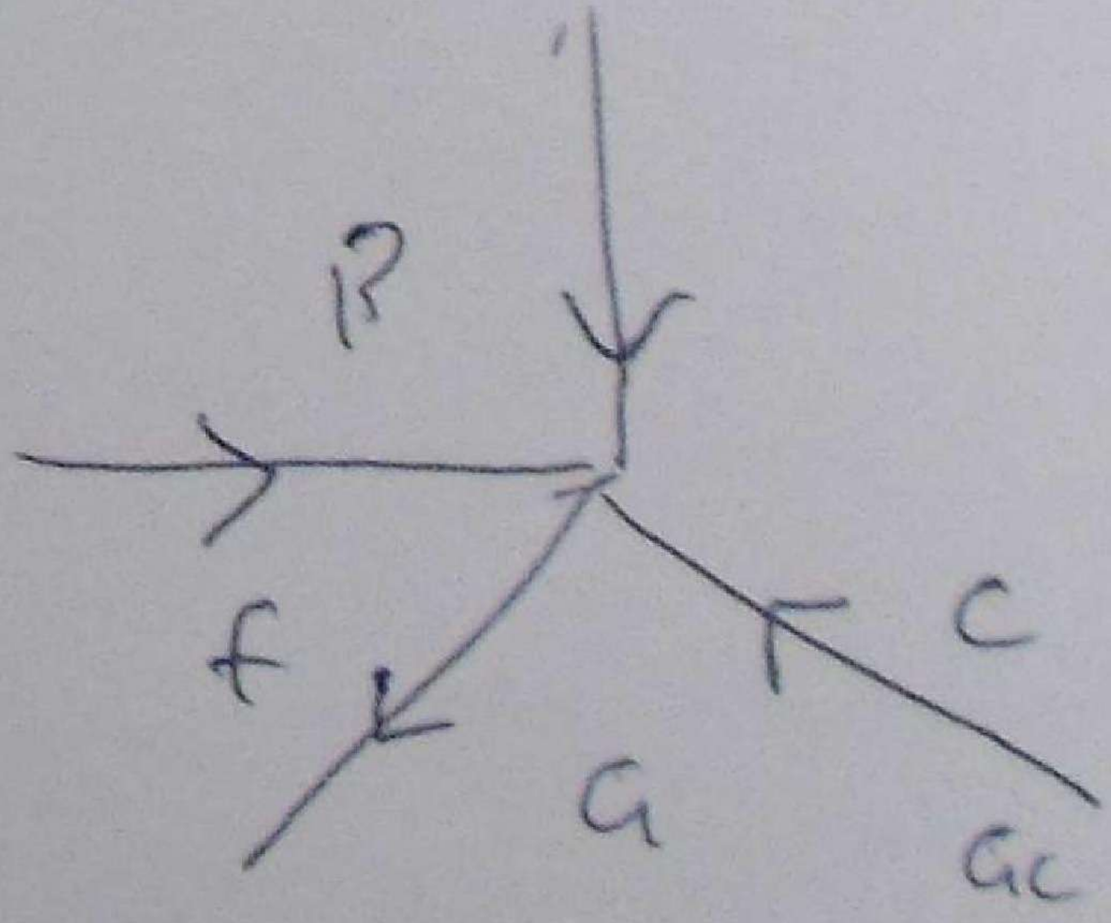


~~FG~~ =

GC = 5 - 9

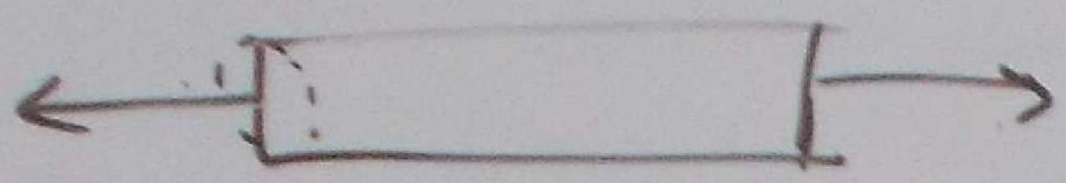
4 UN

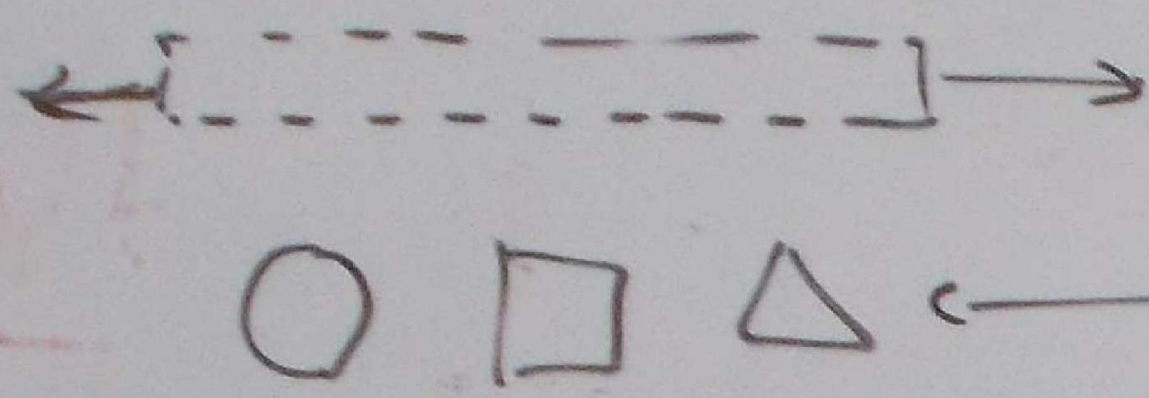
TRANSFER THE DIRECTION



GC = COMPRESSION
5.9 UN

TENSILE STRENGTH





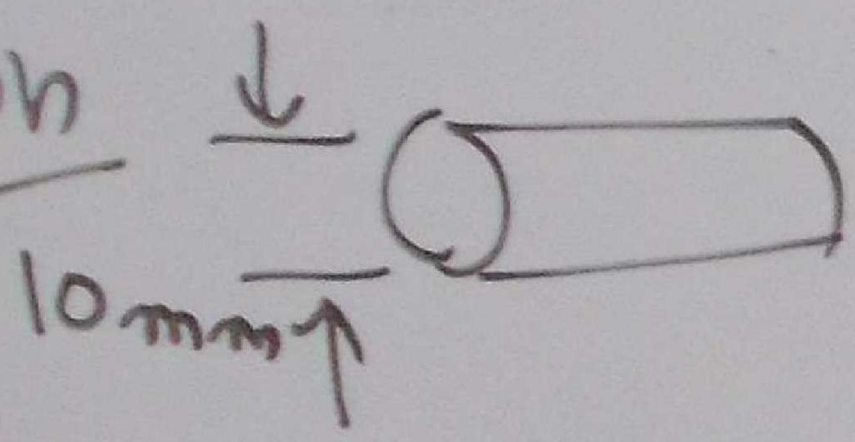
F = MAXIMUM TENSILE FORCE

A = INITIAL CROSS SECTIONAL AREA

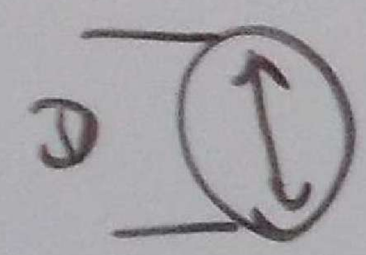
UTS = ULTIMATE TENSILE STRENGTH

$$UTS = \frac{\text{MAXIMUM TENSILE FORCE}}{\text{INITIAL CROSS SECTIONAL AREA}}$$

- Pb



UTS = ?



$$A = \frac{\pi D^2}{4} = \frac{3.1416 \times 10^2}{4} = 78.54\,mm^2$$

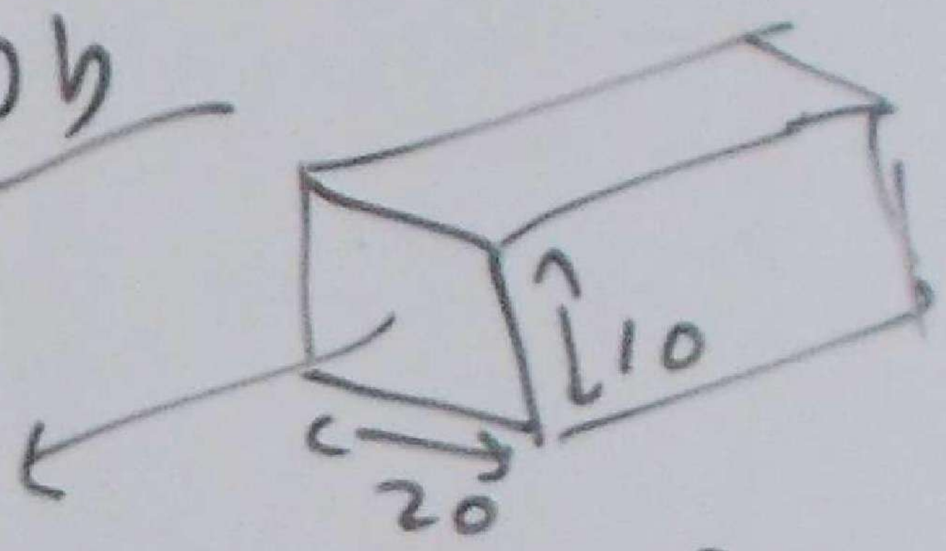
$$UTS = \frac{37 \times 10^3}{78.54} = 471\,N/mm^2$$

$$\text{STRESS} = \frac{\text{FORCE}}{\text{AREA}} \rightarrow f = \frac{F\ (N)}{A\ (mm^2)}$$

MPASCAL

N/mm^2

Pb



$$f = \frac{F}{A} = \frac{18.8 \times 10^3}{20 \times 10}$$

$F = 18.8\,kN$ $\quad f = ?$ $\quad = 94\ N/mm^2$

$= 94\ MPa$

FACTOR OF SAFETY

$$F.S = \frac{\text{ULTIMATE STRESS}}{\text{WORKING STRESS}}$$

$UTS = 470$ MPa, $f = 94$ MPa

$$F_S = \frac{470}{94} = 5$$

Pb TENSILE LOAD (f) = 26 kN HIGH TENSILE STEEL

$SF = 3.5$ WORKING STRESS (f) = ?

$$FS = \frac{UTS}{f} \rightarrow f = \frac{UTS}{FS} = \frac{540}{3.5} = 168.6\ MPa$$

AREA OF HIGH TENSILE STEEL TO WITHSTAND WORKING STRESS 168.6 MPa, TENSILE LOAD 26 KN

$N/mm^2 = MPa$

$D = ?$

$$f = \frac{F}{A} \rightarrow A = \frac{F}{f}$$

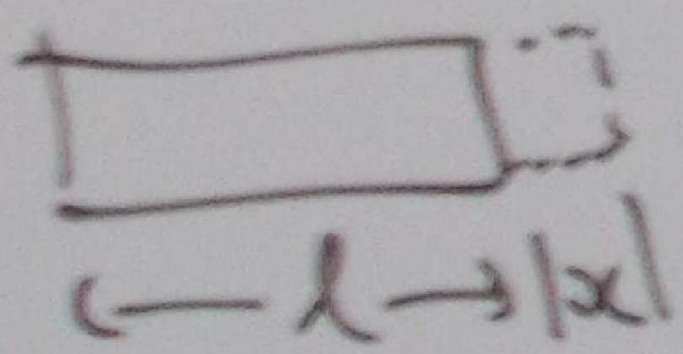
$$= \frac{26 \times 10^3}{168.6}$$

$$= 154.2\ mm^2$$

$$A = \frac{\pi D^2}{4}$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 154.2}{3.1416}} = 14\ mm$$

AXIAL STRAIN IN TENSION



$e = \frac{x}{l}$, e = STRAIN

pb

$l = 2.7\,mm$ $x = 1.27\,mm$

$e = ? \rightarrow e = \frac{x}{l}$

$= \frac{1.27}{2.7\times10^{3}} = 0.47\times10^{-3}$

HOOKE'S LAW

$$\frac{STRESS}{STRAIN} = \frac{f}{e} = E$$ ← YOUNG'S MODULUS OF ELASTICITY

$$E = \frac{f/A}{x/l} = \frac{fl}{xA}$$

pb MILD STEEL BAR

STRESS $f = 94$ MPa, STRAIN $= 4.7\times10^{-3}$

MODULUS OF ELASTICITY $E = ?$

$$E = \frac{f}{e} = \frac{94\,MPa}{0.47\times10^{-3}}$$

$= 200,000\,MPa$

AREA OF HIGH TENSILE STEEL TO WITHSTAND WORKING STRESS 168.6 MPa, TENSILE LOAD 26 KN

$N/mm^2 = MPa$

$D = ?$

$$f = \frac{F}{A} \rightarrow A = \frac{F}{f}$$

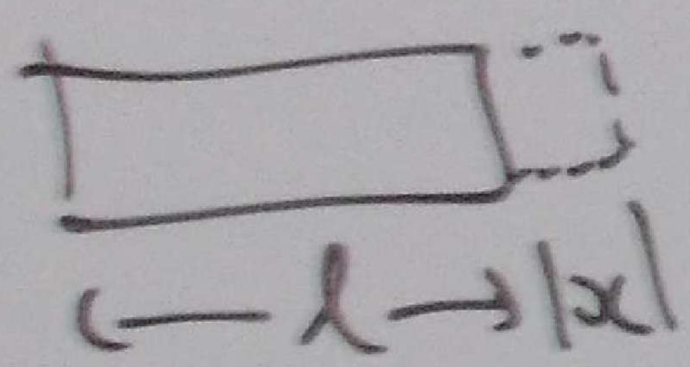
$$= \frac{26 \times 10^3}{168.6}$$

$$= 154.2\ mm^2$$

$$A = \frac{\pi D^2}{4}$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 154.2}{3.1416}} = 14\ mm$$

AXIAL STRAIN IN TENSION



$e = \frac{x}{l}$, e = STRAIN

pb

$l =$ 2.7mm $\quad x$ = 1.27mm

$e = ? \rightarrow e = \dfrac{x}{l}$

$= \dfrac{1.27}{2.7 \times 10^{3}} = 0.47 \times 10^{-3}$

Hooke's Law

$$\frac{\text{STRESS}}{\text{STRAIN}} = \frac{f}{e} = E$$

E ← YOUNG'S MODULUS OF ELASTICITY

$$E = \frac{f/A}{x/l} = \frac{fl}{xA}$$

pb MILD STEEL BAR

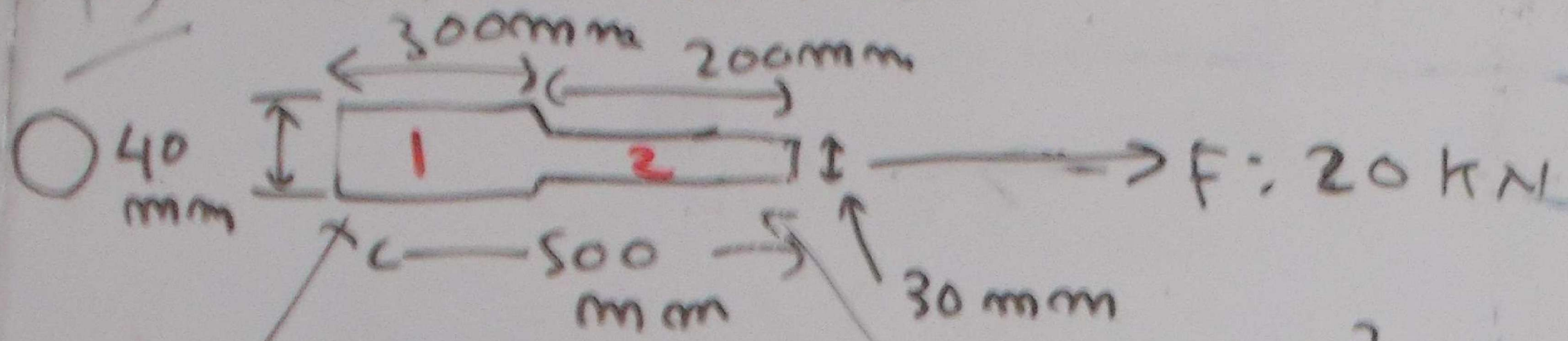
STRESS f = 94 mpa, STRAIN = 4.7×10^{-3}

modulus of E = ?
ELASTICITY

$$E = \frac{f}{e} = \frac{94 \text{ MPa}}{0.47 \times 10^{-3}}$$

$= 200{,}000$ MPc

Pb ALUMINIUM ROD



$f_1 = ?$ $x_1 = ?$ $e_1 = ?$

$f_2 = ?$ $x = ?$ $x_2 = ?$ $e_2 = ?$

(1) $f_1 = \frac{F_1}{A_1} = \frac{20\times10^3}{\frac{\pi}{4}D^2} = \frac{20\times10^3}{\frac{3.1416}{4}(40)^2}$

$E = \frac{f}{e} \rightarrow e_1 = \frac{f_1}{E} = \frac{15.92}{70{,}000} = 0.227\times10^{-3}$ — $= 15.92$ MPa

$e = \frac{x}{l} \rightarrow x_1 = e_1 l_1$

$x_1 = 0.227\times10^{-3}\times300 = 0.0682$ mm

(2) $f_2 = \frac{F_2}{A_2} = \frac{20\times10^3}{\frac{\pi}{4}\times30^2} = 28.29$ mpa

$e_2 = \frac{f_2}{E} = \frac{28.29}{70{,}000} = 0.404\times10^{-3}$

$x_2 = e_2 l_2 = 0.404\times10^{-3}\times200$
$= 0.0808$ mm

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$Y = Y_1 + Y_2 = 0.0682 + 0.0808$

$= 0.149\ mm$

$UTS = 150\ N/mm^2$

$SF = ?$

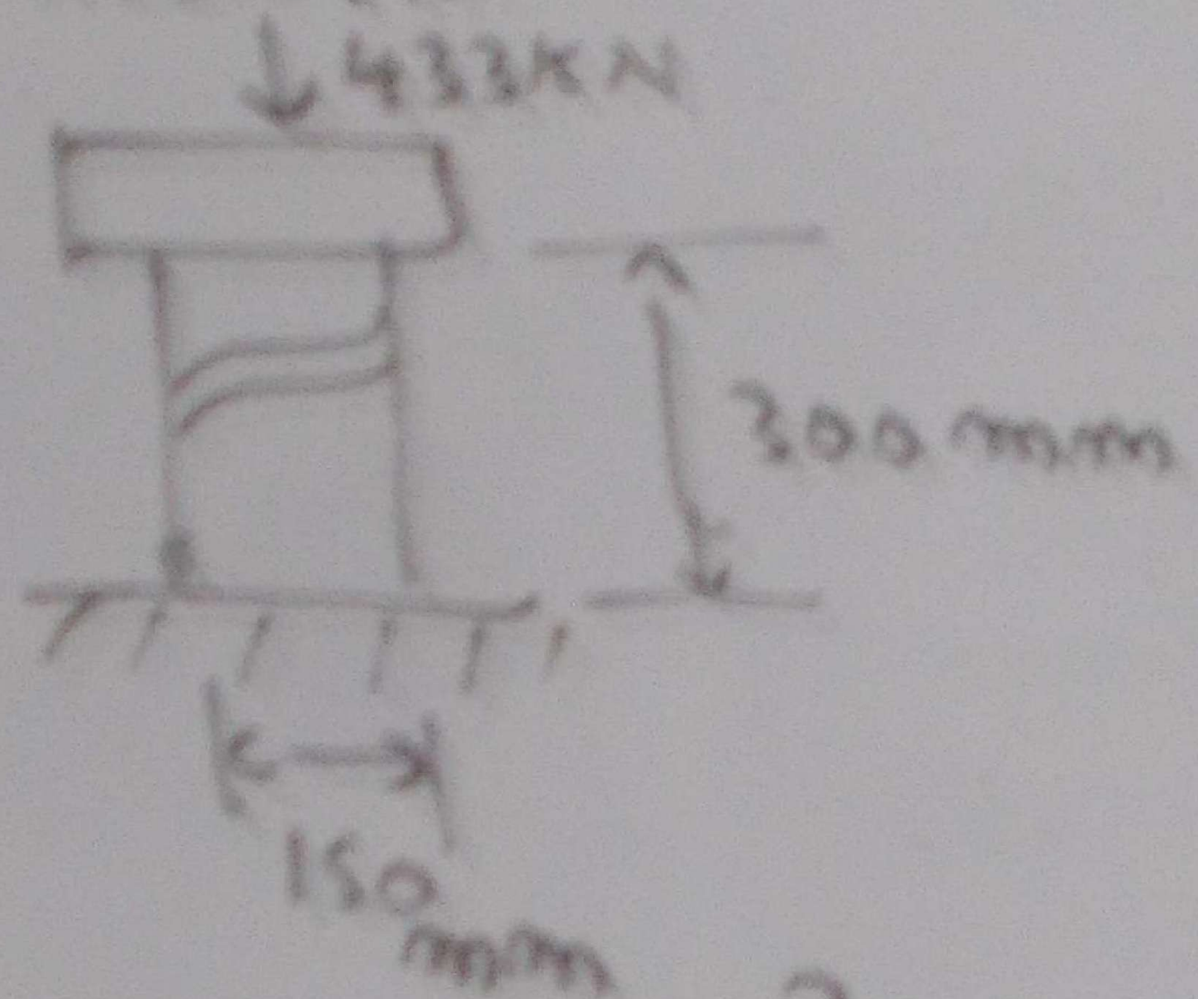
$$SF = \frac{UTS}{f} = \frac{150}{28.29} = 5.3$$

COMPRESSIVE AND THERMAL STRESS

$$UCS = \frac{\text{FORCE CAUSING CRUSHING FAILURE}}{\text{CROSS SECTIONAL AREA}}$$

ULTIMATE COMPRESSIVE STRENGTH

Pb



UCS = ?

$$UCS = \frac{437 \times 10^3}{\frac{\pi}{4}(150)^2} = 24.5 N/mm^2$$

DENSITY, WEIGHT, STRESS

$$\rho = \frac{m}{V}$$

m ← MASS (kg)

V ← VOLUME (m^3)

ρ ↑ DENSITY kg/m^3

WEIGHT = MASS × GRAVITY

$$W = mg$$

Pb W=? TUBULAR STEEL COLUMN 5 m HIGH

OUTSIDE DIAMETER 120 mm

INSIDE DIAMETER 100 mm

$$A = \frac{\pi}{4}\times\left(\frac{120}{1000}\right)^2 - \frac{\pi}{4}\times\left(\frac{100}{1000}\right)^2 = 3.456\times10^{-3}\ m^2$$

$$V = AL = 3.456\times10^{-3}\times 3 = 0.0104\ m^3$$

$$m = \rho V = 7800\times 0.0104 = 80.86\ kg$$

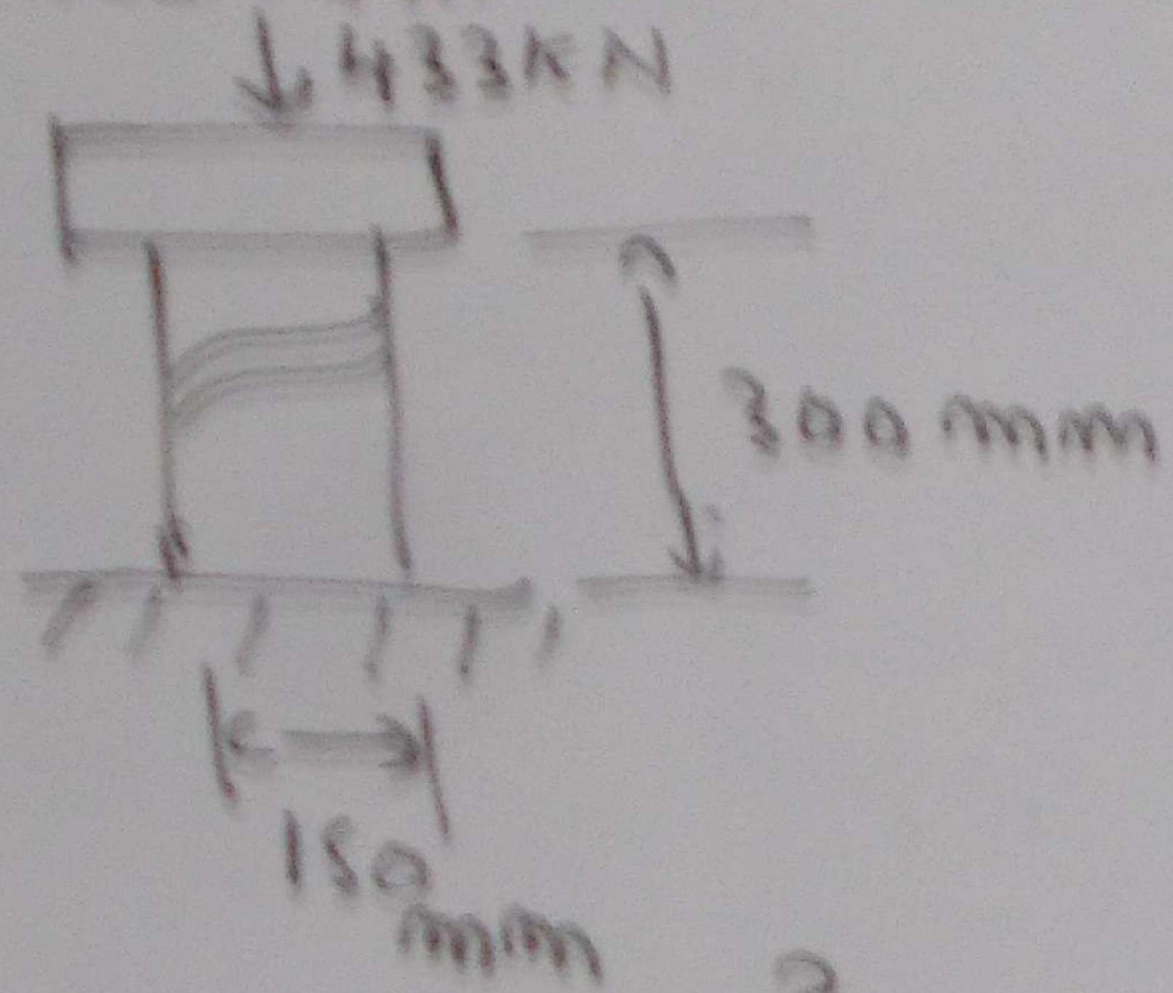
$$W = mg = 80.86\times 9.81 = 793.2\ N$$

COMPRESSIVE AND THERMAL STRESS

$$UCS = \frac{\text{FORCE CAUSING CRUSHING FAILURE}}{\text{CROSS SECTIONAL AREA}}$$

ULTIMATE COMPRESSIVE STRENGTH

Pb



UCS = ?

$$UCS = \frac{437 \times 10^3}{\frac{\pi}{4}(150)^2} = 24.5\ N/mm^2$$

DENSITY, WEIGHT, STRESS

$$\rho = \frac{m}{V}$$

m ← MASS (kg)

V ← VOLUME (m^3)

ρ ↑ DENSITY kg/m^3

WEIGHT = MASS × GRAVITY

$$W = mg$$

pb → W=? TUBULAR STEEL COLUMN 3m HIGH

OUTSIDE DIAMETER 120mm

INSIDE DIAMETER 100mm

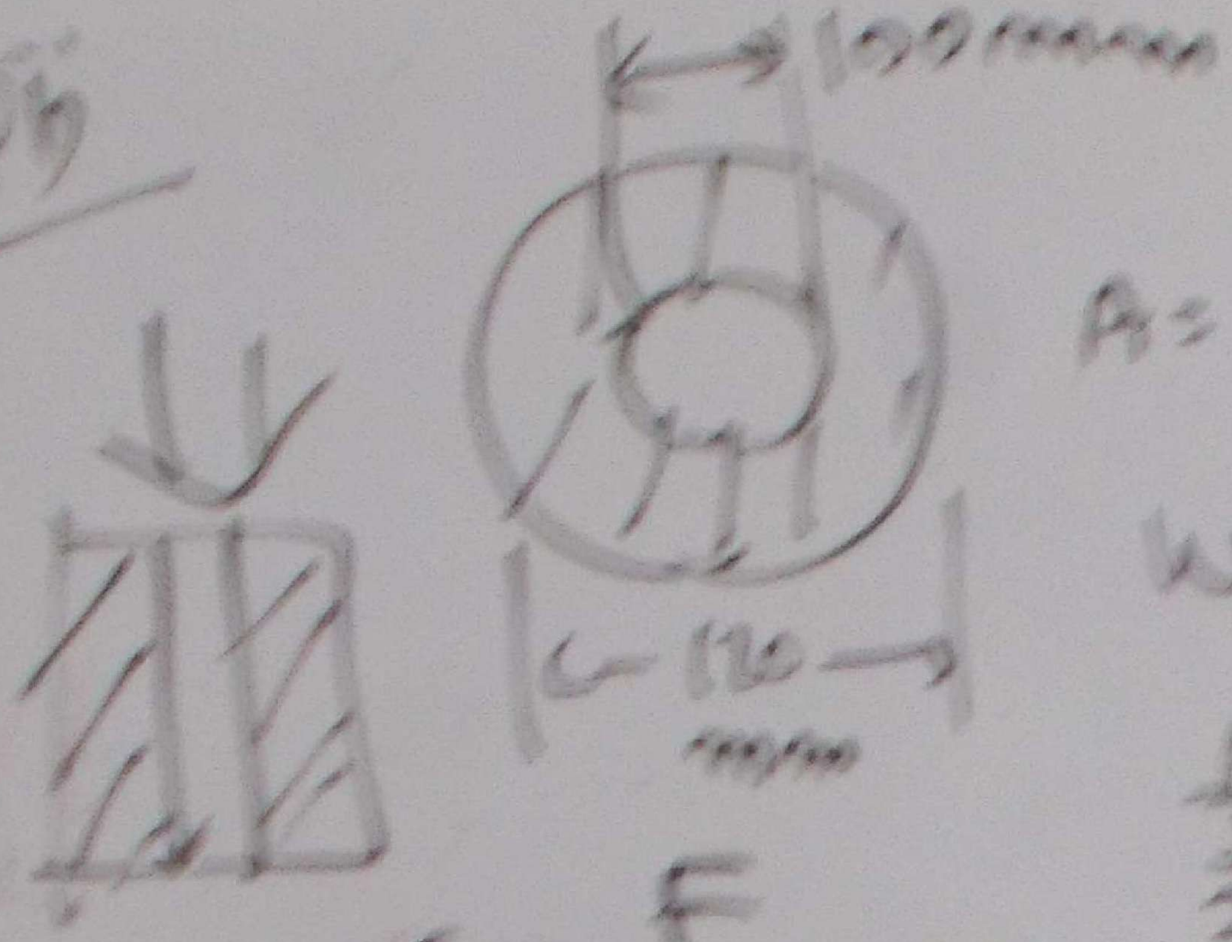
$$A = \frac{\pi}{4}\left(\frac{120}{1000}\right)^2 - \frac{\pi}{4}\left(\frac{100}{1000}\right)^2 = 3.456\times10^{-3}\ m^2$$

$$V = AL = 3.456\times10^{-3} \times 3 = 0.0104\ m^3$$

$$m = \rho V = 7800 \times 0.0104 = 80.86\ kg$$

$$W = mg = 80.86 \times 9.81 = 793.2\ N$$

P9



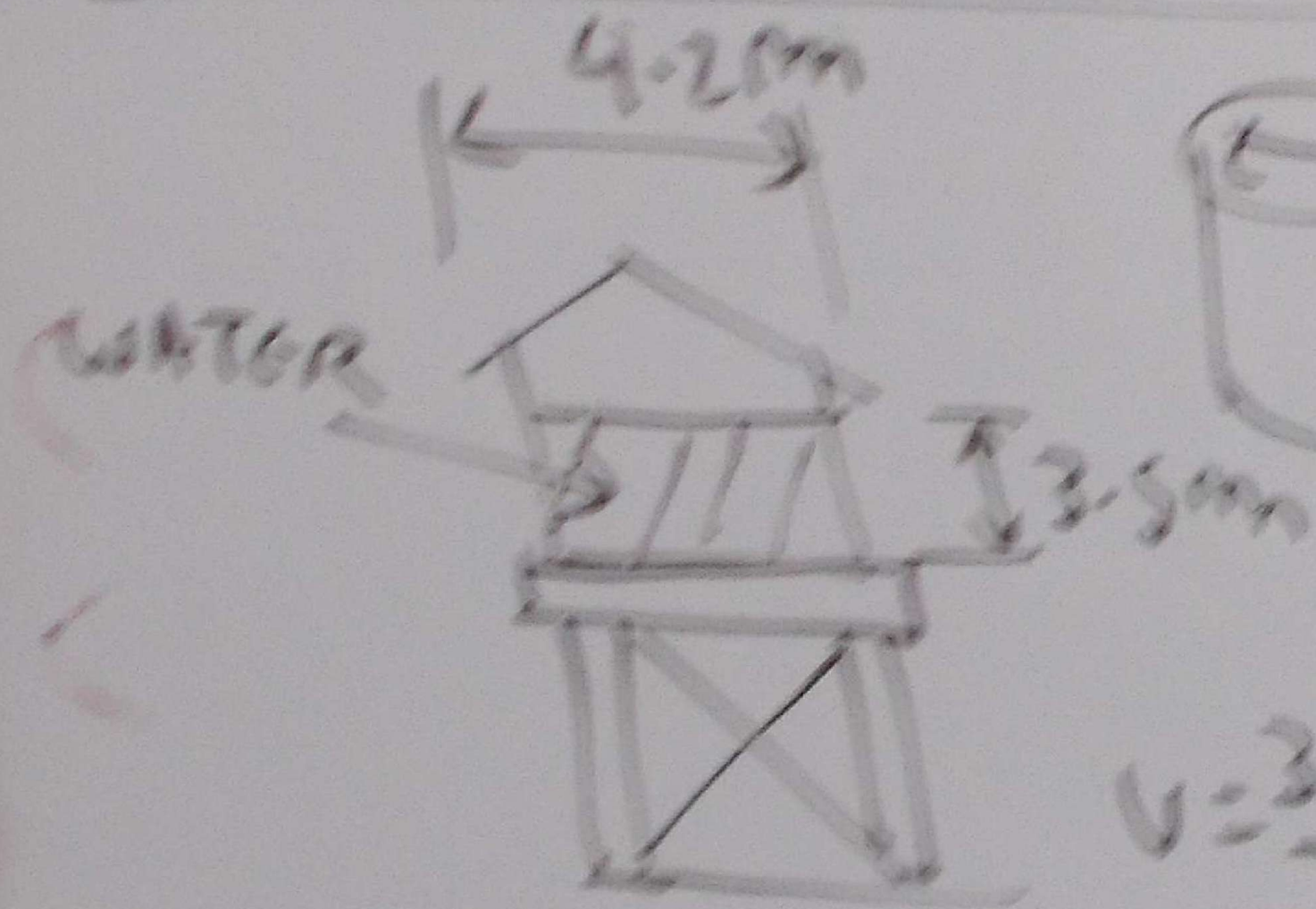
$A = 3.456 \times 10^{-3}\ m^2$

$W = 793.3\ N$

$f = ?$

$$f = \frac{F}{A} = \frac{793.3}{3.456 \times 10^{-3} \times (10^{-3})^2}$$

$$= \frac{793.3}{3.456 \times 10^{-3} \times 10^{-6}} = 0.23\ MPa$$



$$V = \frac{\pi D^2}{4} \times l$$

$$V = \frac{3.1416 \times 4.2^2}{4} \times 3.5$$

TANK = 2300 kg

= 48440 kg

TOTAL MASS = 48490 + 2300

= 50790 kg

WEIGHT = mg

= 50790 × 9.81

= 498250 N

CSA = 3456 mm²

TOTAL CSA = 4 × 3456 mm²

= 13824 mm²

$$f = \frac{F}{A} = \frac{498250}{13824}$$

= 36 MPa

$$E = \frac{Fl}{Ax}$$

$x = el = 0.00042 \times 300$

$= 0.126$ mm

Pb F = 140 kN d = 80 mm l = 300 mm

BRASS

f = ? e = ? x = ?

$$A = \frac{\pi}{4}(80)^2$$

$= 5027$ mm²

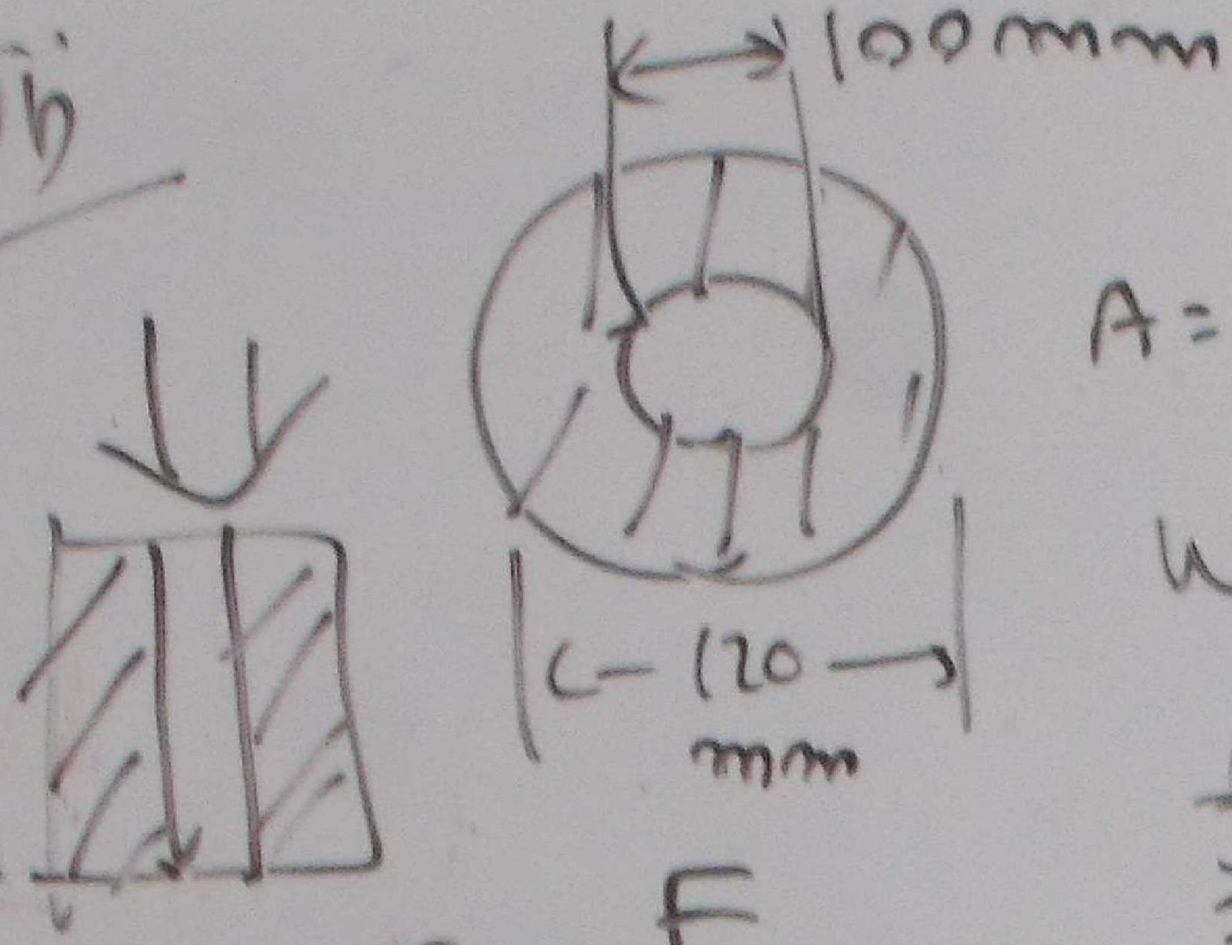
$$f = \frac{F}{A} = \frac{140 \times 10^3}{5027}$$

= 37.8 MPa

$$e = \frac{f}{E} = \frac{37.8}{90000}$$

= 0.00042

Pb



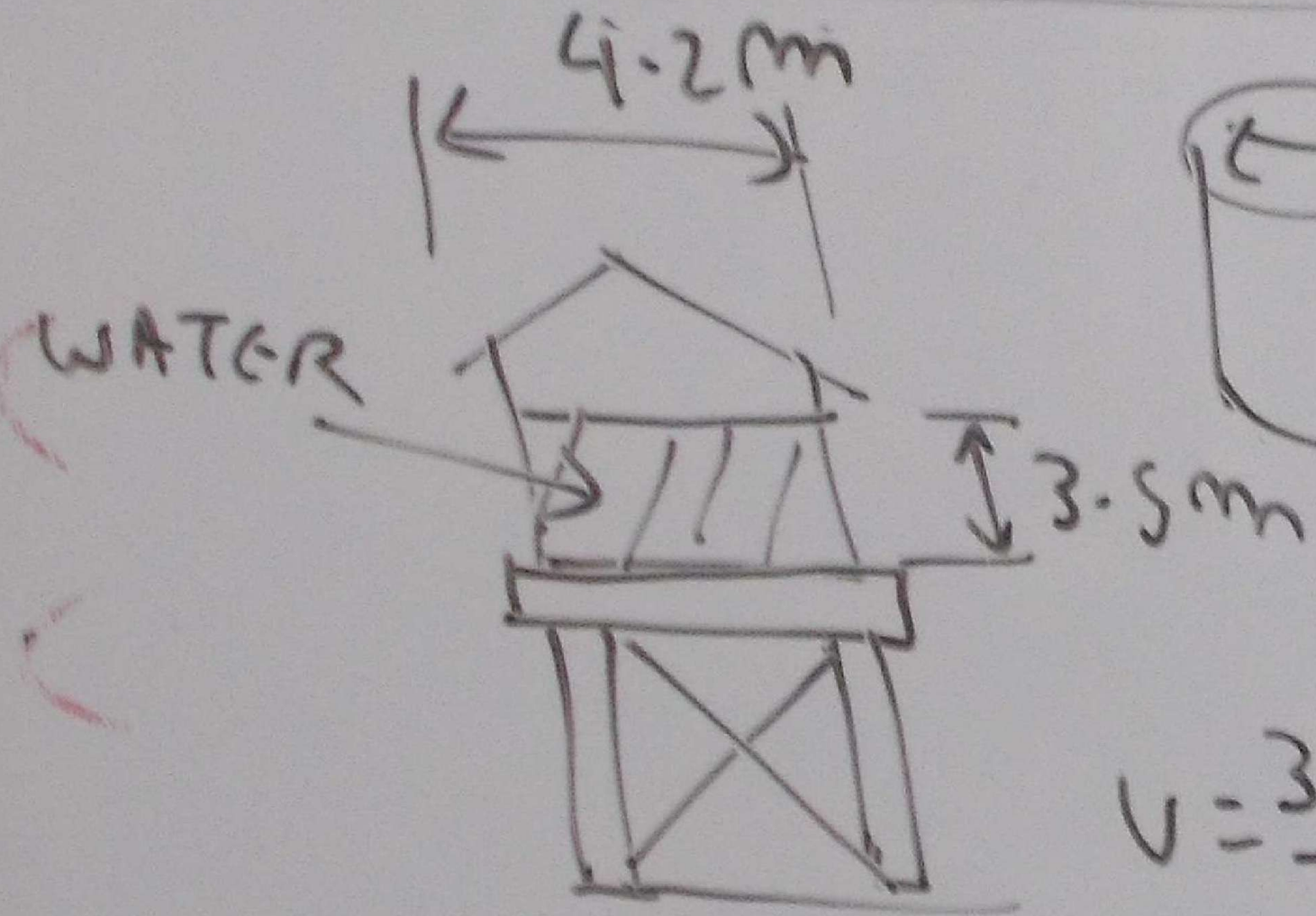
$A = 3.456 \times 10^{-3}\ m^2$

$W = 793.3\ N$

$f = ?$

$$f = \frac{F}{A} = \frac{793.3}{3.456 \times 10^{-3} \times (10^{3})^2}$$

$$= \frac{793.3}{3.456 \times 10^{-3} \times 10^{6}} = 0.23\ mpa$$



$$V = \frac{\pi D^2}{4} \times l$$

$$V = \frac{3.1416 \times 4.2^2}{4} \times 3.5$$

$= 48490\ kg$

TANK = 2300 kg

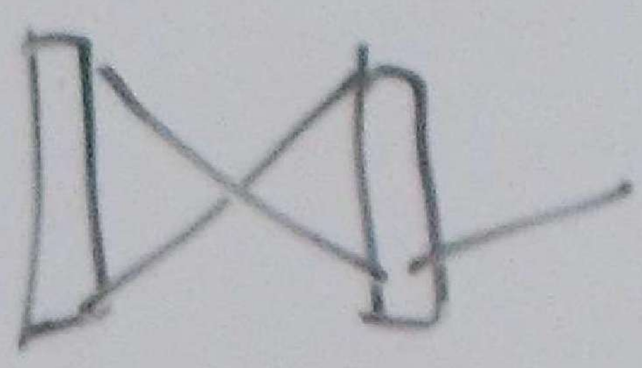
TOTAL MASS = 48490 + 2300

= 50790 kg

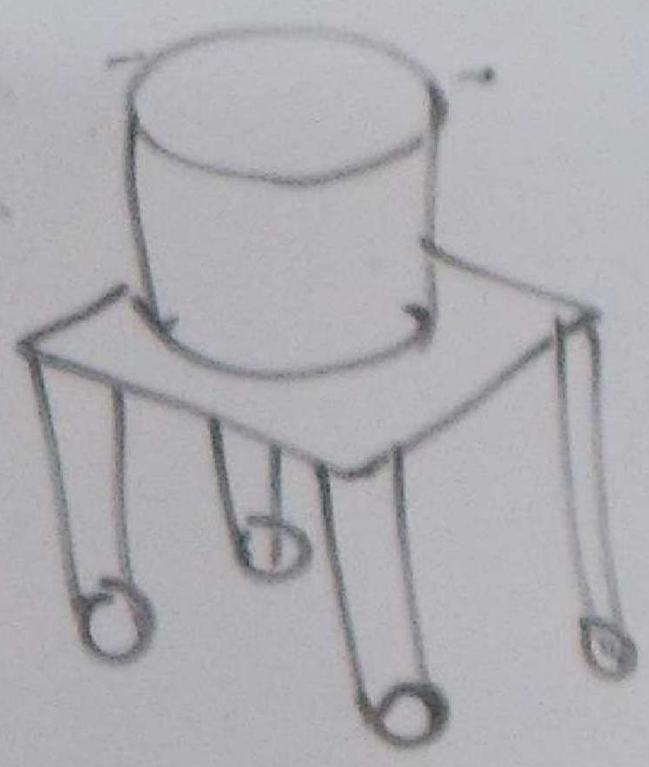
WEIGHT = m g

= 50790 × 9.81

= 498250 N



CSA = 3456 mm^2



TOTAL CSA = 4 × 3456 mm^2

= 13824 mm^2

$$f = \frac{F}{A} = \frac{498250}{13824}$$

= 36 MPa

$$E = \frac{Fl}{Ax}$$

$x = el = 0.00042 \times 300$

= 0.126 mm

Pb F = 140 kN d = 80 mm l = 300 mm

BRASS

f = ? e = ? x = ?

$A = \frac{\pi}{4}(80)^2$

= 5027 mm^2

$f = \frac{F}{A} = \frac{140 \times 10^3}{5027}$

= 37.8 MPa

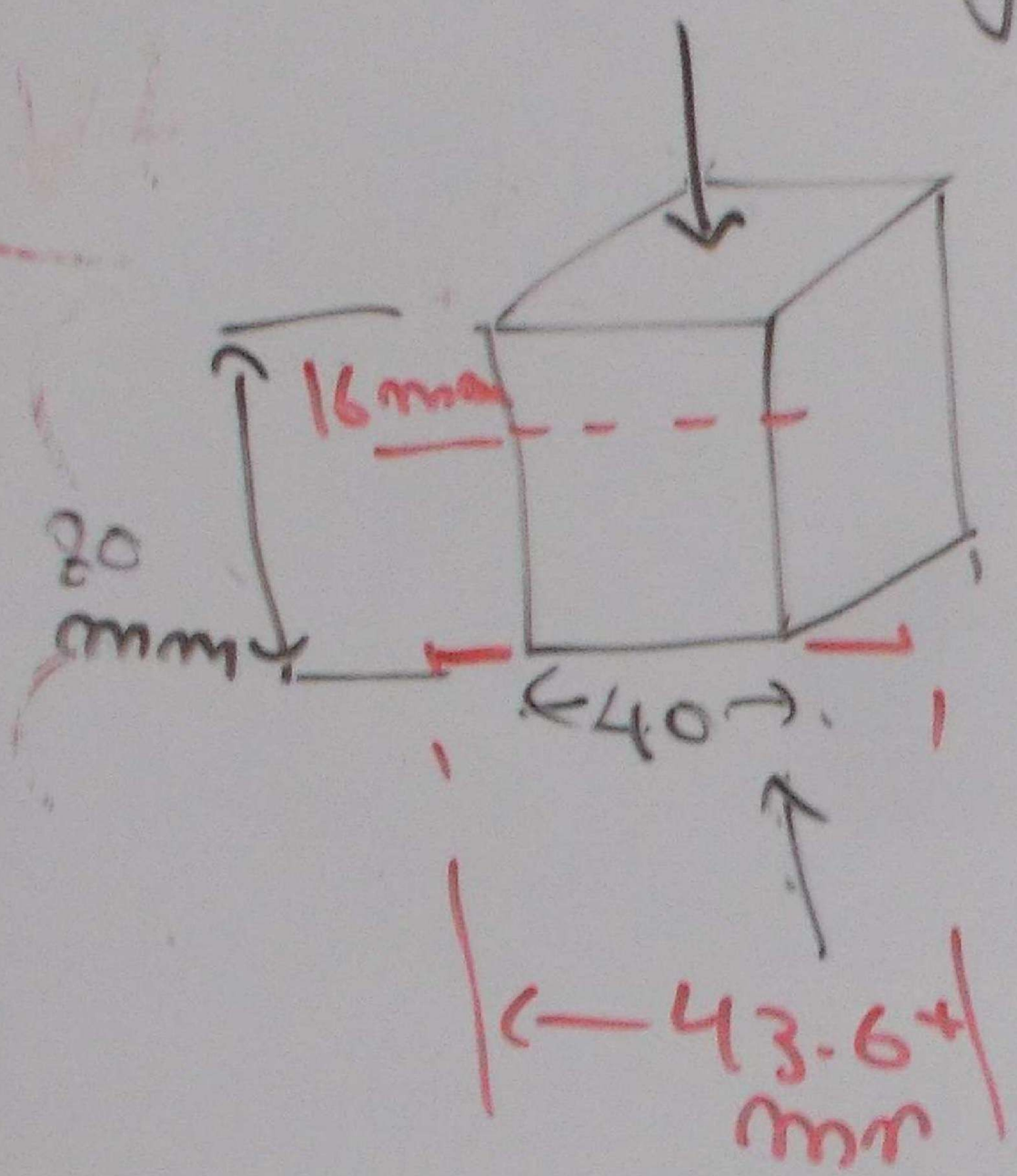
$e = \frac{f}{E} = \frac{37.8}{90000}$

= 0.00042

POISSION RATIO

$$\text{POISSION RATIO} = \frac{\text{LATERAL STRAIN}}{\text{AXIAL STRAIN}}$$

ν



e_A = AXIAL STRAIN

$$= \frac{16}{80} = 0.2$$

e_l = LATERAL STRAIN

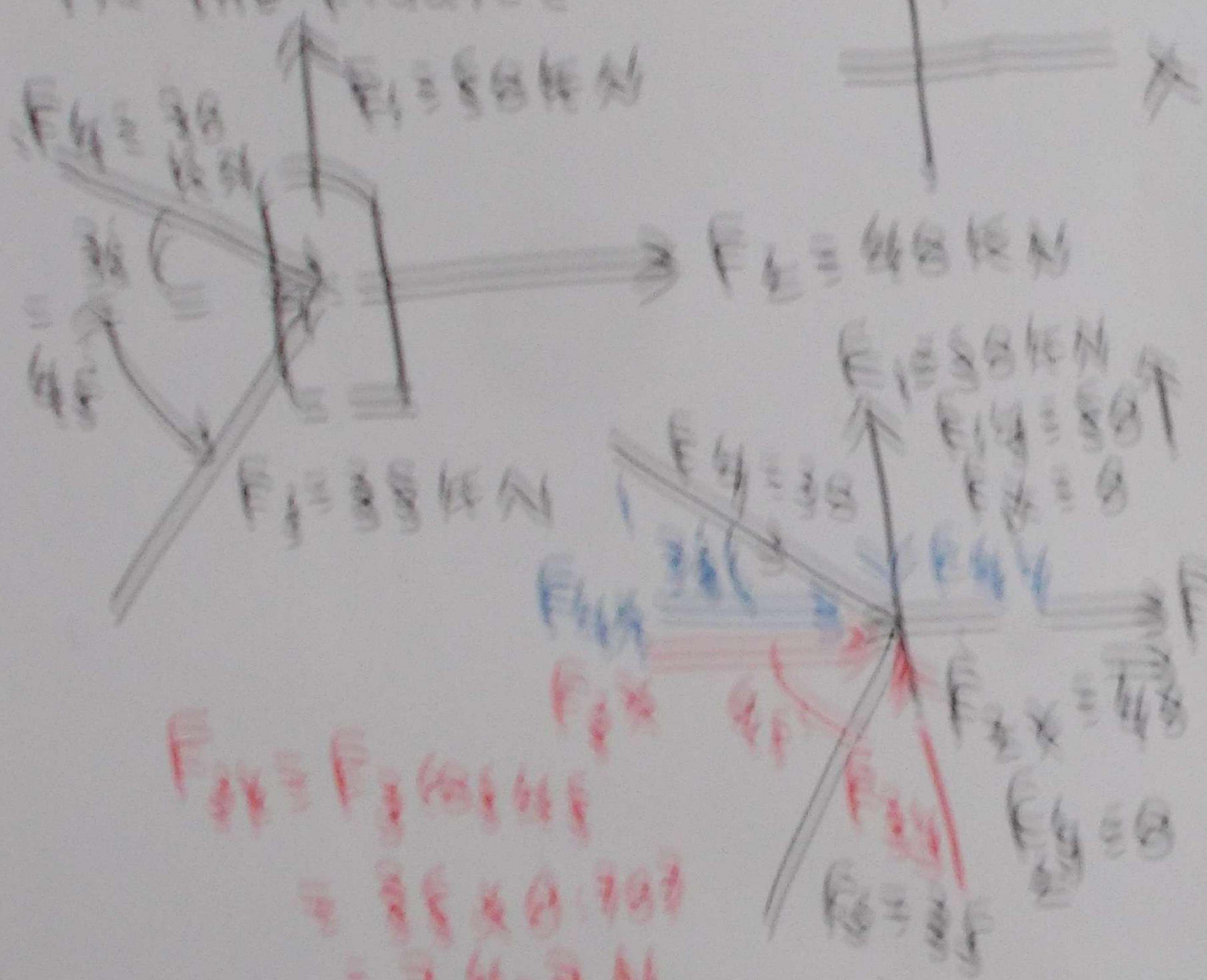
$$x = 43.6 - 40$$

$$= 3.6$$

$$e_l = \frac{3.6}{40} = 0.09$$

$$\nu = \frac{e_l}{e_A} = \frac{0.09}{0.2} = 0.45$$

Pb DETERMINE THE RESULTANT FORCE RETING ON THE BARGE DUE TO THE COMBINED EFFORT OF FOUR TUG BOATS IN THE FIGURE



$F_{4x} = F_4 \cos 45$

$= 38 \times 0.707$

$= 26.9\ N$

$F_{4y} = F_4 \sin 45$

$= 38 \times 0.707$

$= 26.9\ N \uparrow$

$$\Delta l = \alpha L_0 \Delta t$$

Pb ALUMINIUM

$L_0 = 1.2\,m \quad t_1 = 15^\circ C \quad t_2 = 40^\circ C$

$\Delta l = ? \quad \Delta t = t_2 - t_1 = 40 - 15 = 25\,C$

$$\Delta l = \alpha L_0 \Delta t = 0.000024 \times 1.2 \times 25$$
$$= 0.72\,mm$$

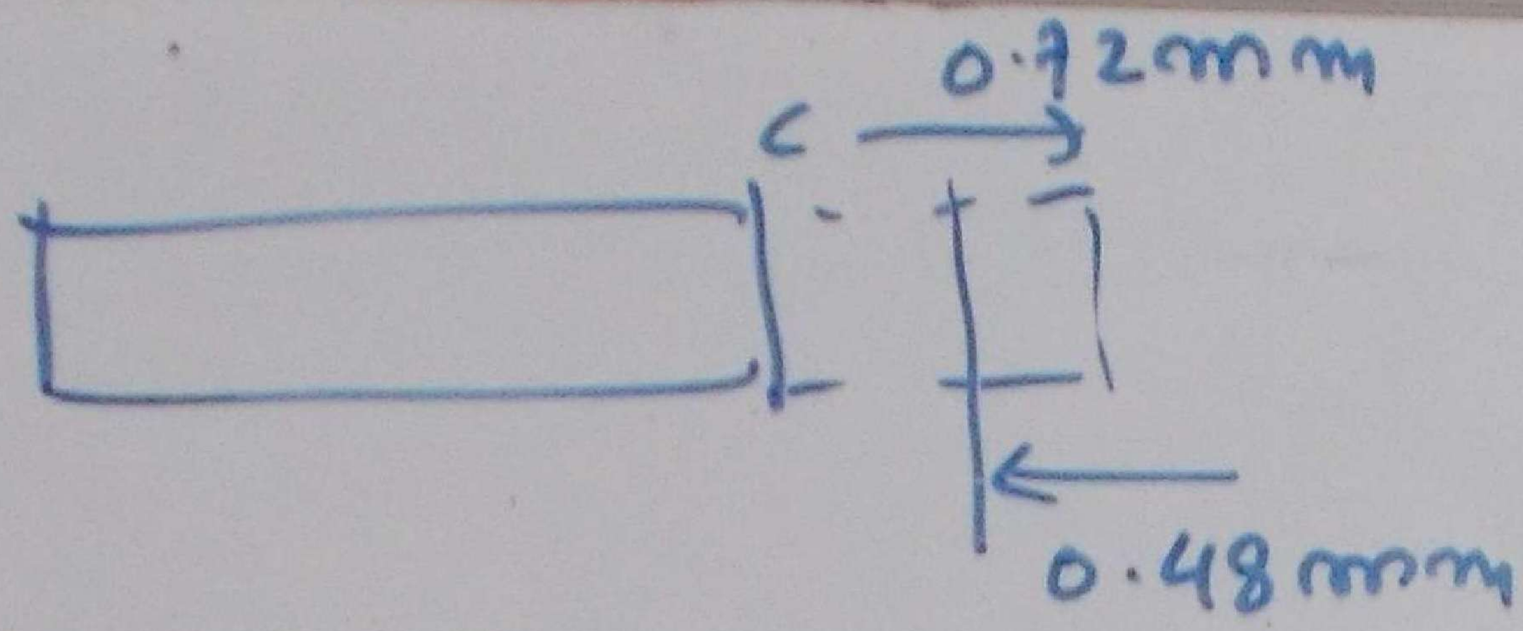
$$L = L_0 + \Delta l = 1.2 + 0.72 \times 10^{-3} =$$
$$= 1.2 \times 10^{3} + 0.72 = 1200.72\,mm$$

THERMAL STRESS

$e = \frac{x}{l} \quad f = E \times e$ ALUMINIUM

$$e = \frac{0.72}{1200} = 0.0006$$

$$f = e E = 0.0006 \times 70000$$
$$= 42\,MPa$$



$$x = 0.72 - 0.48 = 0.24\ mm$$

$$e = \frac{x}{l} = \frac{0.24}{1200} = 0.0002$$

$$f = E\,e = 70000 \times 0.0002$$

$$= 14\ mpa$$

$$\text{STRESS } f = E\,e = E\,\frac{\Delta L_1 - \Delta L_2}{L_0}$$

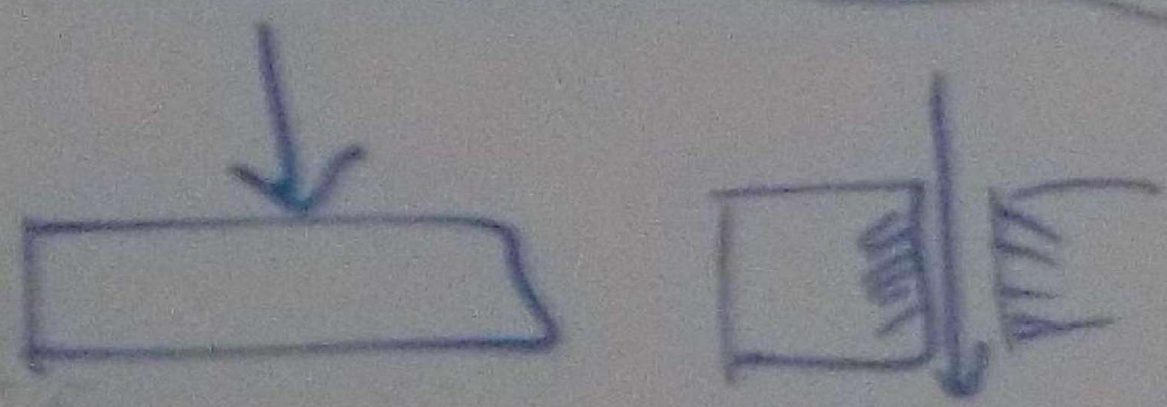
$$\text{STRESS } f = E\,\frac{\Delta L}{L_0}$$

$$f = E\,\frac{L_0 \cdot \alpha\,\Delta t}{L_0}$$

$$\boxed{f = E \alpha \Delta t}$$

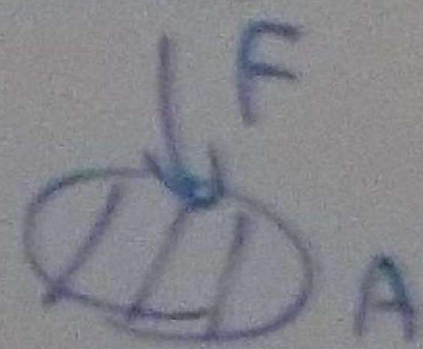
THERMAL STRESS

SHEAR AND TORSIONAL STRESSES

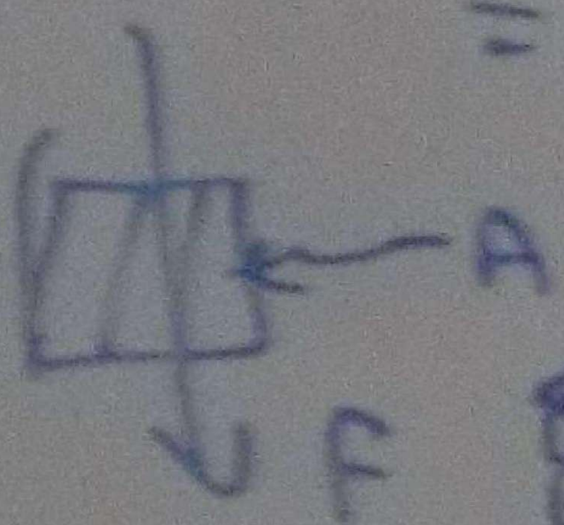


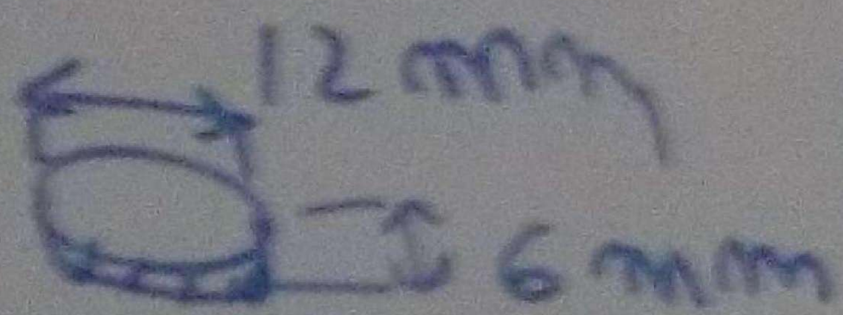
SHEAR STRESS

$$\text{STRESS} = \frac{\text{FORCE}}{\text{AREA}}$$



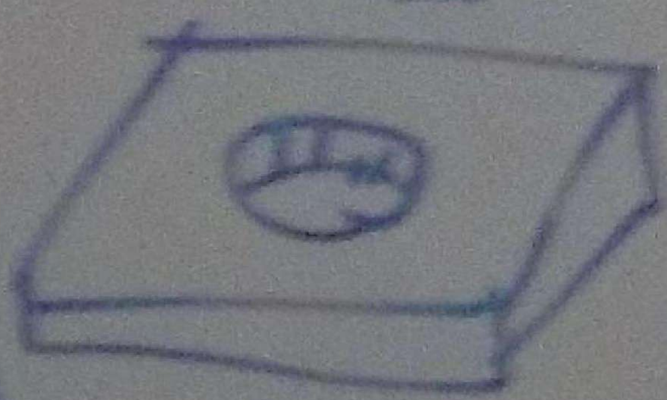
$$= \frac{F}{A}$$





FORCE = ?

Pb



MILD STEEL

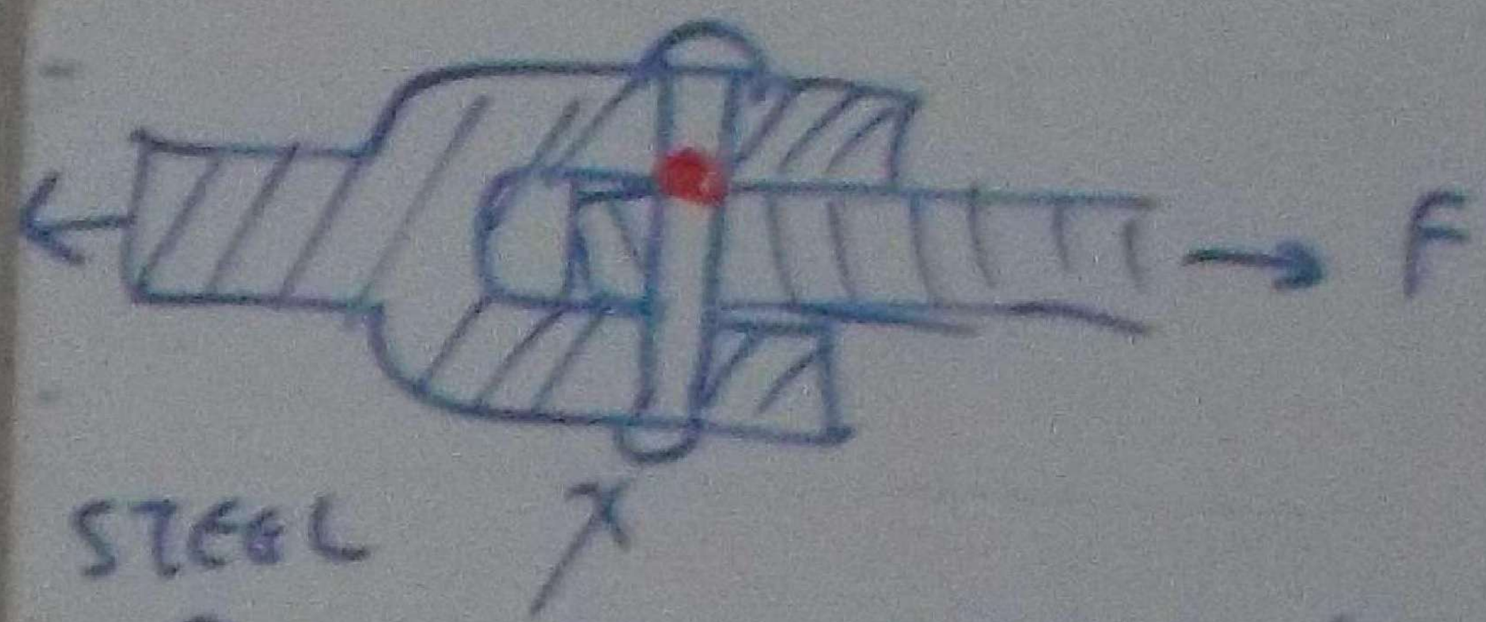
SHEAR STRESS = 360 N/mm²

$\pi D t$ = SHEAR AREA

$3.1416 \times 12 \times 6 = 226.2\ mm^2$

F = STRESS × AREA = 360 × 226.2

= 81.4 KN



$$f = \frac{F}{A}$$

DIAMETER = 10mm

$F_{max} = 11.3kN$

SHEAR STRESS, SF = ?

$$A = \frac{\pi}{4} D^2 = \frac{3.146}{4} \times 10^2 \times 2 = 157 mm^2$$

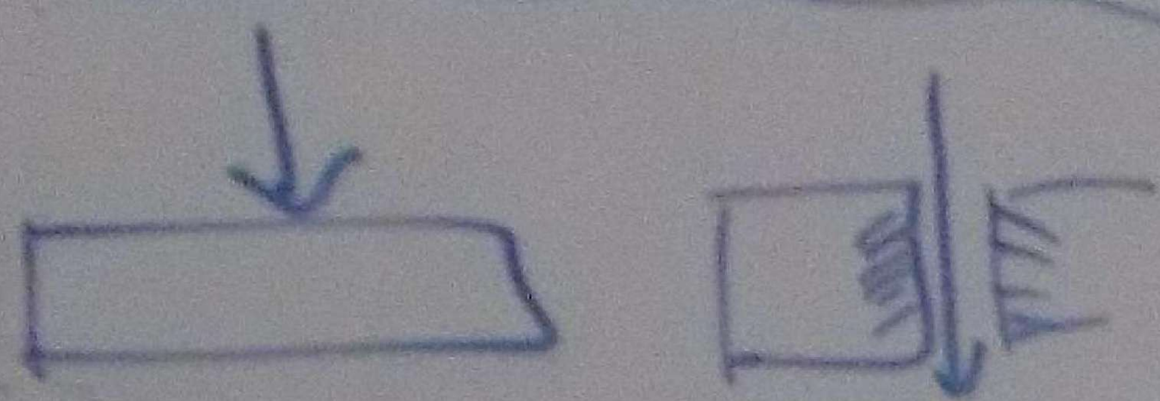
$$f = \frac{F}{A} = \frac{11.3 \times 10^3}{157} = 72 MPa$$

ULTIMATE SHEAR STRESS = 360MPa

USS

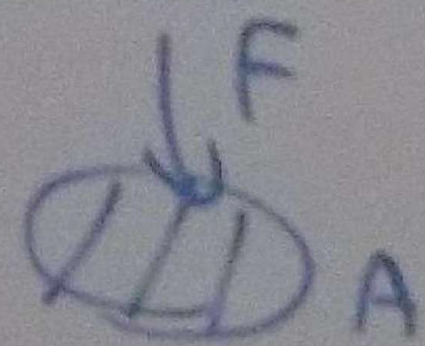
$$Sf = \frac{USS}{f} = \frac{360}{72} = 5$$

SHEAR AND TORSIONAL STRESSES

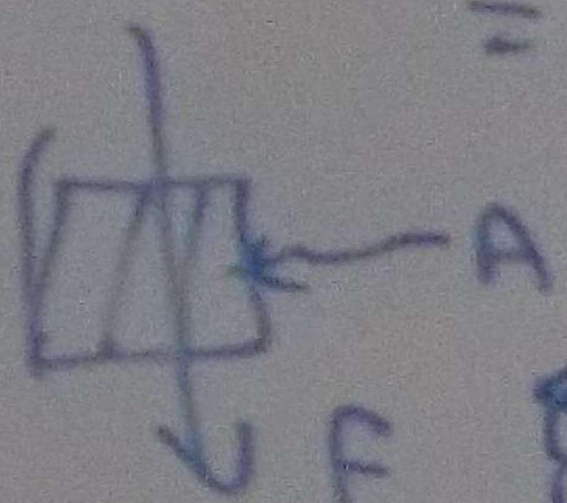


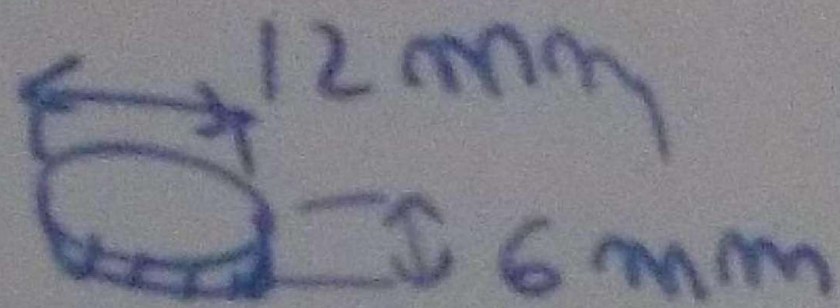
SHEAR STRESS

$$\text{STRESS} = \frac{\text{FORCE}}{\text{AREA}}$$



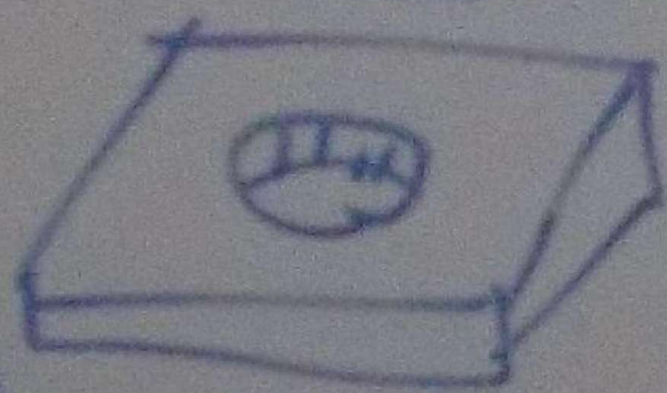
$$= \frac{F}{A}$$





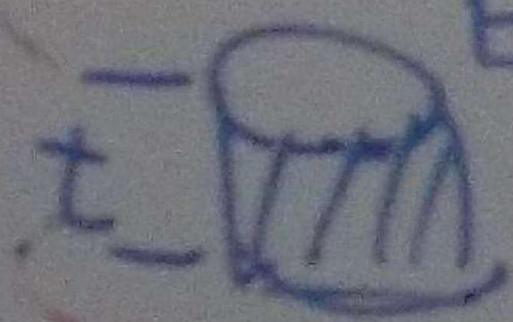
FORCE = ?

Pb



MILD STEEL

SHEAR STRESS = 360 N/mm²

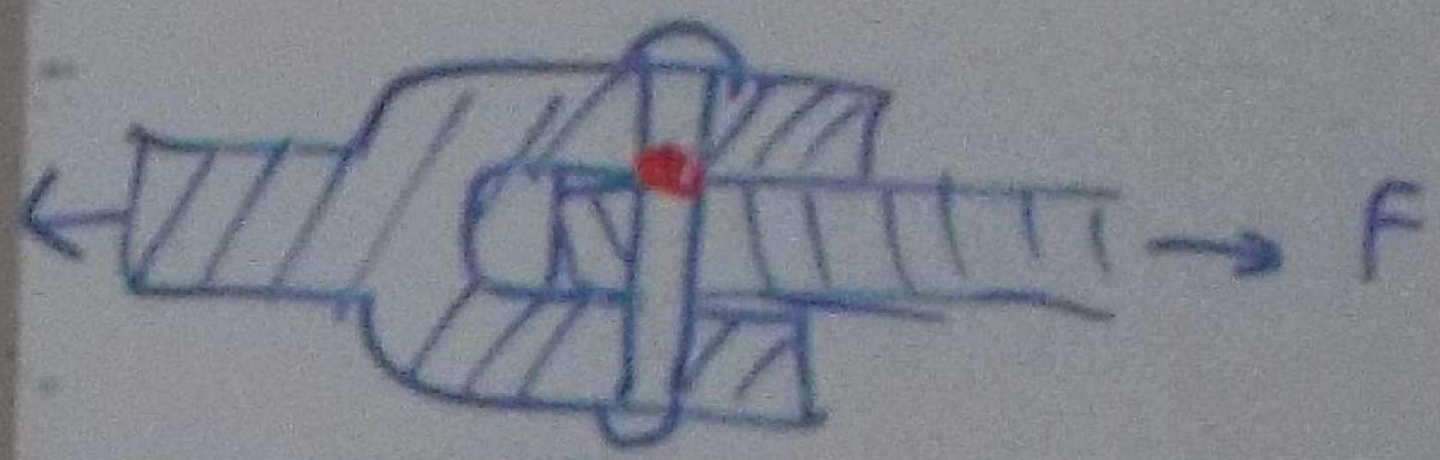


$\pi D t$ = SHEAR AREA

$3.1416 \times 12 \times 6 = 226.2\ mm^2$

F = STRESS × AREA = 360 × 226.2

= 81.4 kN

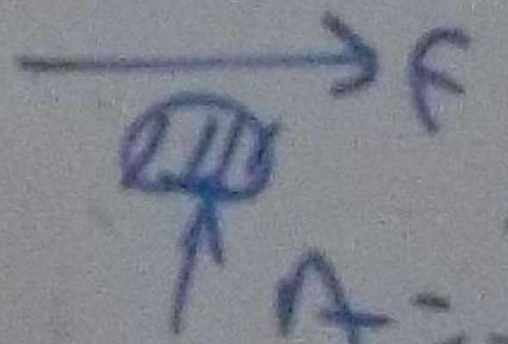


$$f = \frac{F}{A}$$

STEEL PIN DIAMETER = 10 mm

$F_{max} = 11.3\,kN$

SHEAR STRESS, SF = ?



$$A = \frac{\pi}{4} D^2 = \frac{3.1416}{4} \times 10^2 \times 2 = 157\,mm^2$$

$$f = \frac{F}{A} = \frac{11.3 \times 10^3}{157} = 72\,MPa$$

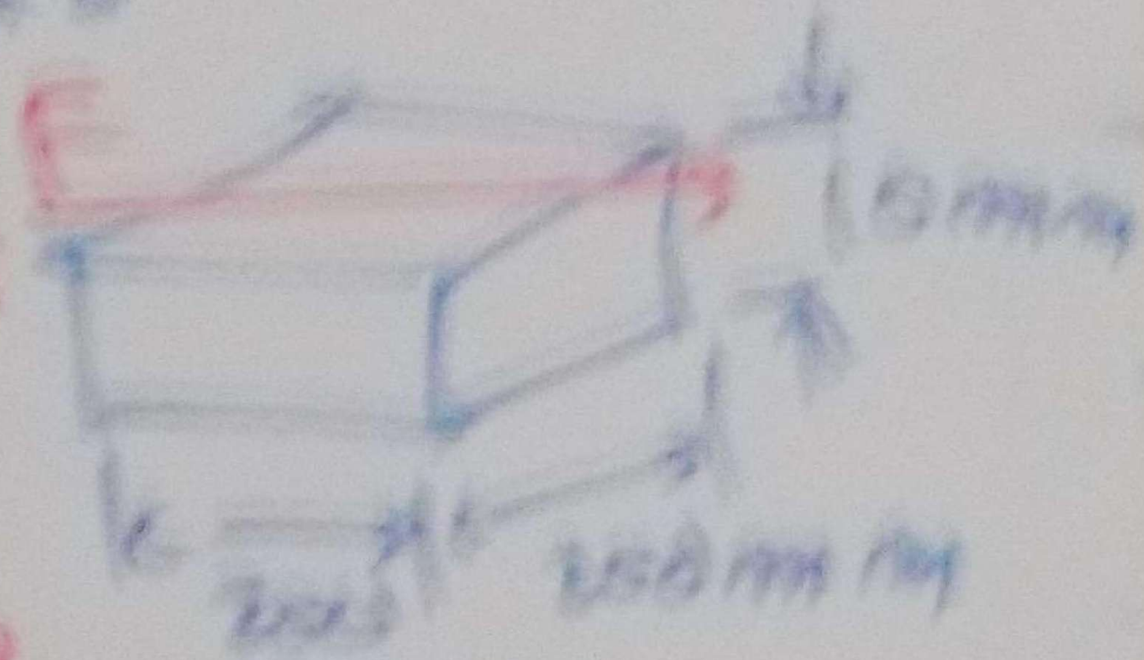
ULTIMATE SHEAR STRESS = 360 MPa

USS

$$Sf = \frac{USS}{f} = \frac{360}{72} = 5$$

Q4 ANTI VIBRATION MOUNTING FOR MACHINE

RUBBER PAD



HORIZONTAL FORCE (600N)

SHEAR DEFORMATIONS? $G = 1.5$ MPa

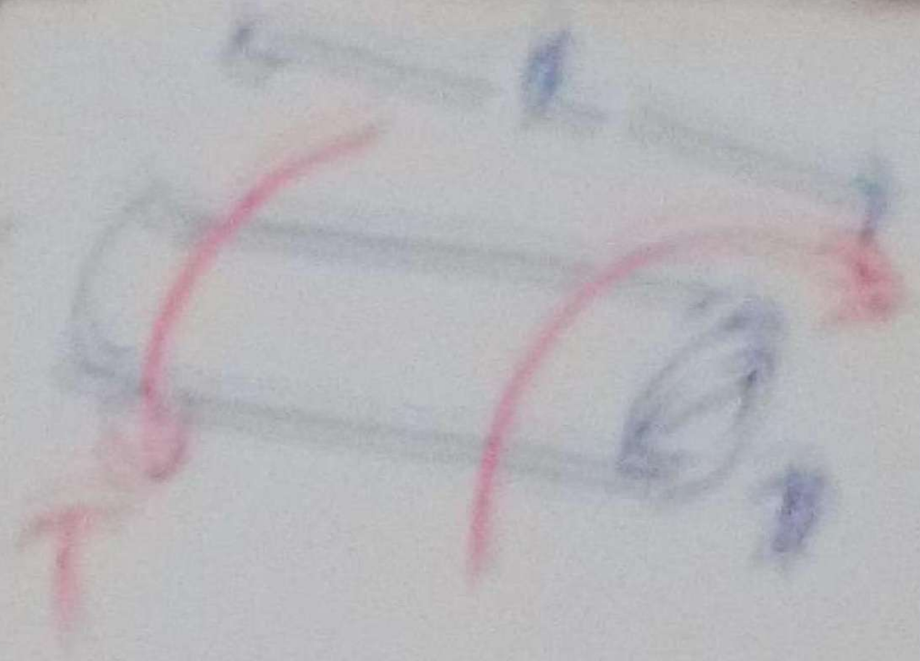
$$f = \frac{F}{A} = \frac{600}{200 \times 200} = 0.015 \text{ MPa}$$

$$G = \frac{f}{e} \rightarrow e = \frac{f}{G} = \frac{0.015}{1.5} = 0.01$$

$$e = \frac{x}{y} \rightarrow x = e\,y$$

$$= 0.01 \times 10$$

$$= 0.1 \text{ mm}$$



TORSIONAL STRESS

$$\tau_s = \frac{Tr}{J}$$

T = TORQUE

r = RADIUS OF CYLINDER

J = POLAR MOMENT OF INERTIA

$r = \frac{D}{2}$

τ_s = MAXIMUM TORSIONAL SHEAR STRESS

TORSIONAL SHEAR STRAIN

$$e_s = \frac{\theta r}{l}$$

θ = TOTAL ANGLE

l = LENGTH OF SHAFT

$$\theta = \frac{Tl}{JG}$$

$$J = \frac{\pi D^4}{32}$$

Q7) TORSIONAL SHEAR STRESS?

SHAFT DIAMETER 50 mm

TORQUE = 1400 N-m

$$f_{fs} = \frac{T r}{J}$$

$$J = \frac{\pi D^4}{32} = \frac{3.1416 \times 50^4}{32} = 613.6 \times 10^3 \text{ mm}^4$$

$$r = D/2 = \frac{50}{2} = 25 \text{ mm}$$

$$f_{fs} = \frac{1400 \times 25}{613.6 \times 10^3} = 57 \text{ MPa}$$

$F_{4x} = F_4 \cos 30 = 30 \cos 30 = 26 N$ →

$F_{4y} = F_4 \sin 30 = 30 \sin 30 = 15 N$ ↓

$F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$ → + ← −

$= 0 + (+48) + (+24.7) + (+18)$

$= 48 + 24.7 + 18 = 90.7 N$ →

$F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$ ↑ + ↓ −

$= (+50) + (0) + (+24.7) + (-18)$

$= 50 + 24.7 - 18$

$= 54.7 N$ ↑

$F_y = 54.7$

$F_x = 90.7$

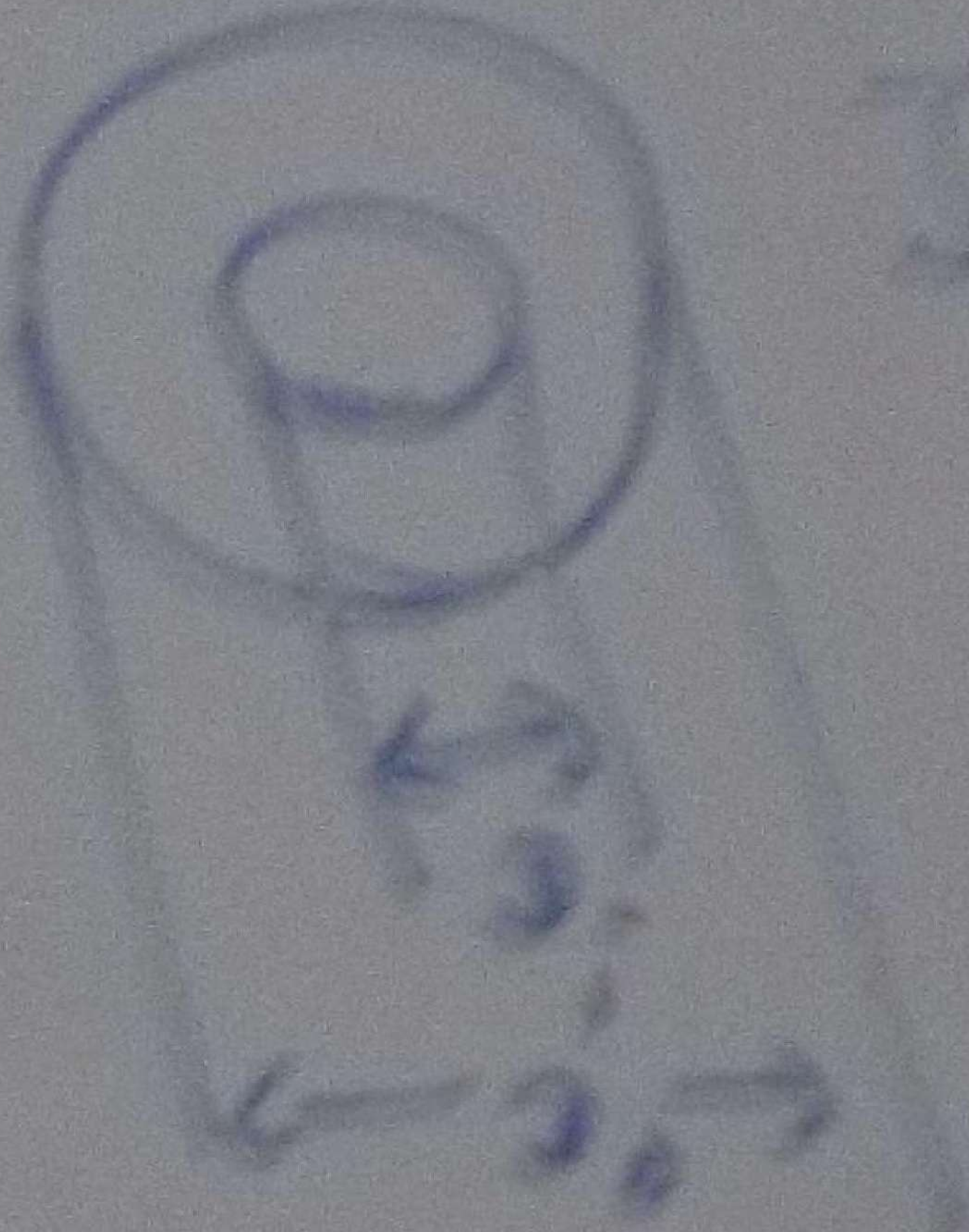
$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{90.7^2 + 54.7^2}$

$= 105.9 N$

$\theta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{54.7}{90.7} = 31.1$

HOLLOW SHAFT

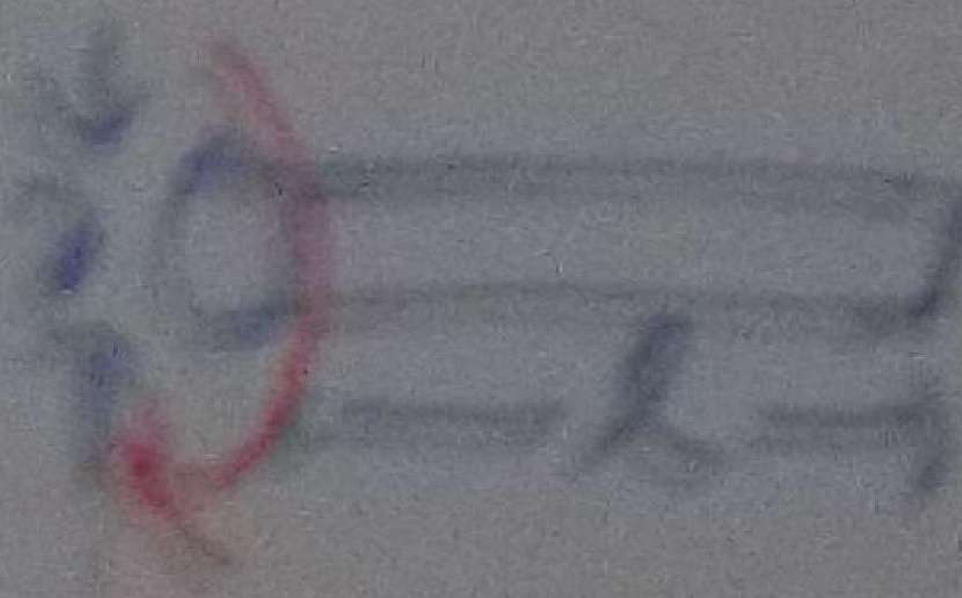


$$J = \frac{\pi (D_o^4 - D_i^4)}{32}$$

PB

$T = 1400$ N-mm

$J = 613.6 \times 10^3$



$l = 1530$ mm

$\theta = ?$

$G_{steel} = 80{,}000$

$$\theta = \frac{T l}{J G} = \frac{1400 \times 1530}{613.6 \times 10^3 \times 80{,}000}$$

$\theta = 0.04364$ RAD

$$\theta_{DEG} = \frac{0.04364 \times 180}{3.1416} = 2.5^\circ$$

$$J = \frac{\pi D^4}{32}$$

Pb TORSIONAL SHEAR STRESS = ?

SHAFT DIAMETER 50mm

TORQUE = 1400 N-m

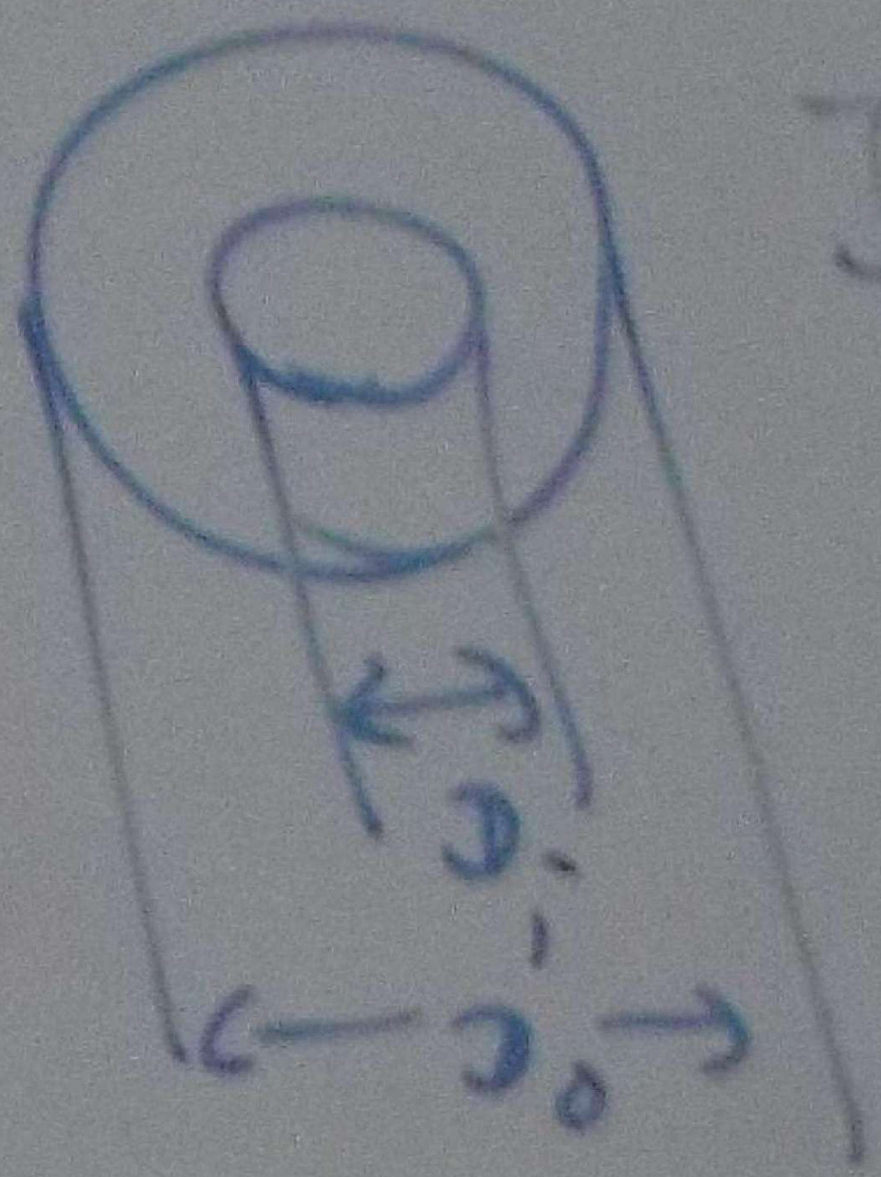
$$f_{fs} = \frac{T r}{J}$$

$$J = \frac{\pi D^4}{32} = \frac{3.1416 \times 50^4}{32} = 613.6 \times 10^3 \ mm^4$$

$$r = D/2 = \frac{50}{2} = 25 mm$$

$$f_{fs} = \frac{1400 \times 25}{613.6 \times 10^3} = 57 MPa$$

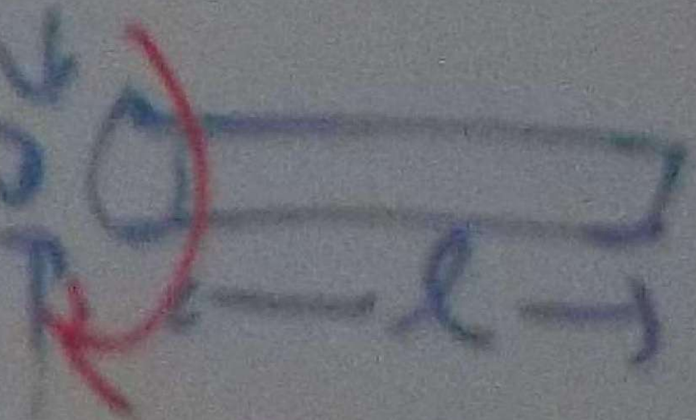
- HOLLOW SHAFT



$$J = \frac{\pi (D_o^4 - D_i^4)}{32}$$

Pb $T = 1400\ N\text{-}m$

$J = 613.6 \times 10^3$



$l = 1530\ mm$

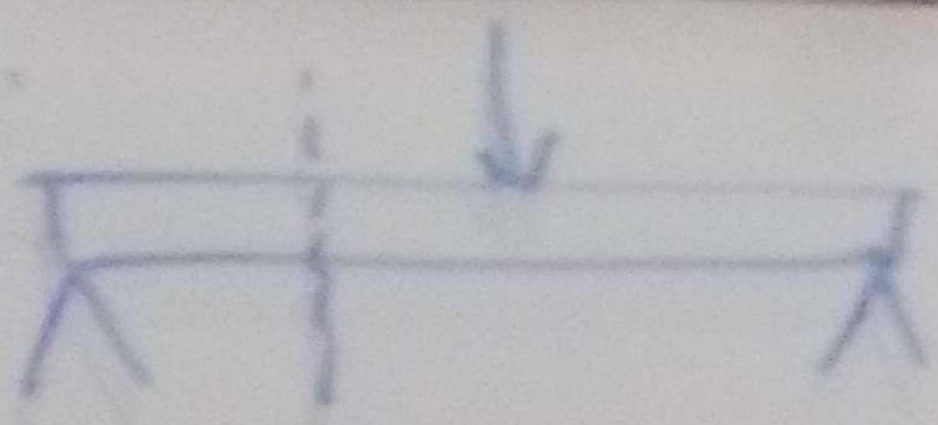
$\theta = ?$

$G_{steel} = 80{,}000$

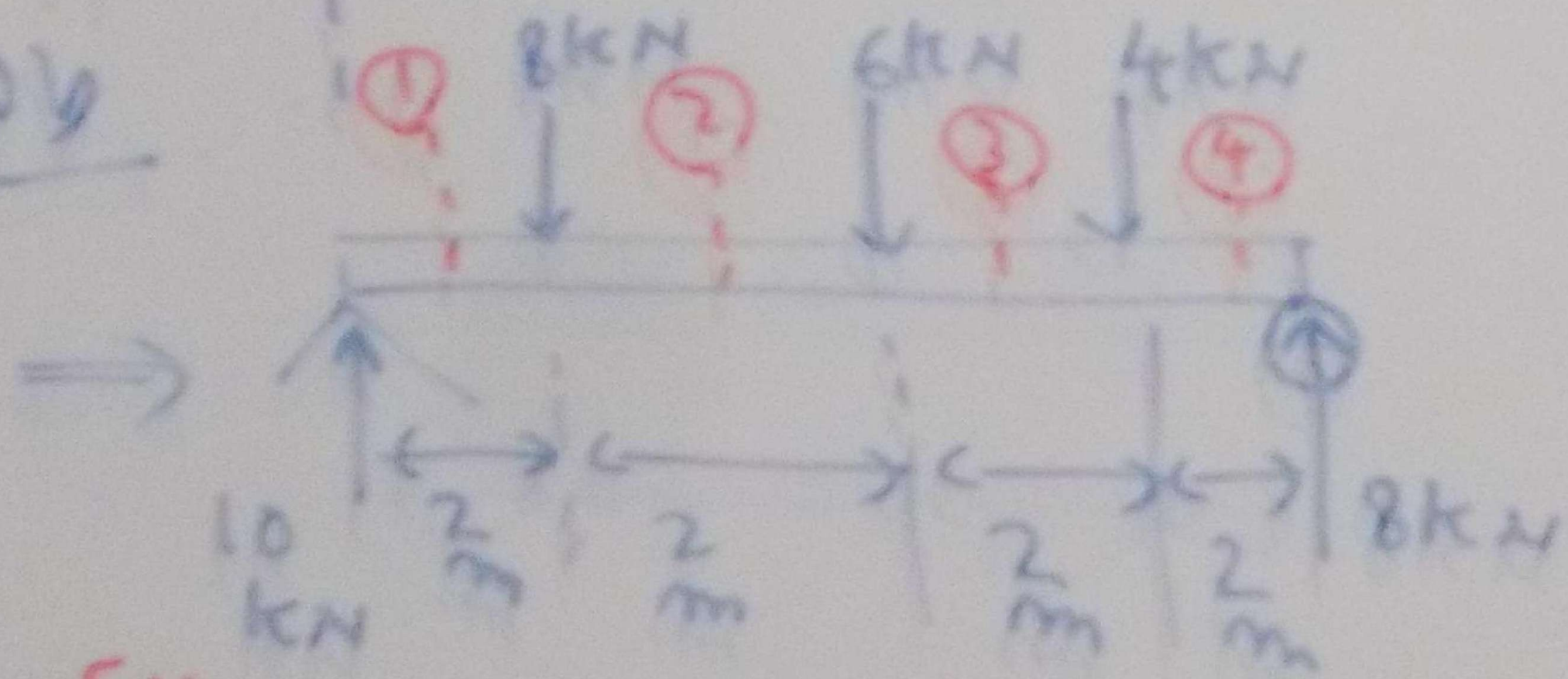
$$\theta = \frac{Tl}{JG} = \frac{1400 \times 1530}{613.6 \times 10^3 \times 80{,}000}$$

$\theta = 0.04364\ RAD$

$$\theta_{DEG} = \frac{0.04364 \times 180}{3.1416} = 2.5^\circ$$

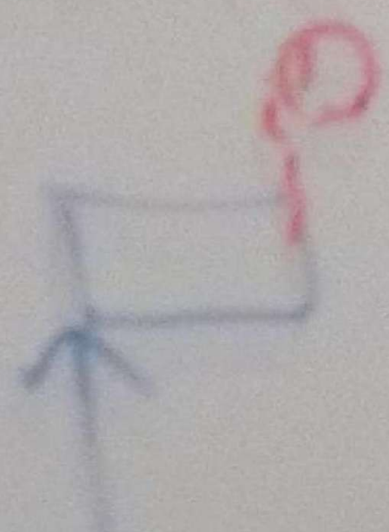


Pb



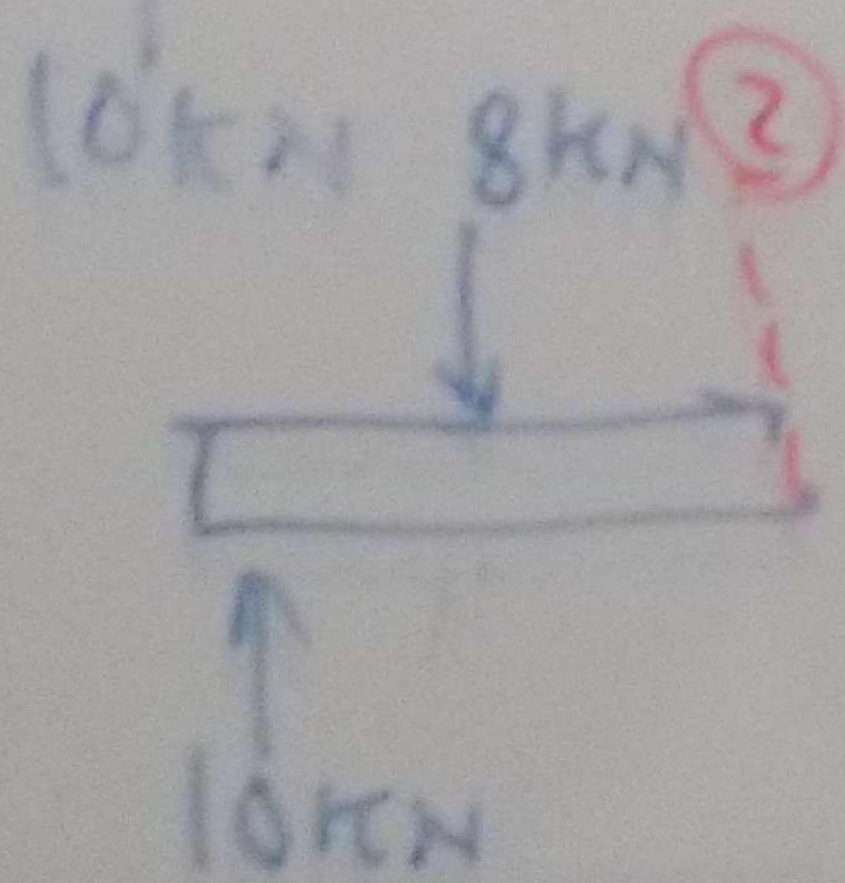
SHEAR FORCE :?

↑+



$SF_1 = \Sigma F_y = 10kN \uparrow$

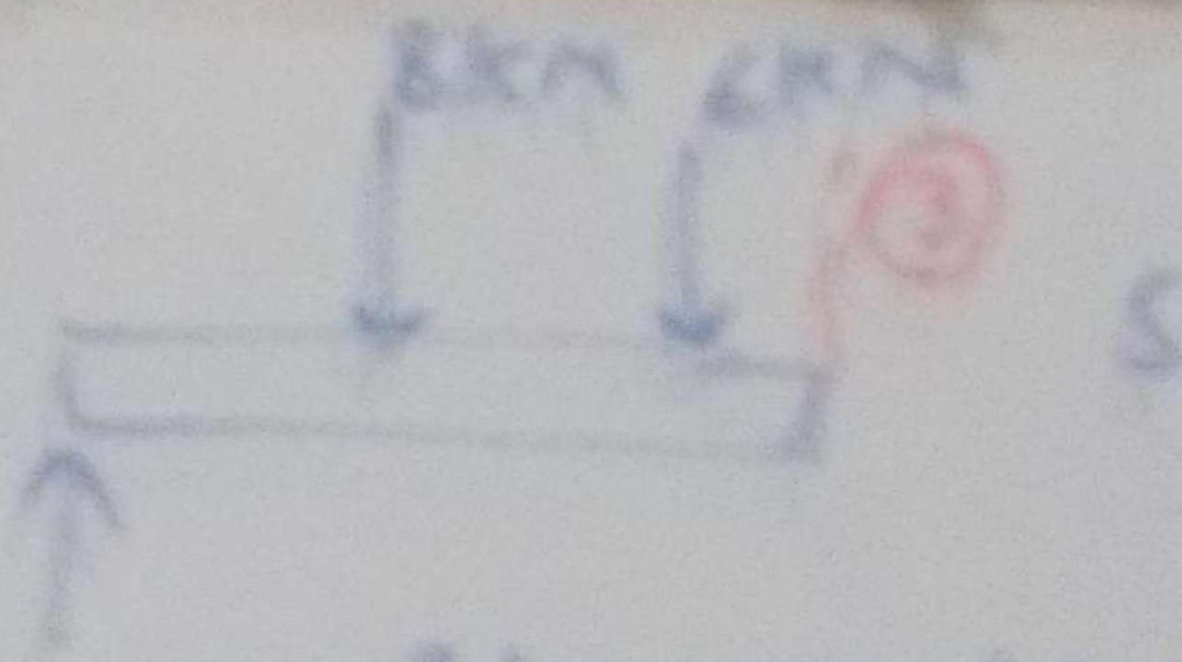
↑+ ↓−



$SF_2 = \Sigma F_y$

$= 10 - 8$

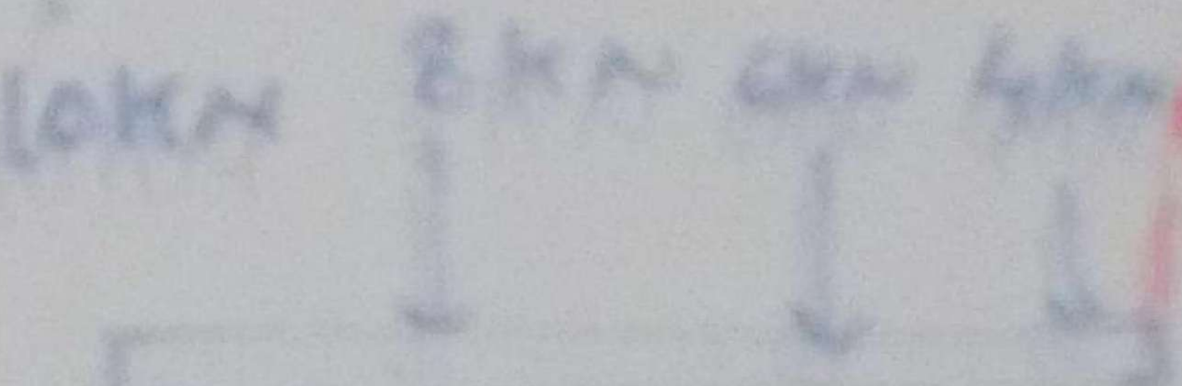
$= 2kN \uparrow$



③ $SF_B = \Sigma F_y$

$= 10 - 8 - 6$

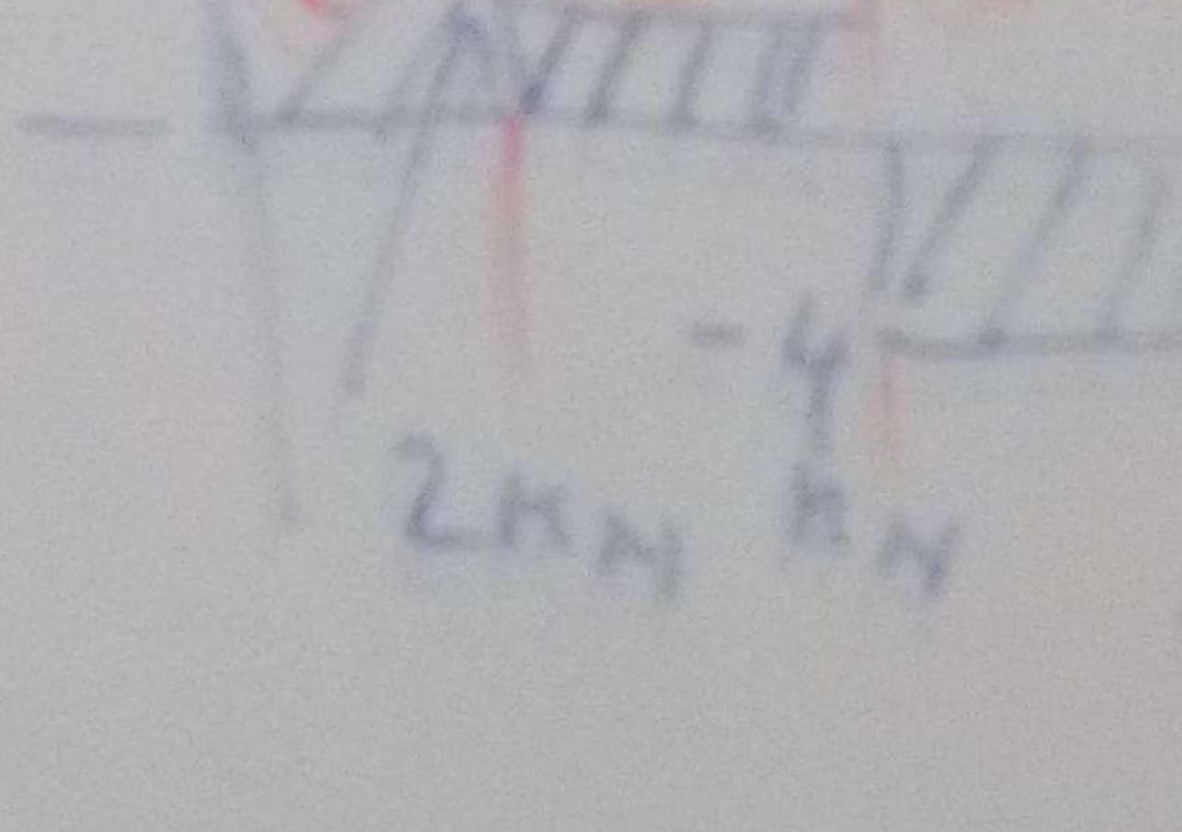
$= -4\,kN$

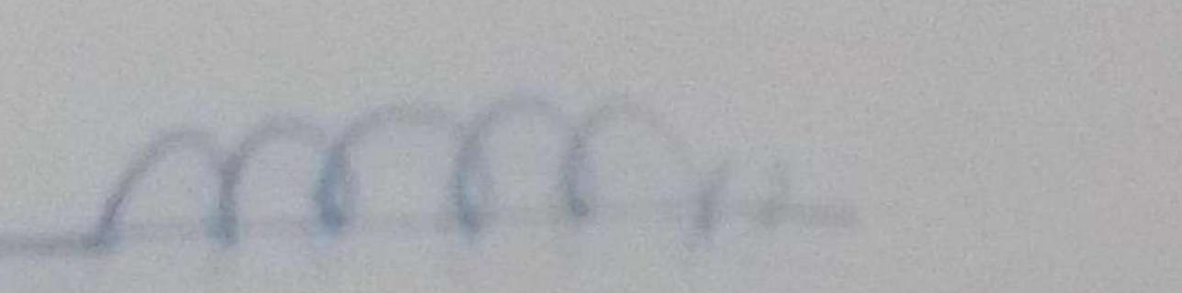


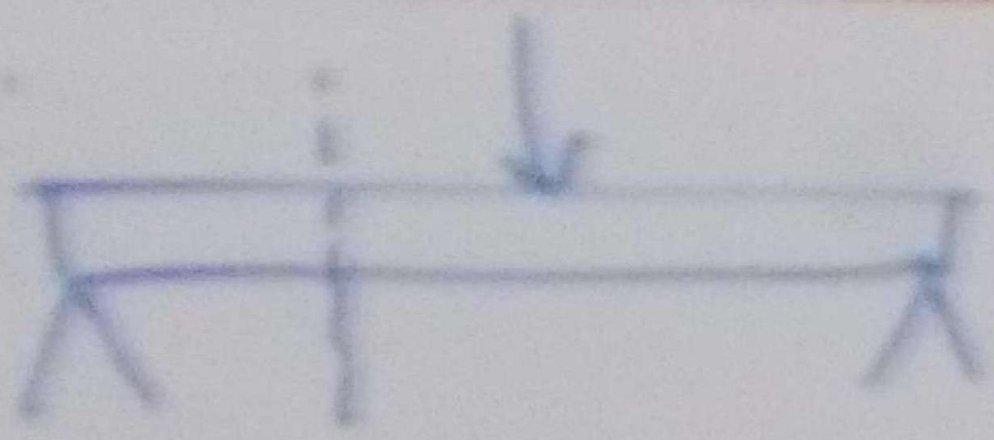
④ $SF_C = \Sigma F_y$

$= 10 - 8 - 6 - 4$

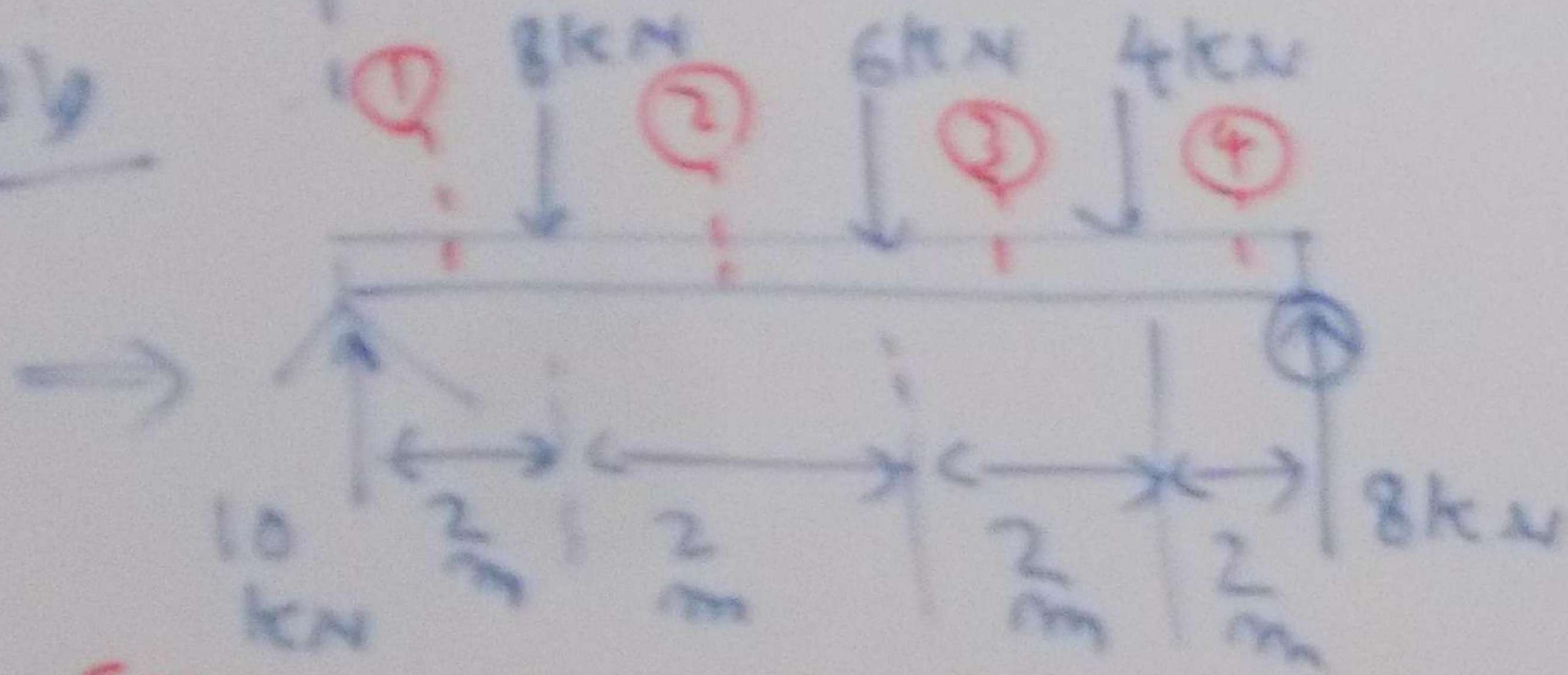
$= -8\,kN$



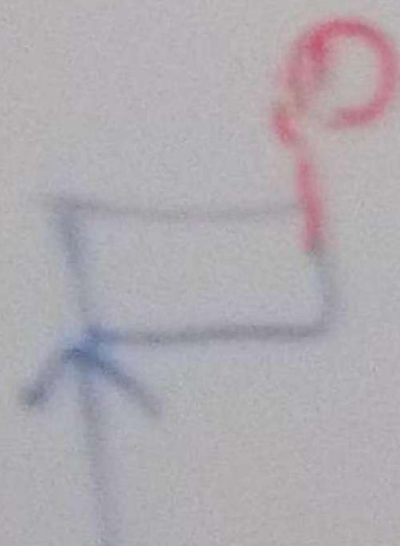




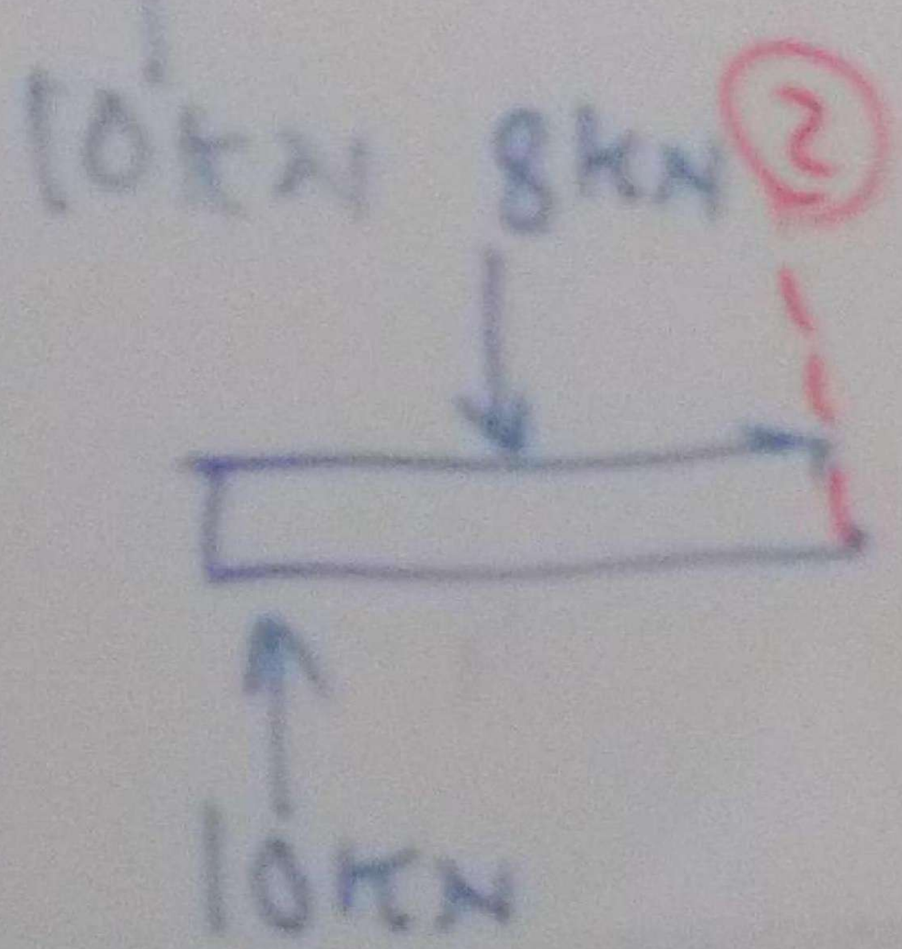
Pb



SHEAR FORCE :-



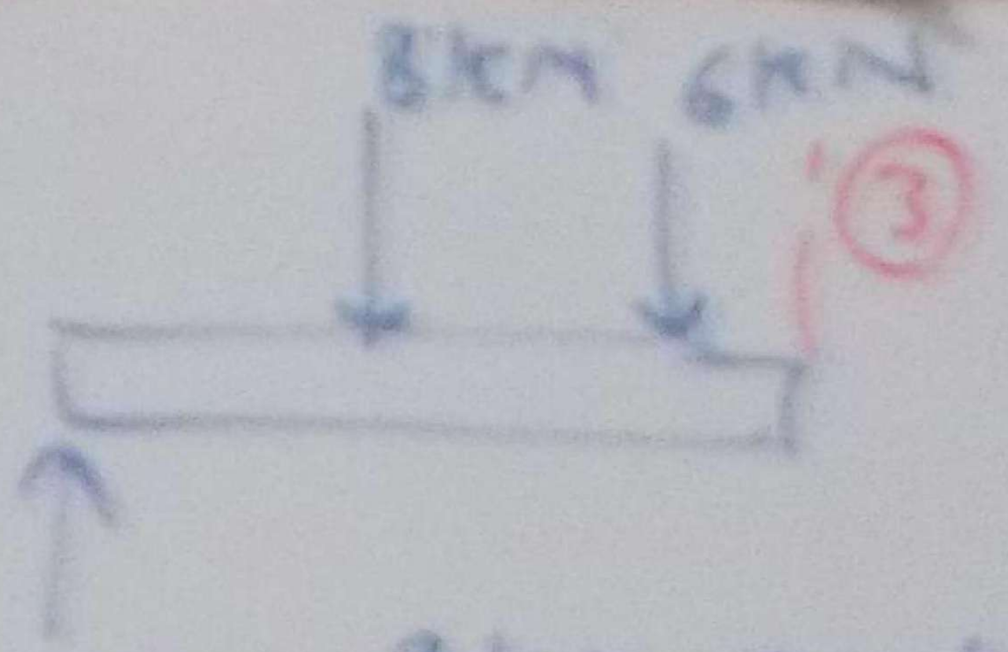
$SF_1 = \Sigma F_y = 10kN \uparrow$



$SF_2 = \Sigma F_y$

$= 10 - 8$

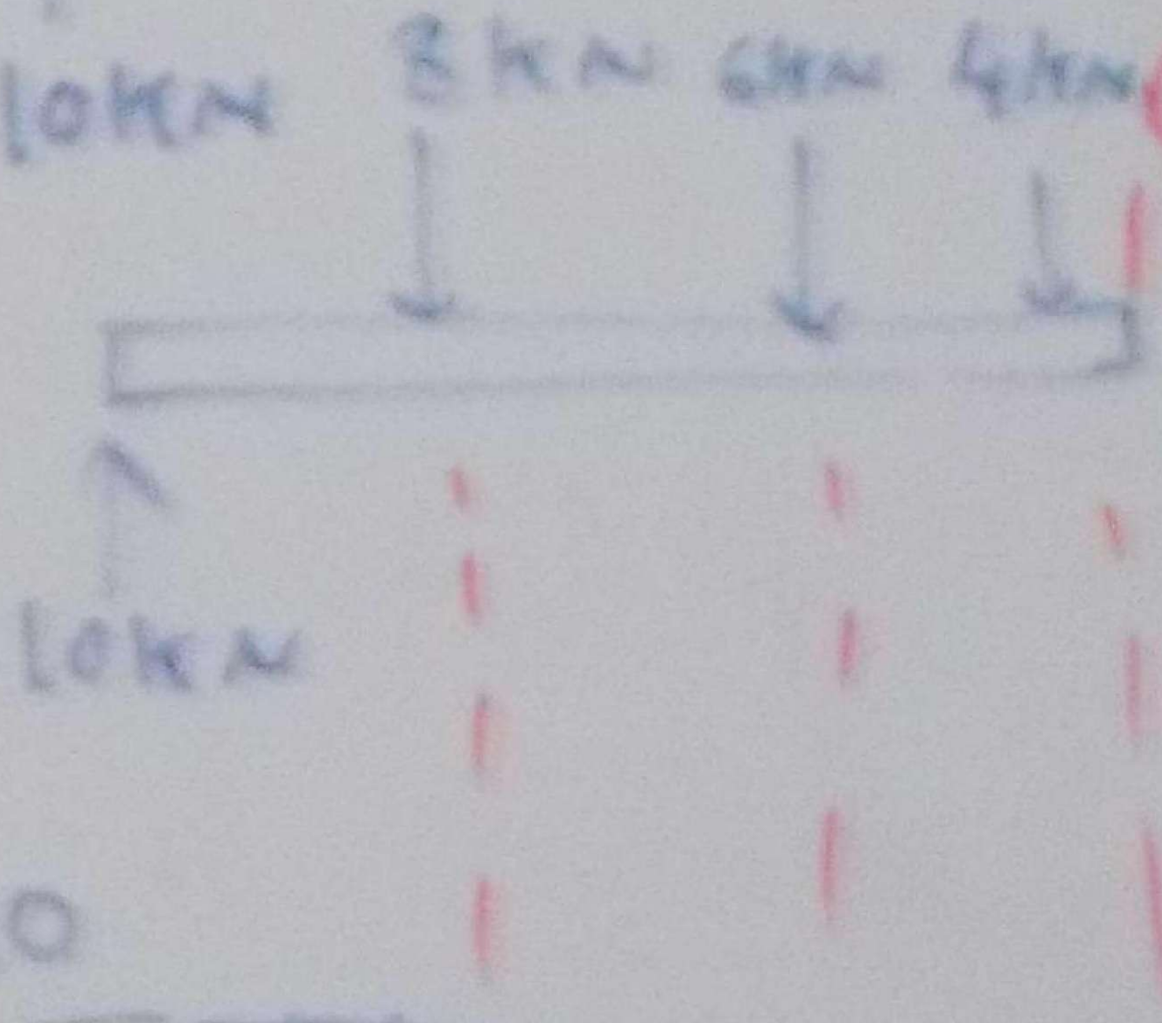
$= 2kN \uparrow$



$SF_3 = \Sigma F_y$

$= 10 - 8 - 6$

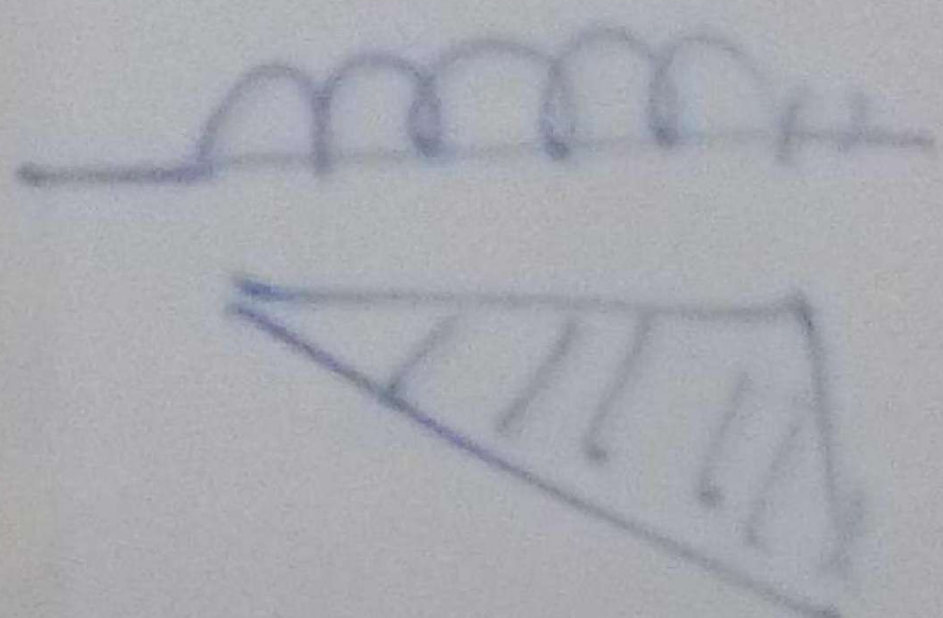
$= -4\,KN \downarrow$



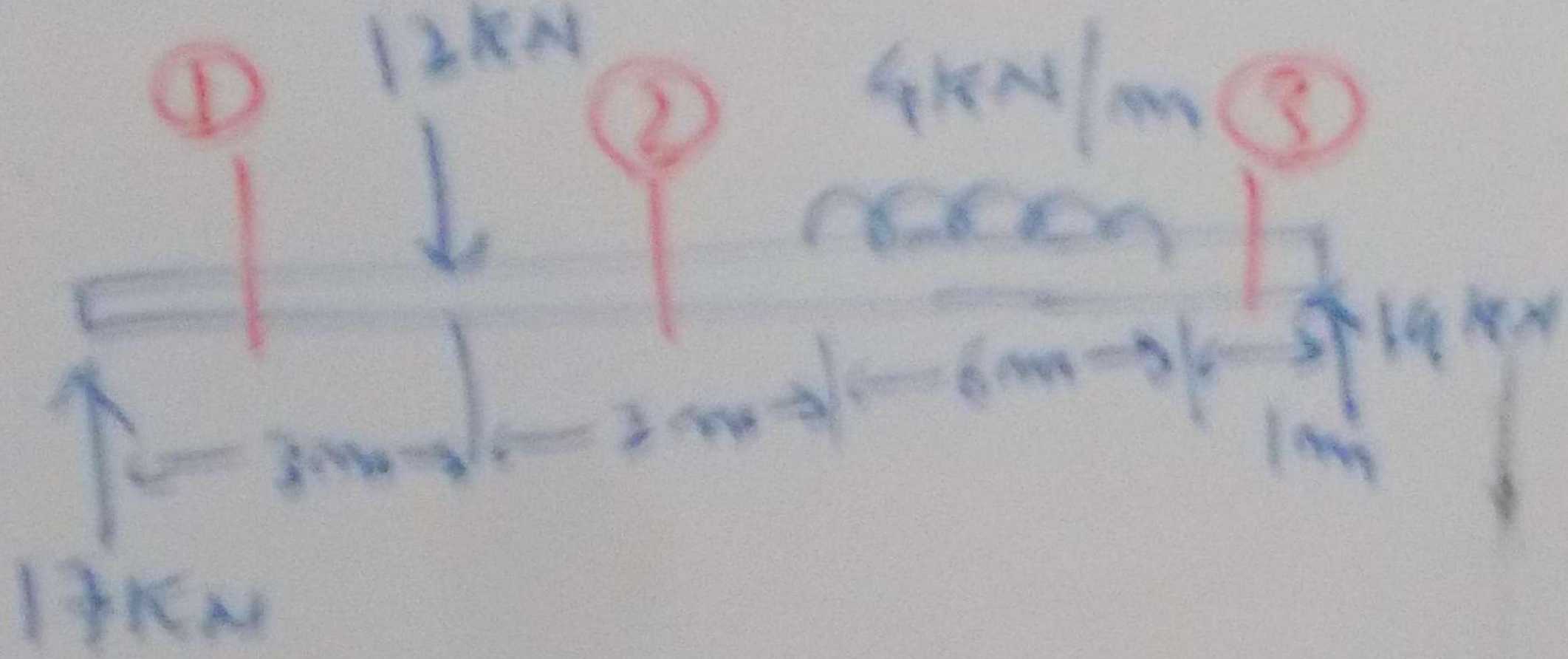
$SF_4 = \Sigma F_y$

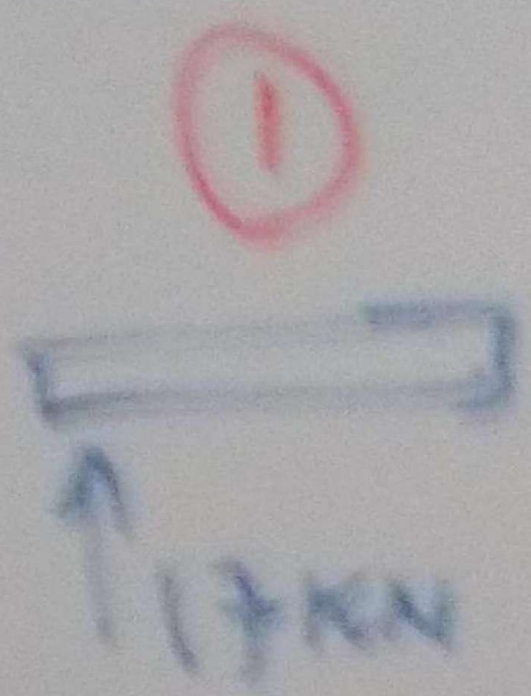
$= 10 - 8 - 6 - 4$

$= -8\,KN \downarrow$

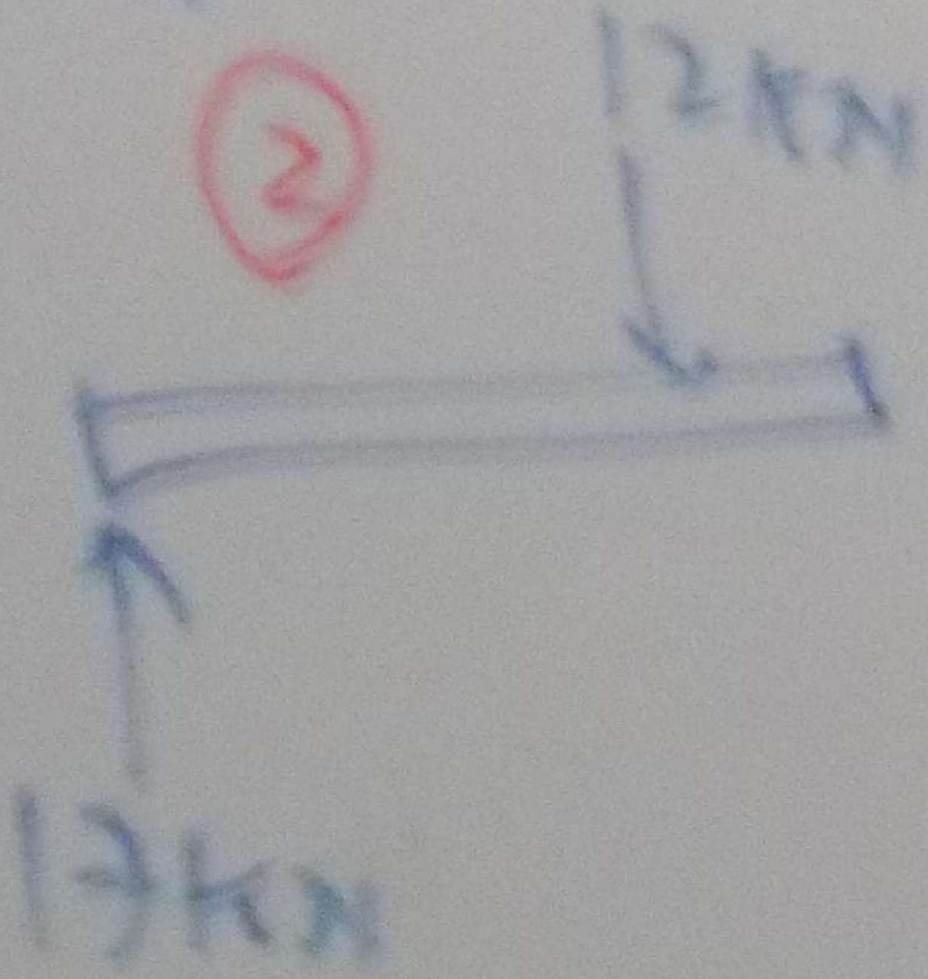


SHEAR FORCE DUE TO DISTRIBUTED LOAD





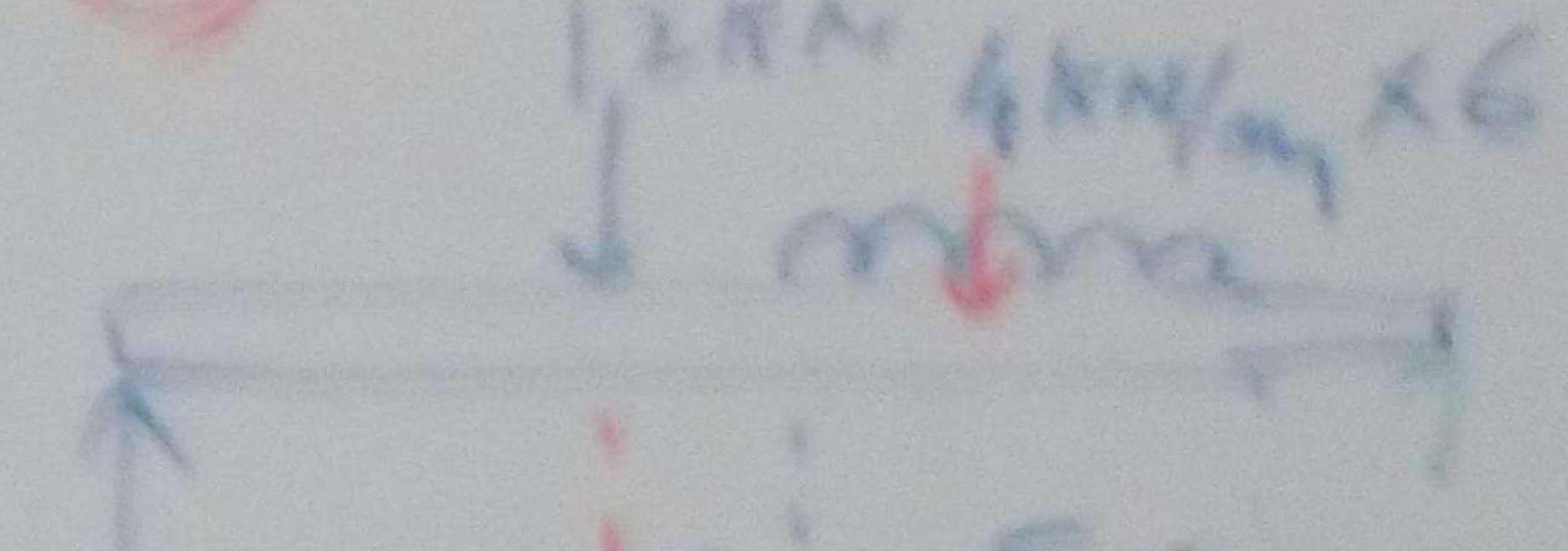
$SF_1 = \Sigma F_y = 17\,KN \uparrow$



$SF_2 = \Sigma F_y$

$= 17 - 12 = 5\,KN \uparrow$

(3)

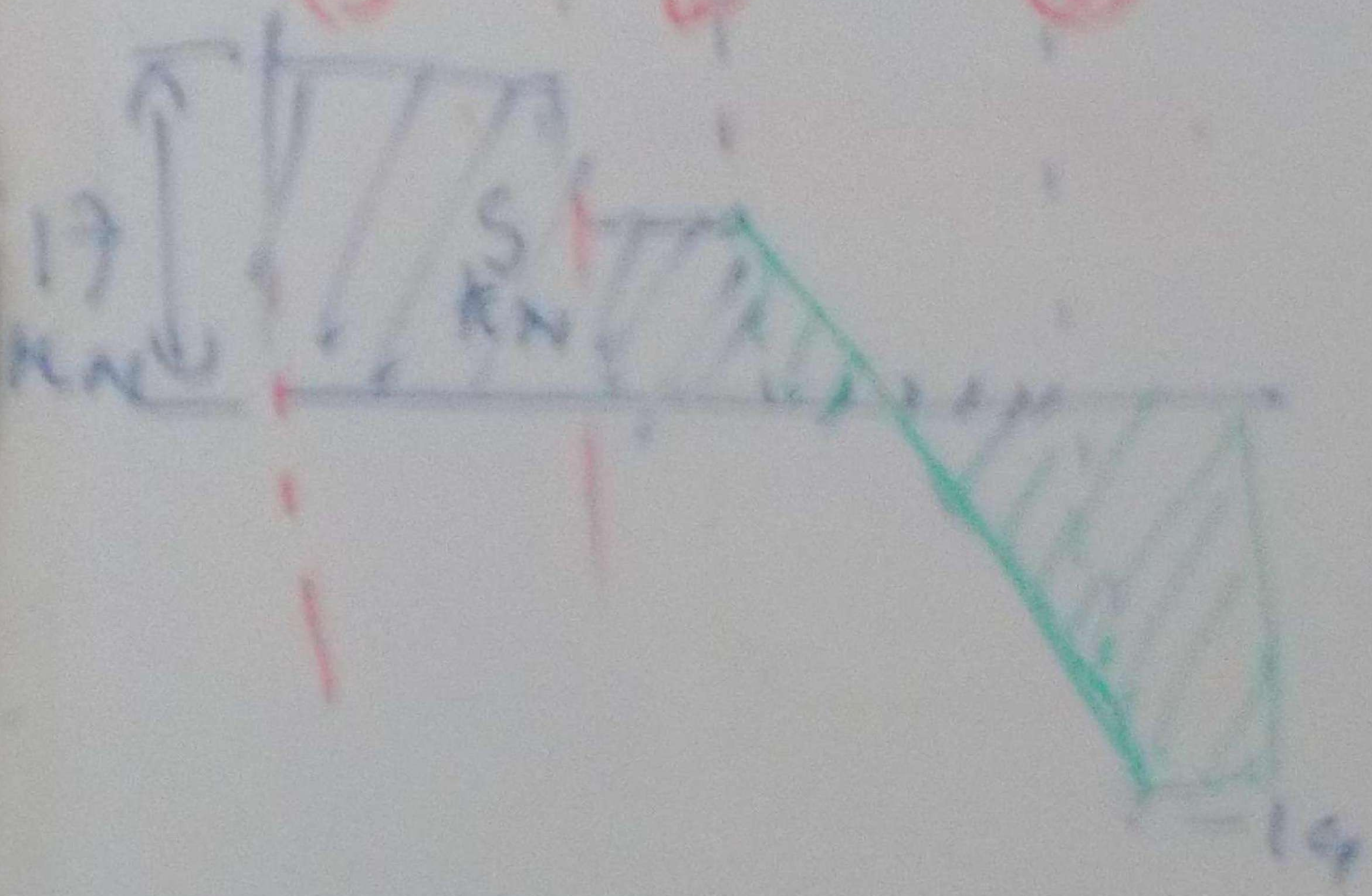


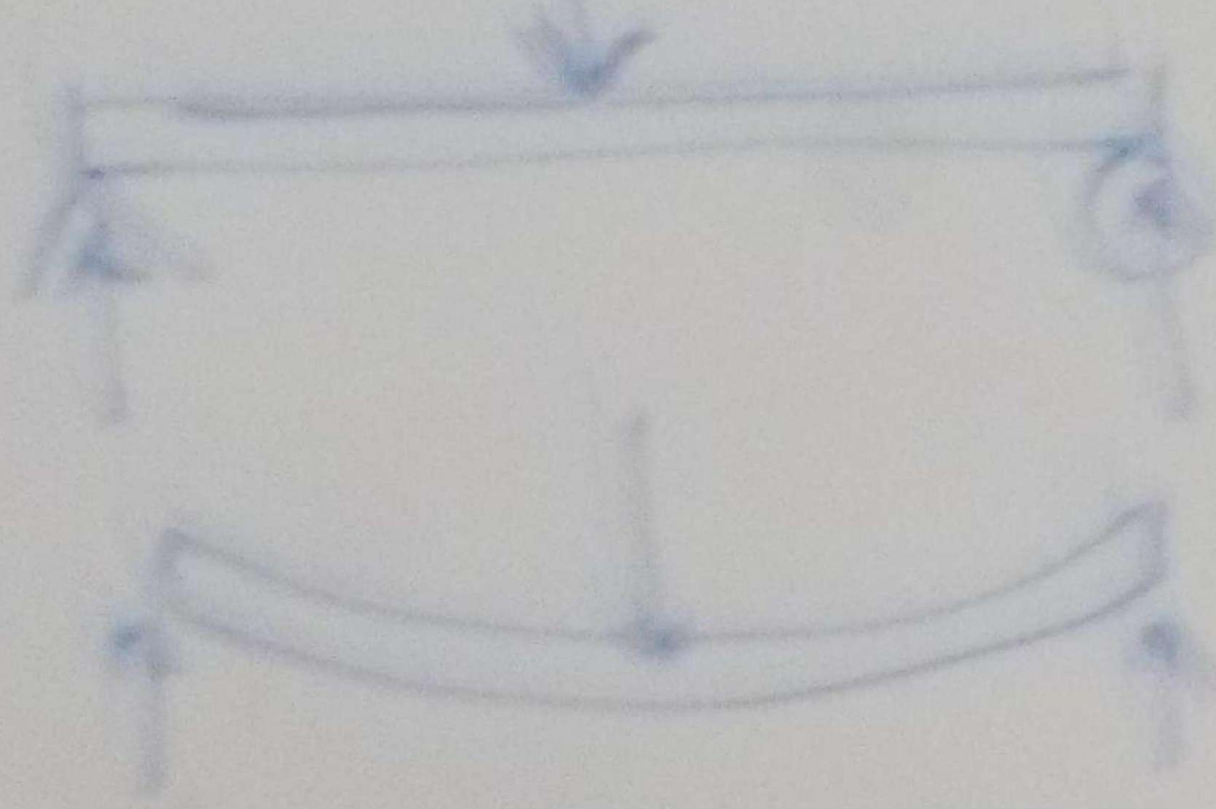
$SF_3 = \Sigma F_y$

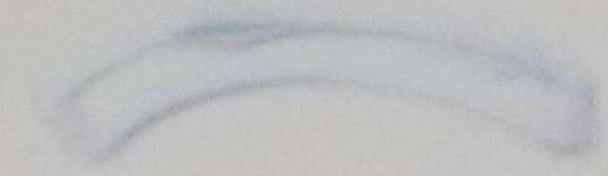
$= 17 - 12 - 4\times6$

$= -14KN$

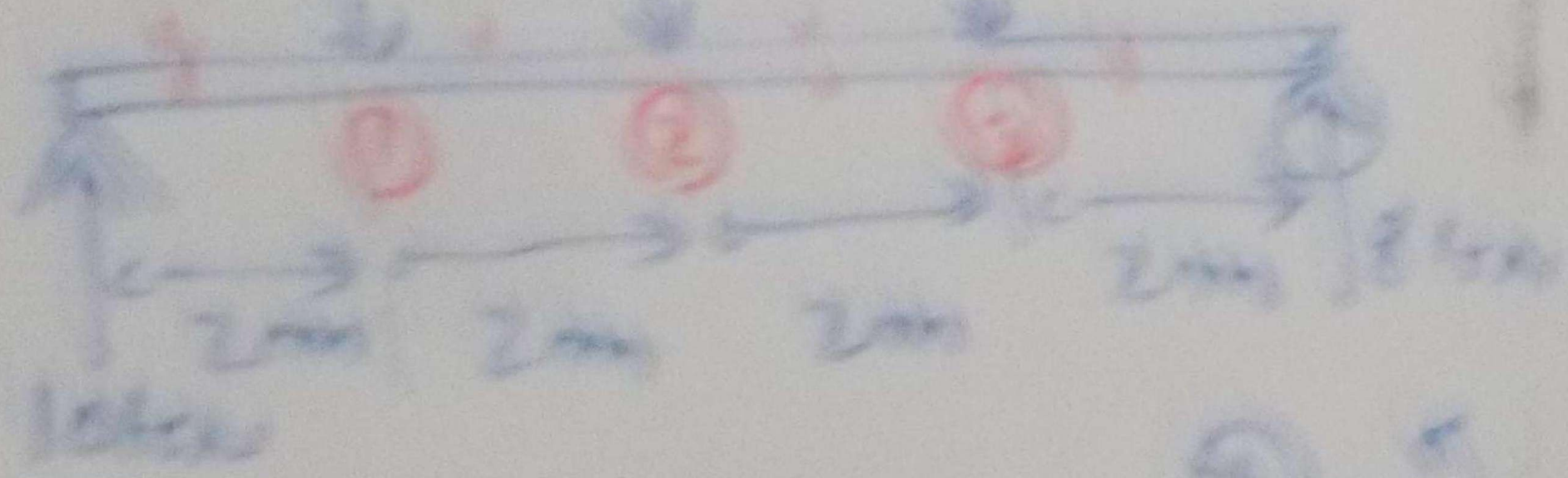
(AT END → $-14 + 14 = 0$)



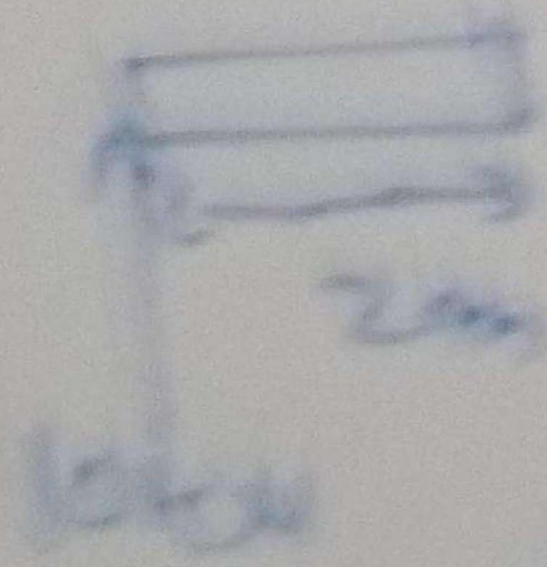




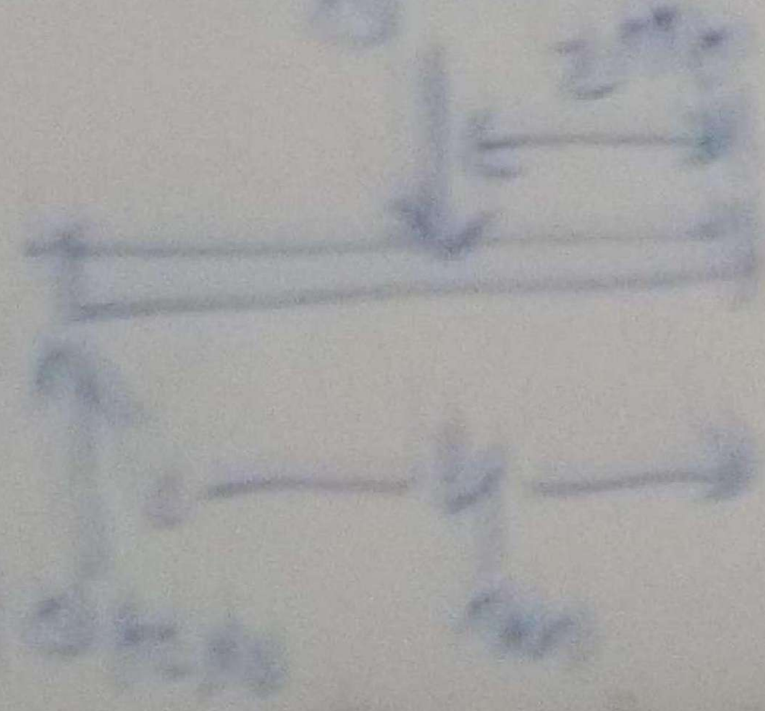
(b)



(1)



(2)



NOTE:

[illegible]

[illegible]

[illegible]

(N) [illegible] ∈ M [illegible]

[illegible] ∈ [illegible]

[illegible]

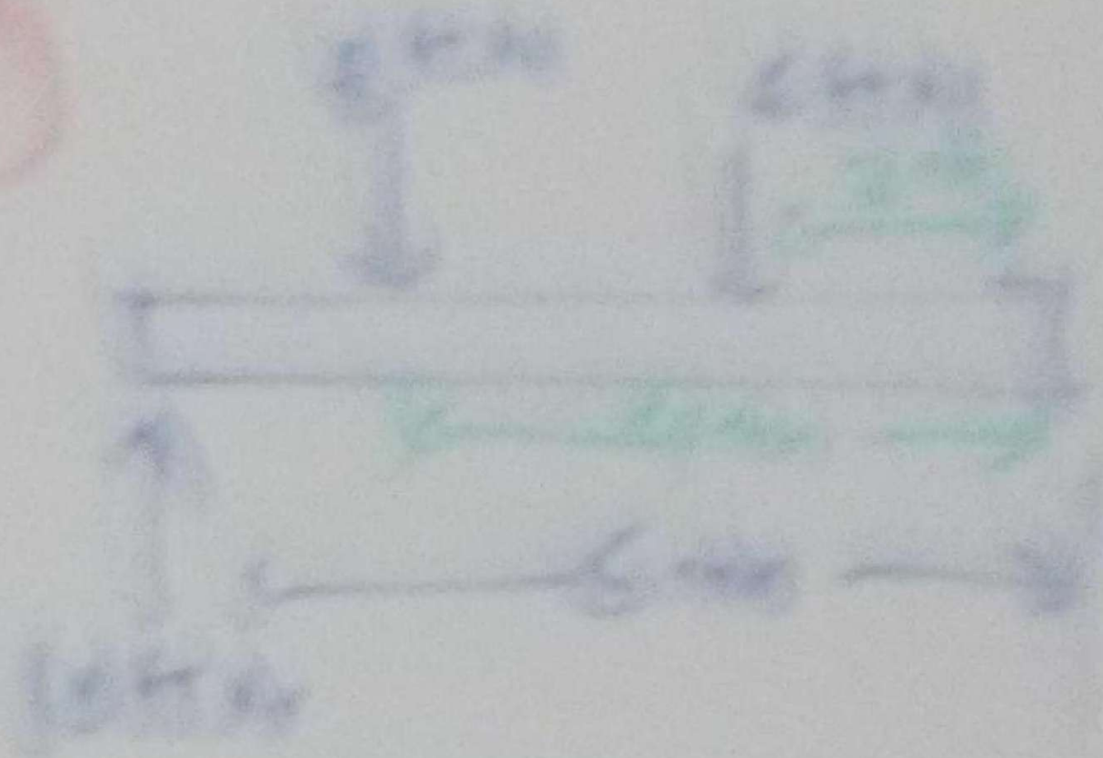
[illegible]

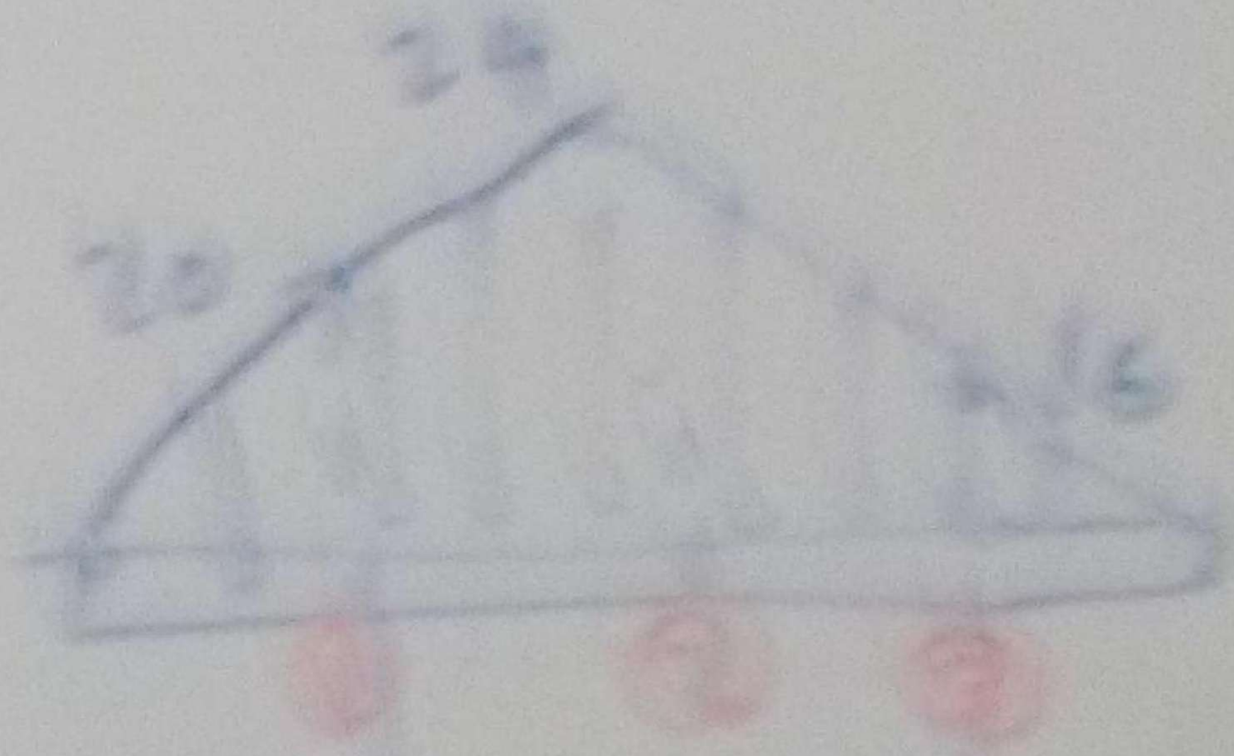
[illegible]

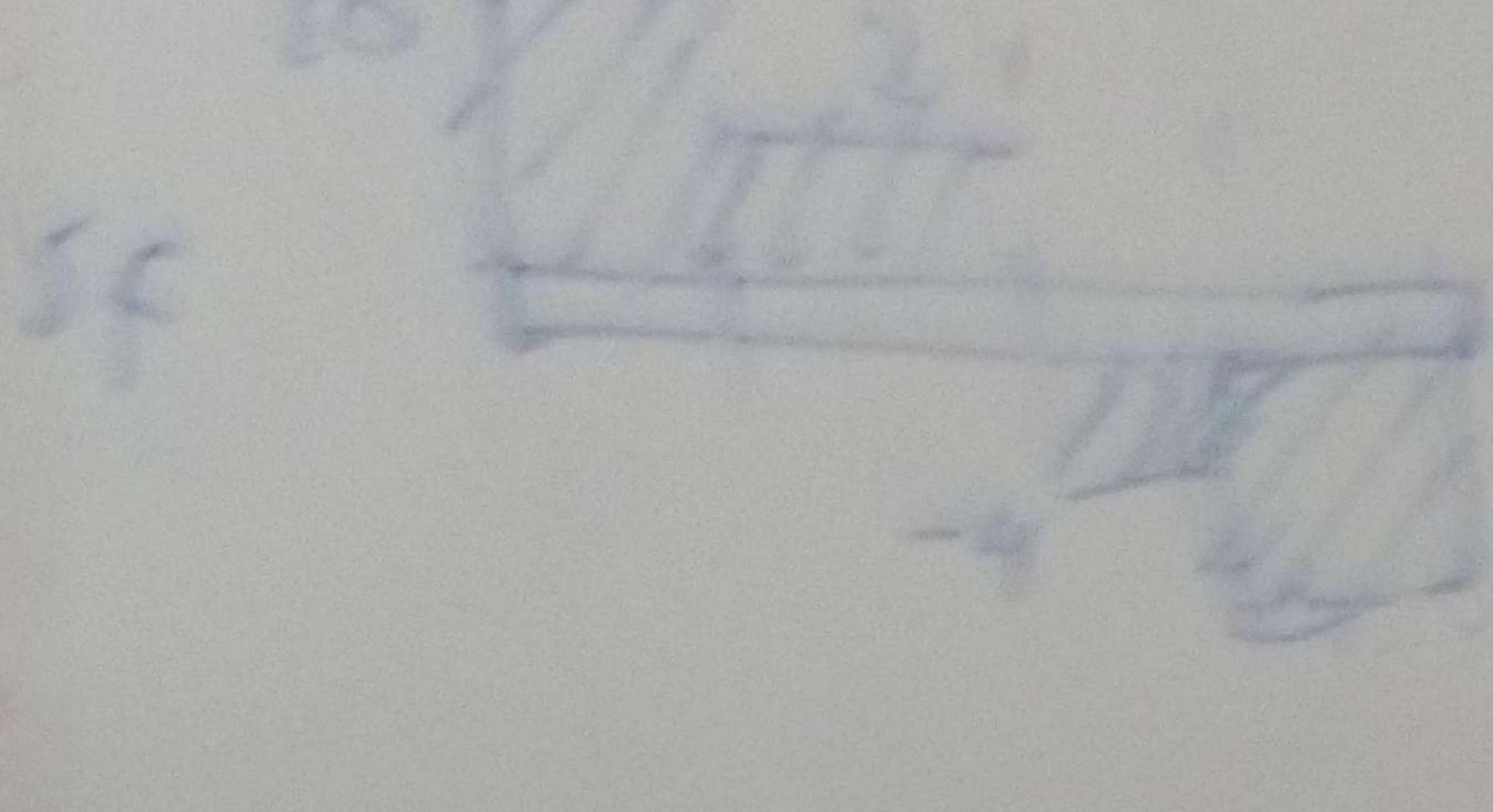
[illegible]

[illegible]

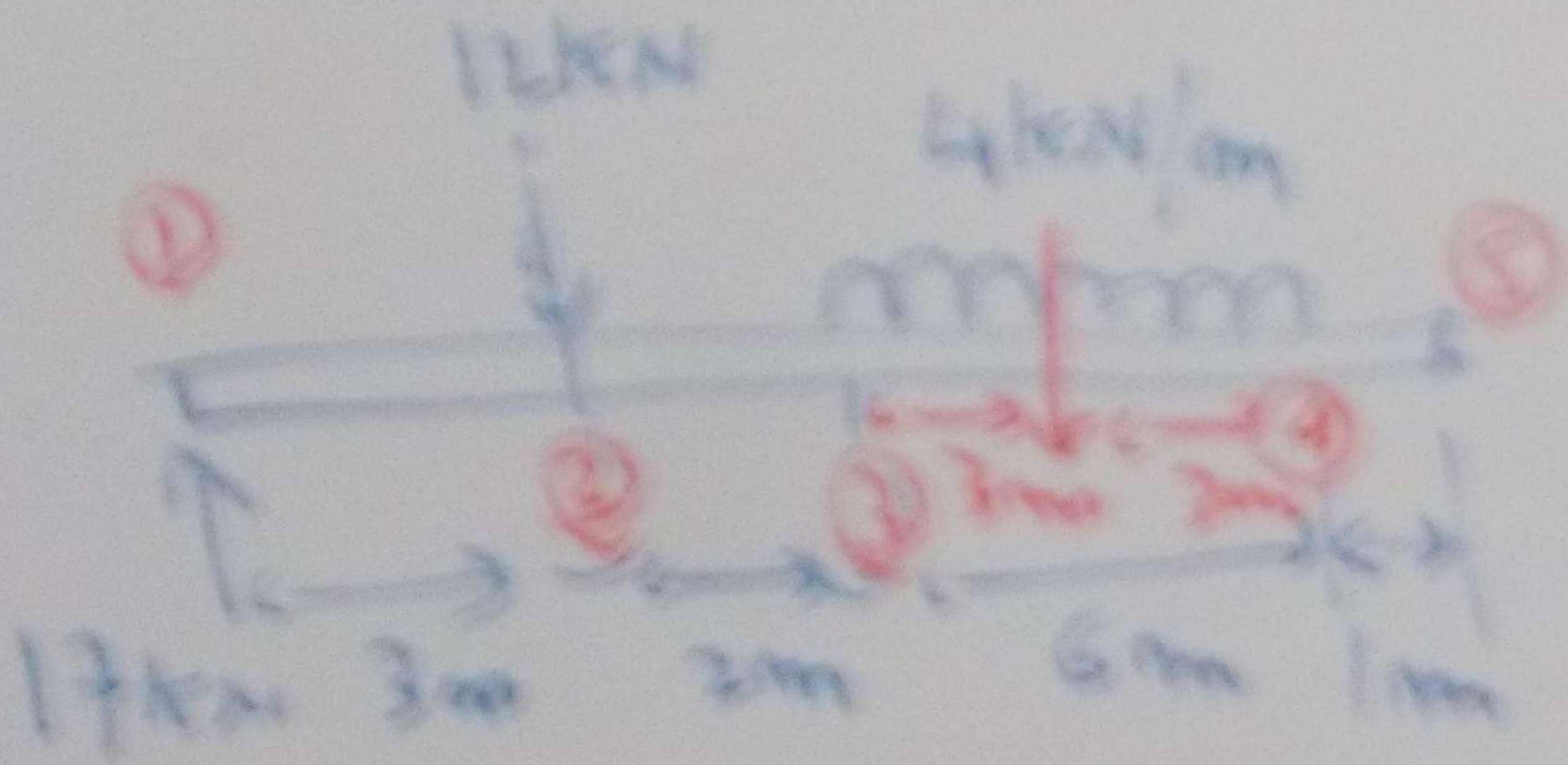
3







BENDING MOMENT DIAGRAM OF UNIFORMLY DISTRIBUTED LOAD



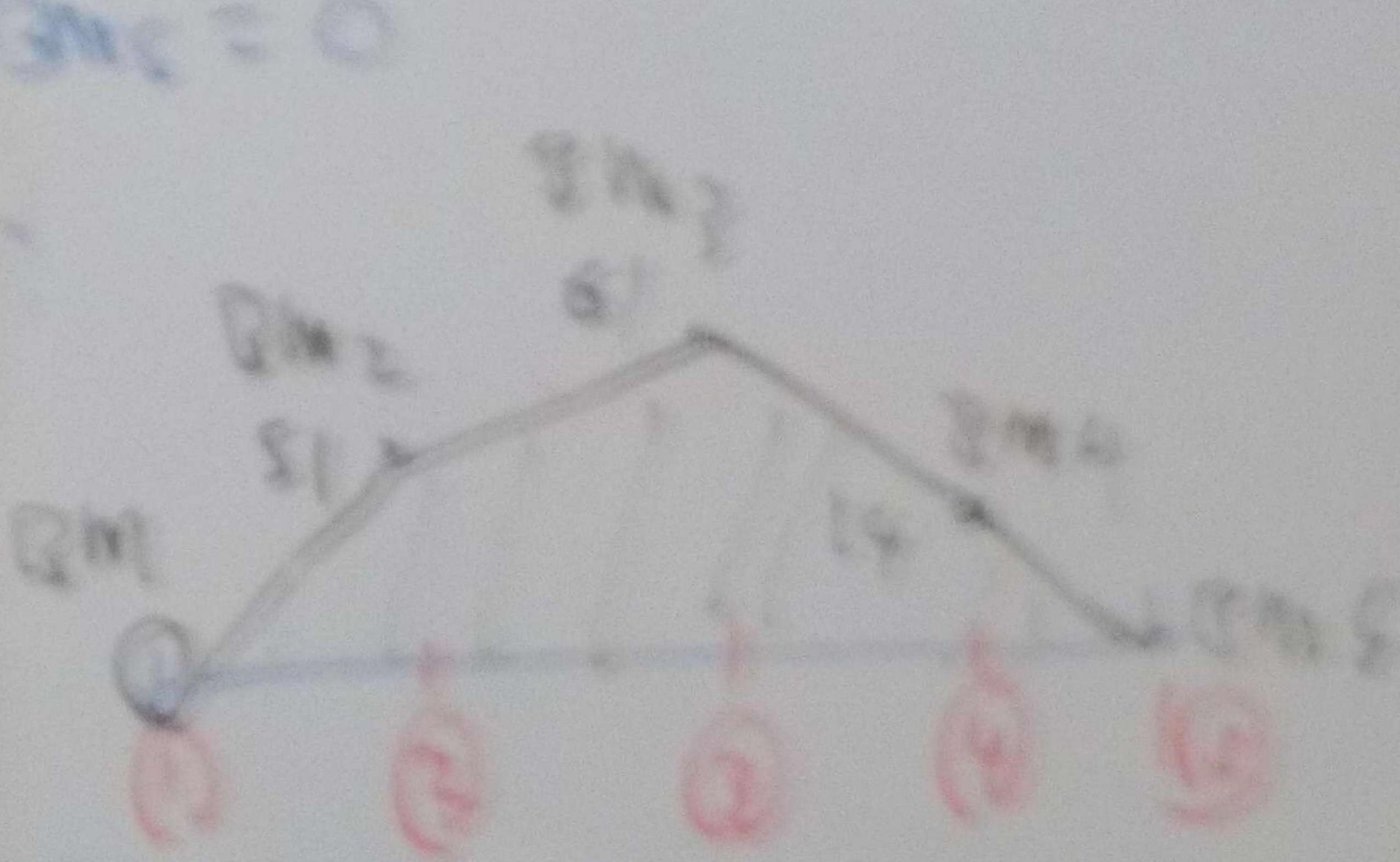
① $BM_1 = 0$

② $BM_2 = 17 \times 3 = 51$ kN-m

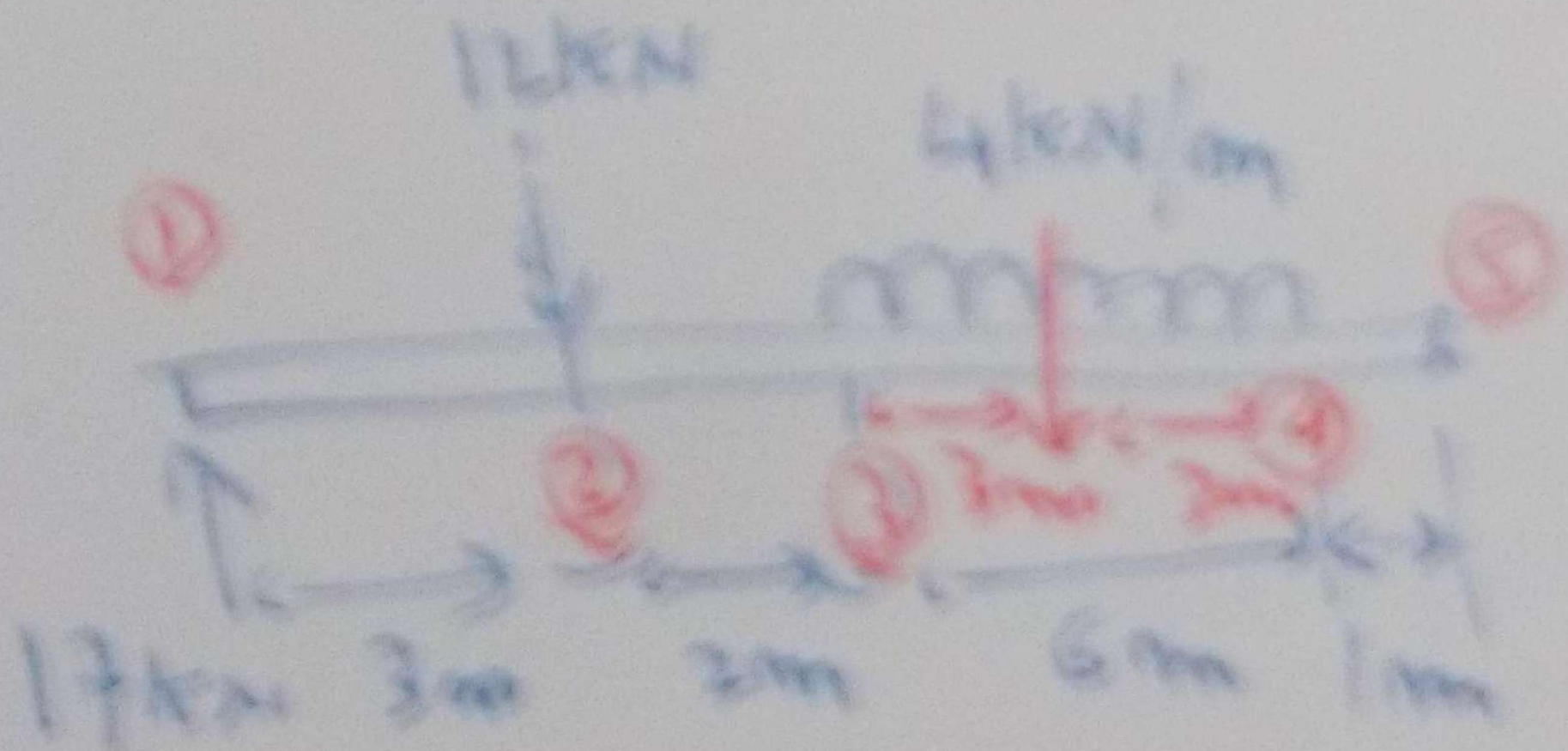
③ $BM_3 = 17 \times 5 - 12 \times 2$
$= 61$ kN-m

④ $17(3+2+6) - 12(2+6) - 4 \times 6 \times 3$

$BM_4 = 17 \times 11 - 12 \times 8 - 4 \times 6 \times 3$



BENDING MOMENT DIAGRAM OF UNIFORMLY DISTRIBUTED LOAD



① $BM_1 = 0$

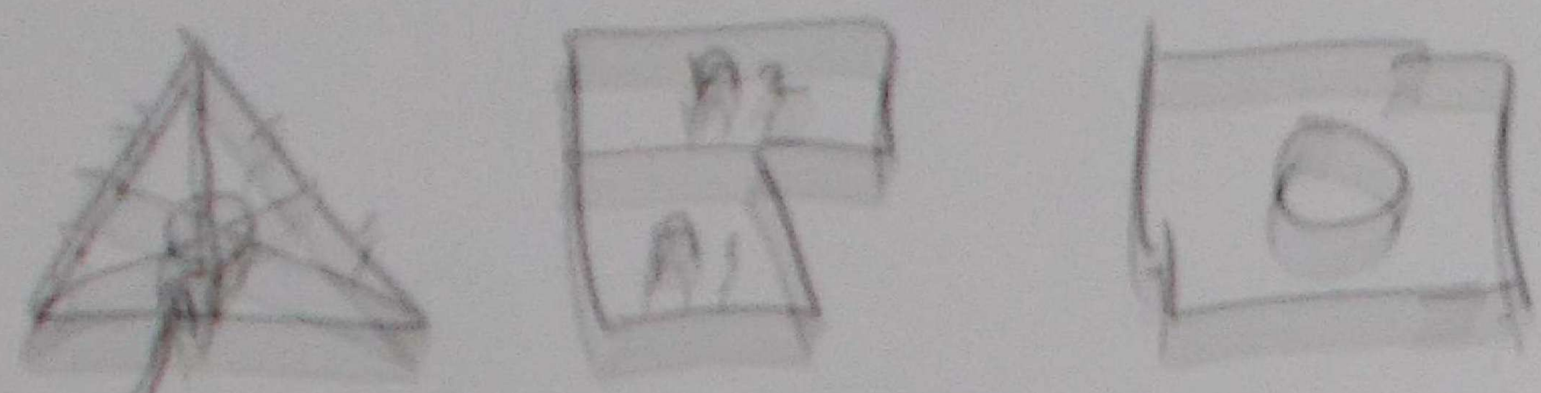
② $BM_2 = 17 \times 3 = 51$ kN-m

③ $BM_3 = 17 \times 5 - 12 \times 2$
$= 61$ kN-m

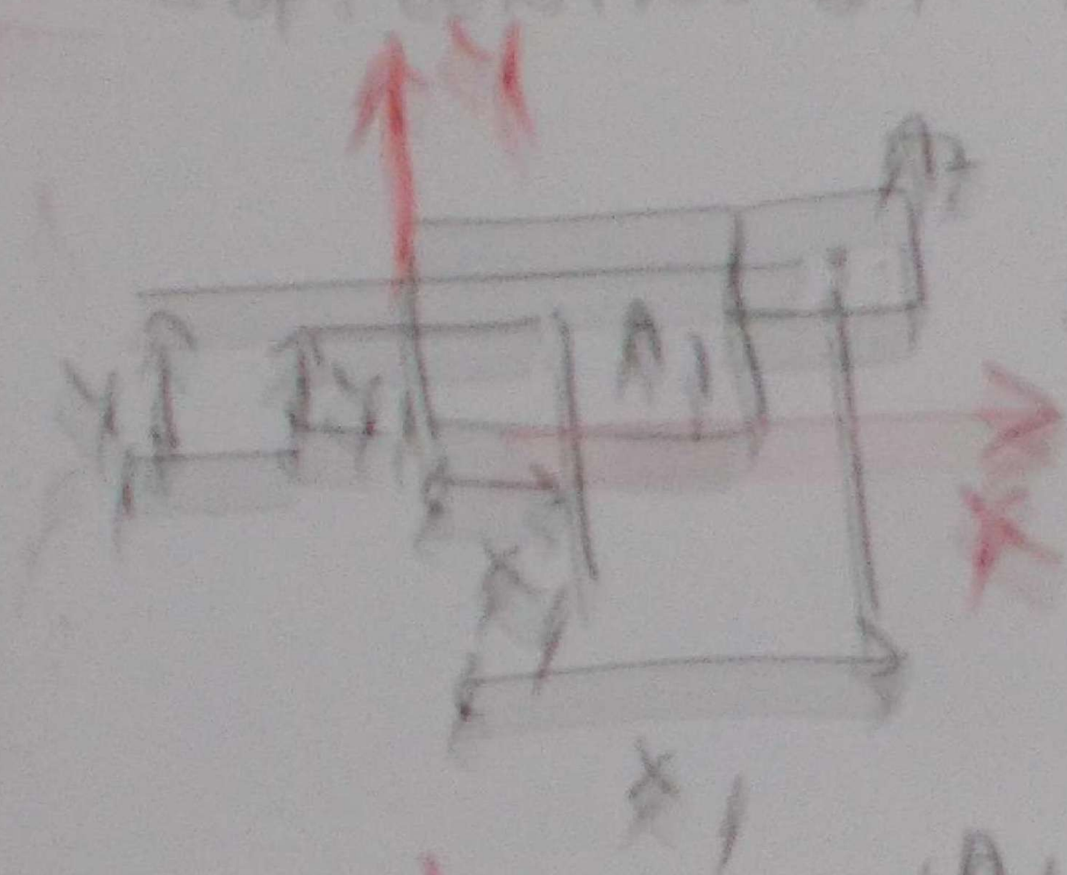
④ $17(3+2+6) - 12(2+6) - 4 \times 6 \times 3$

$BM_4 = 17 \times 11 - 12 \times 8 - 4 \times 6 \times 3$

BENDING STRESSES

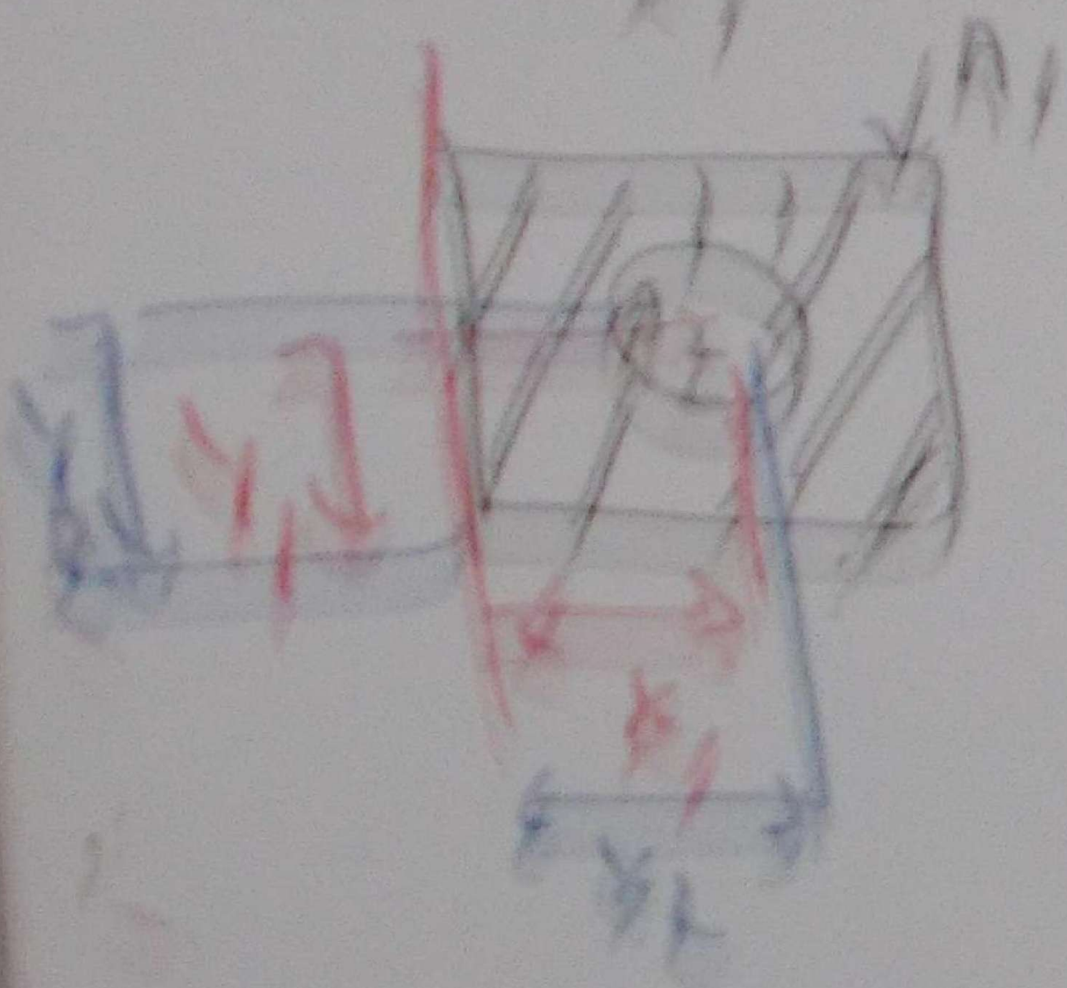


CG = CENTRE OF GRAVITY



$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{\sum A x}{\sum A}$$

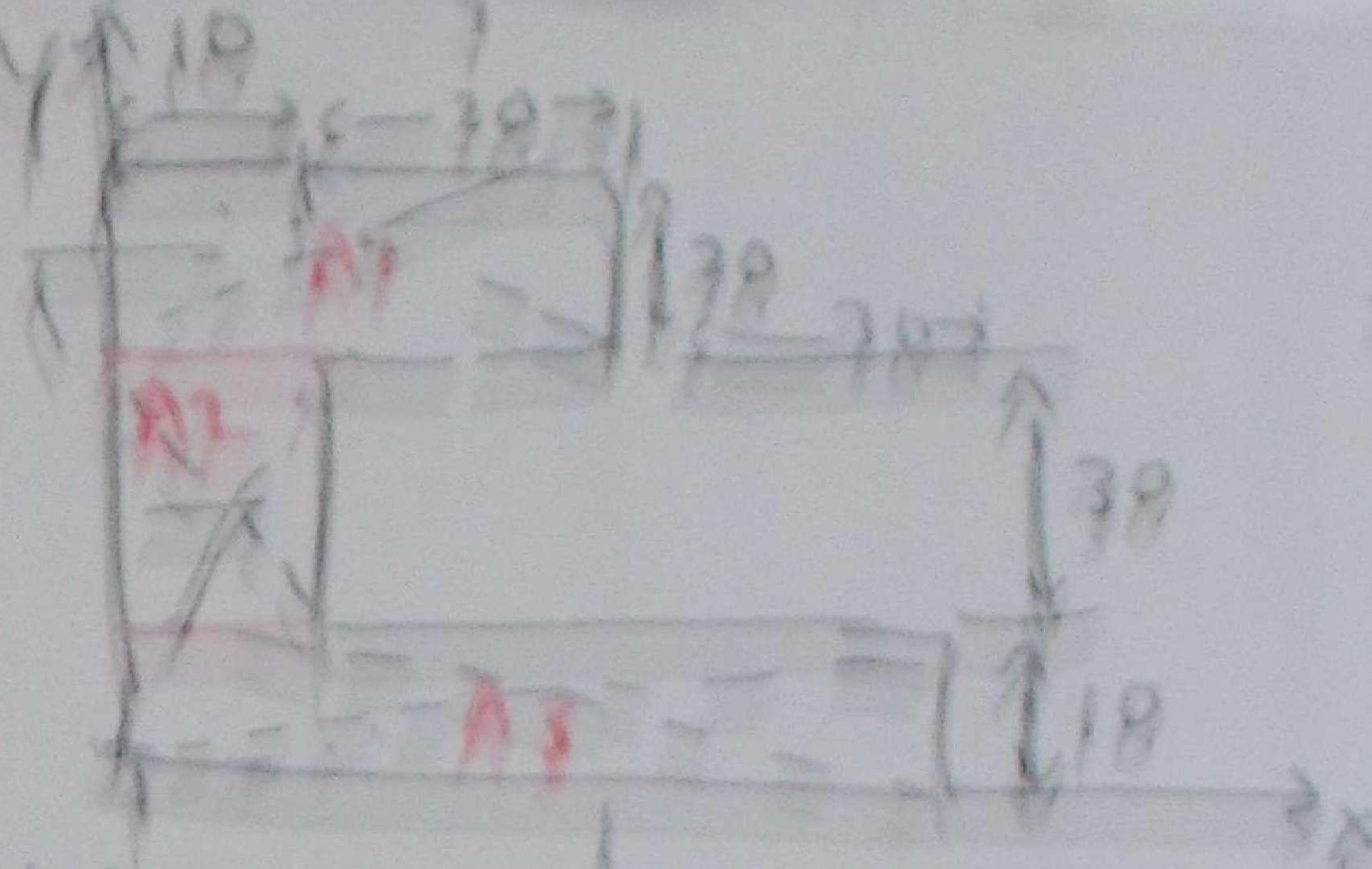
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\sum A y}{\sum A}$$



$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

Ex)



$\bar{x}, \bar{y} = ?$

$x_1 = 15, \quad y_1 = 50 \qquad A_1 = 30 \times 20 = 600$

$x_2 = 5 \quad y_2 = 35 \qquad A_2 = 30 \times 10 = 300$

$x_3 = 30 \quad y_3 = 5 \qquad A_3 = 60 \times 10 = 600$

$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{600 \times 15 + 300 \times 5 + 600 \times 30}{600 + 300 + 600}$$

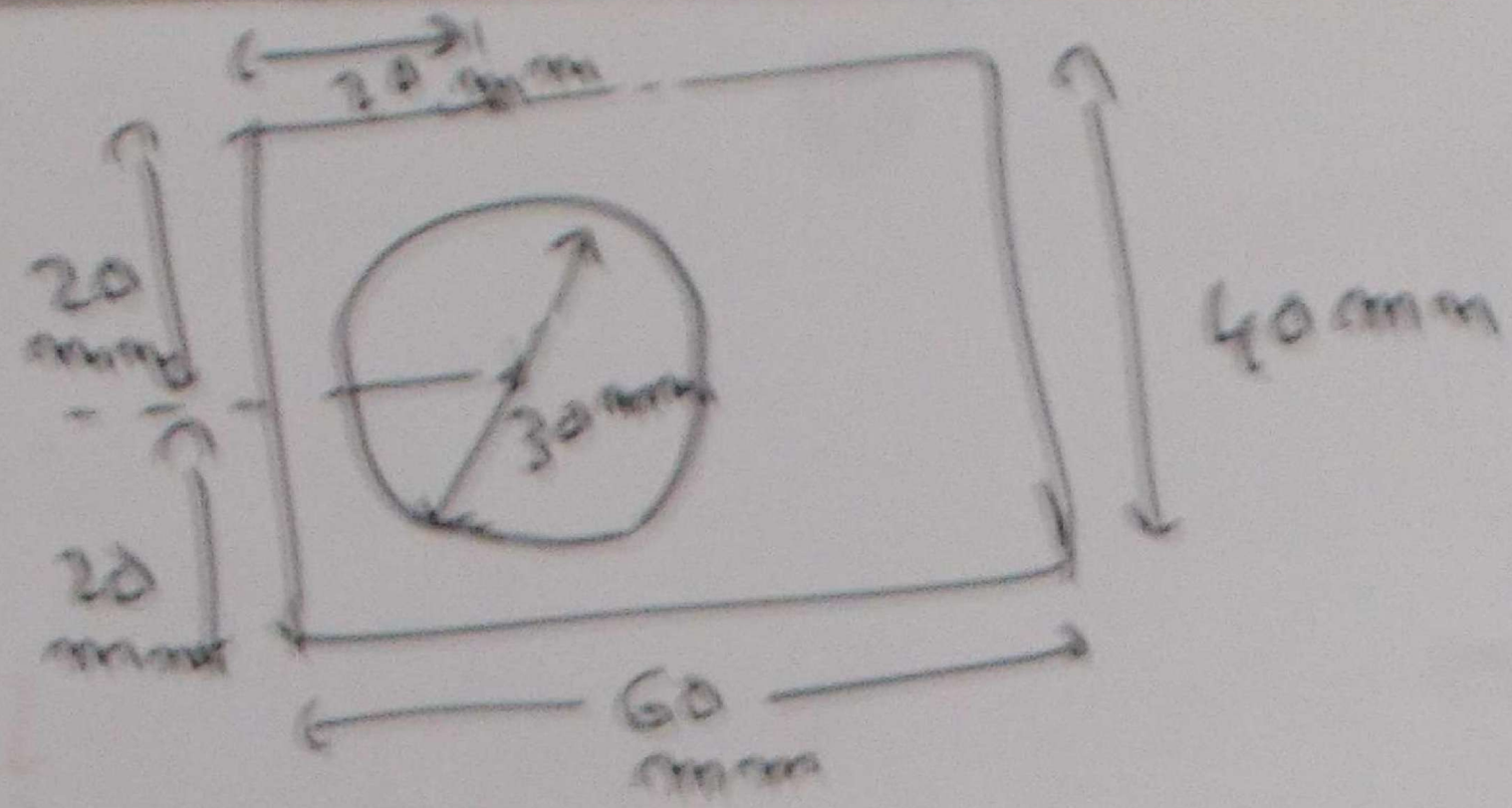
$$= 19 \text{ mm}$$

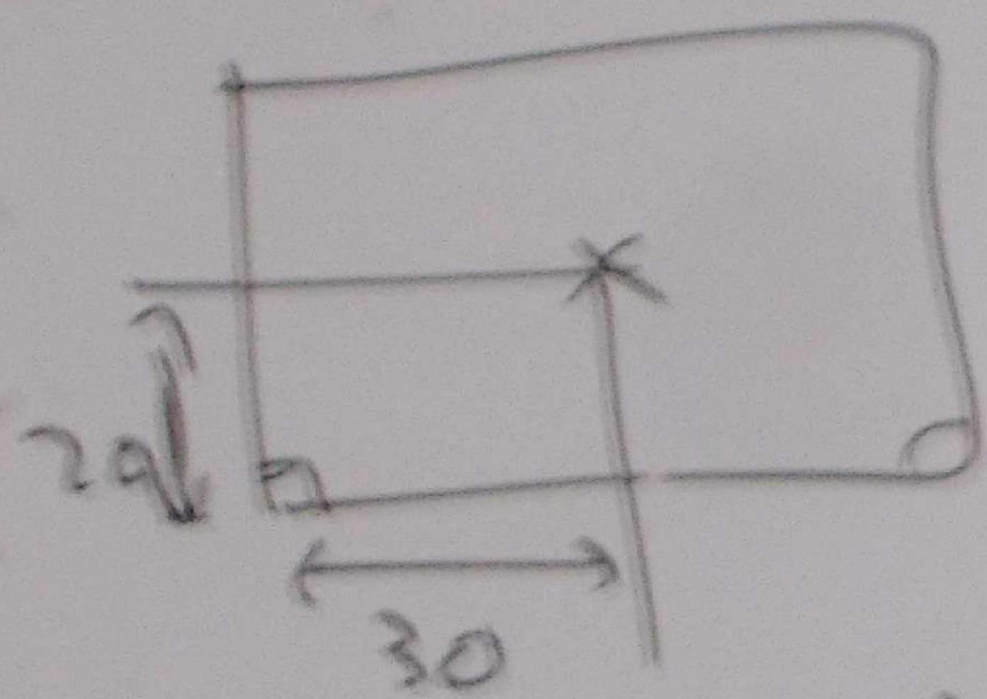
$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{600 \times 50 + 300 \times 35 + 600 \times 5}{600 + 300 + 600}$$

$$= 29 \text{ mm}$$

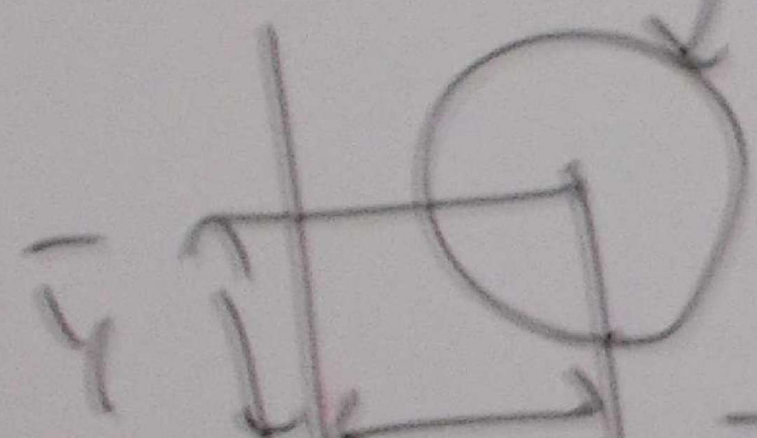
Pb





$X_1 = 30$ $A_1 = 60 \times 40$

$Y_1 = 20$ $= 2400$



$X_2 = 20$ $A_2 = \frac{\pi}{4} D^2$

$Y_2 = 20$ $= \frac{3.1416 \times 30^2}{4}$

$\bar{Y} = 20$

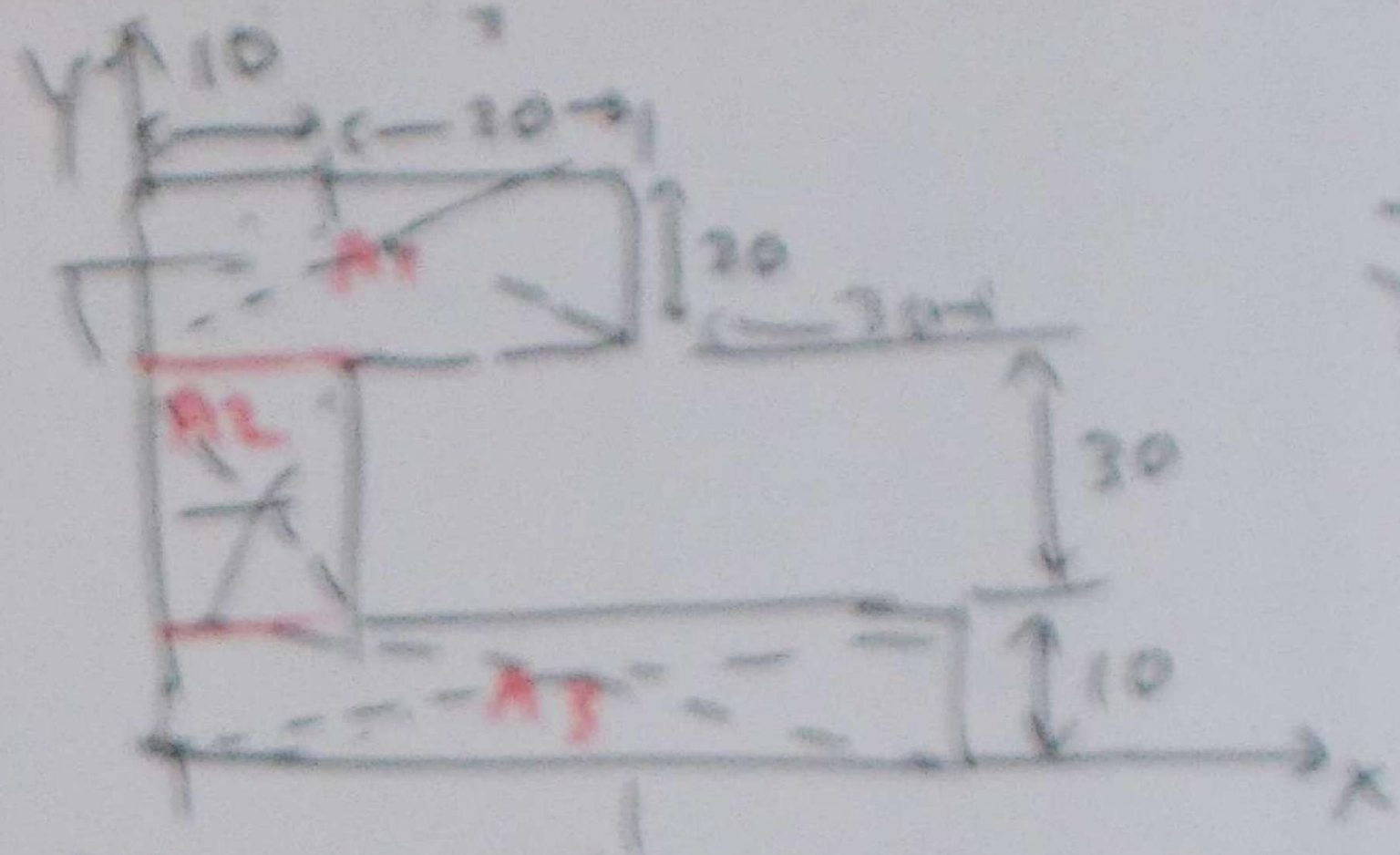
$$\bar{X} = \frac{A_1 X_1 - A_2 X_2}{A_1 - A_2}$$

$$= \frac{2400 \times 30 - \frac{3.1416}{4} \times 30^2 \times 20}{2400 - \frac{3.1416}{4} \times 30^2}$$

$= 34.2 mm$

$\bar{Y} = 20 mm$

P4



$\bar{X}, \bar{Y} = ?$

$X_1 = 15 \quad Y_1 = 50 \quad A_1 = 30 \times 20 = 600$

$X_2 = 5 \quad Y_2 = 25 \quad A_2 = 30 \times 10 = 300$

$X_3 = 30 \quad Y_3 = 5 \quad A_3 = 60 \times 10 = 600$

$$\bar{X} = \frac{\sum AX}{\sum A} = \frac{A_1X_1 + A_2X_2 + A_3X_3}{A_1 + A_2 + A_3}$$

$$= \frac{600 \times 15 + 300 \times 5 + 600 \times 30}{600 + 300 + 600}$$

$$= 19 \text{ mm}$$

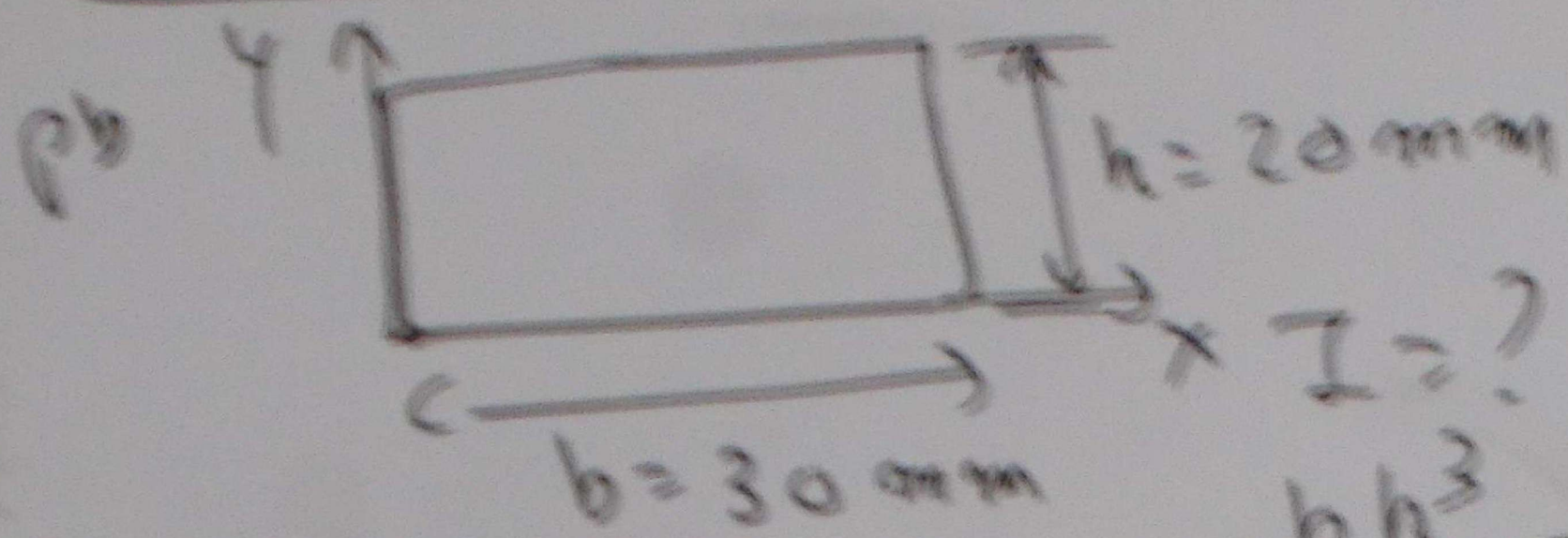
$$\bar{Y} = \frac{\sum AY}{\sum A} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

$$= \frac{600 \times 50 + 300 \times 25 + 600 \times 5}{600 + 300 + 600}$$

$$= 27 \text{ mm}$$

MOMENT OF INERTIA

Pb

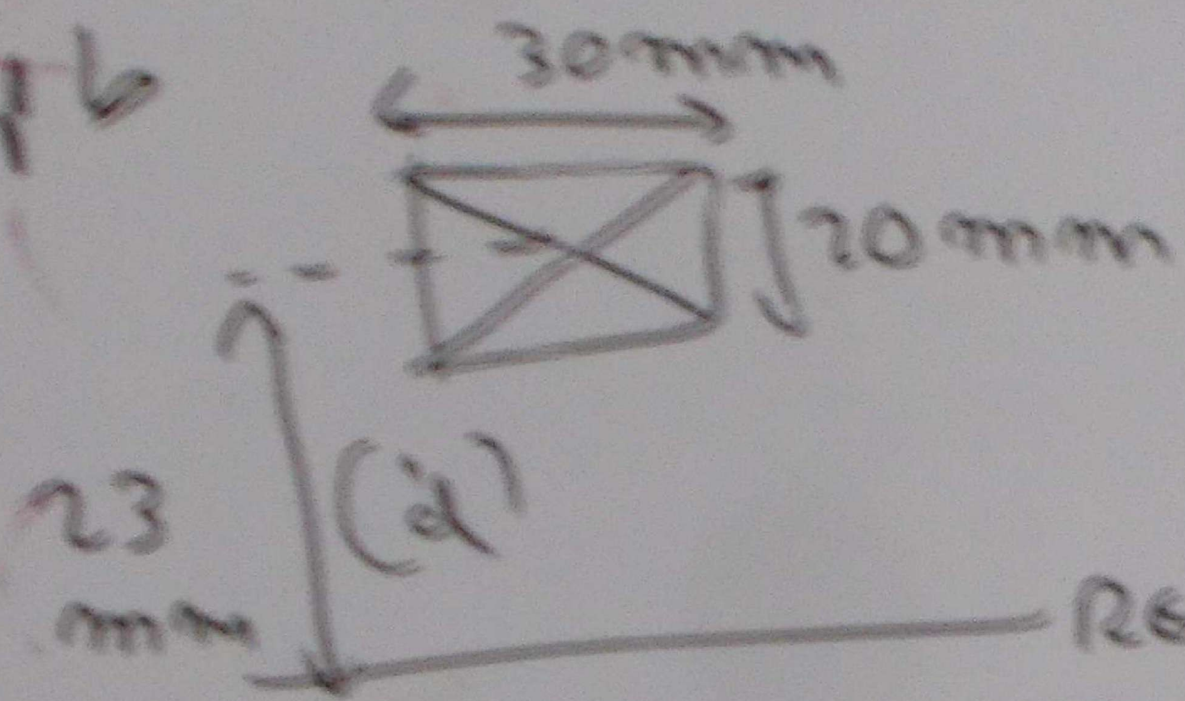


$I = ?$

$$I_c = \frac{bh^3}{12} = \frac{30 \times 20^3}{12}$$

$$= 20{,}000 \text{ mm}^4$$

Pb



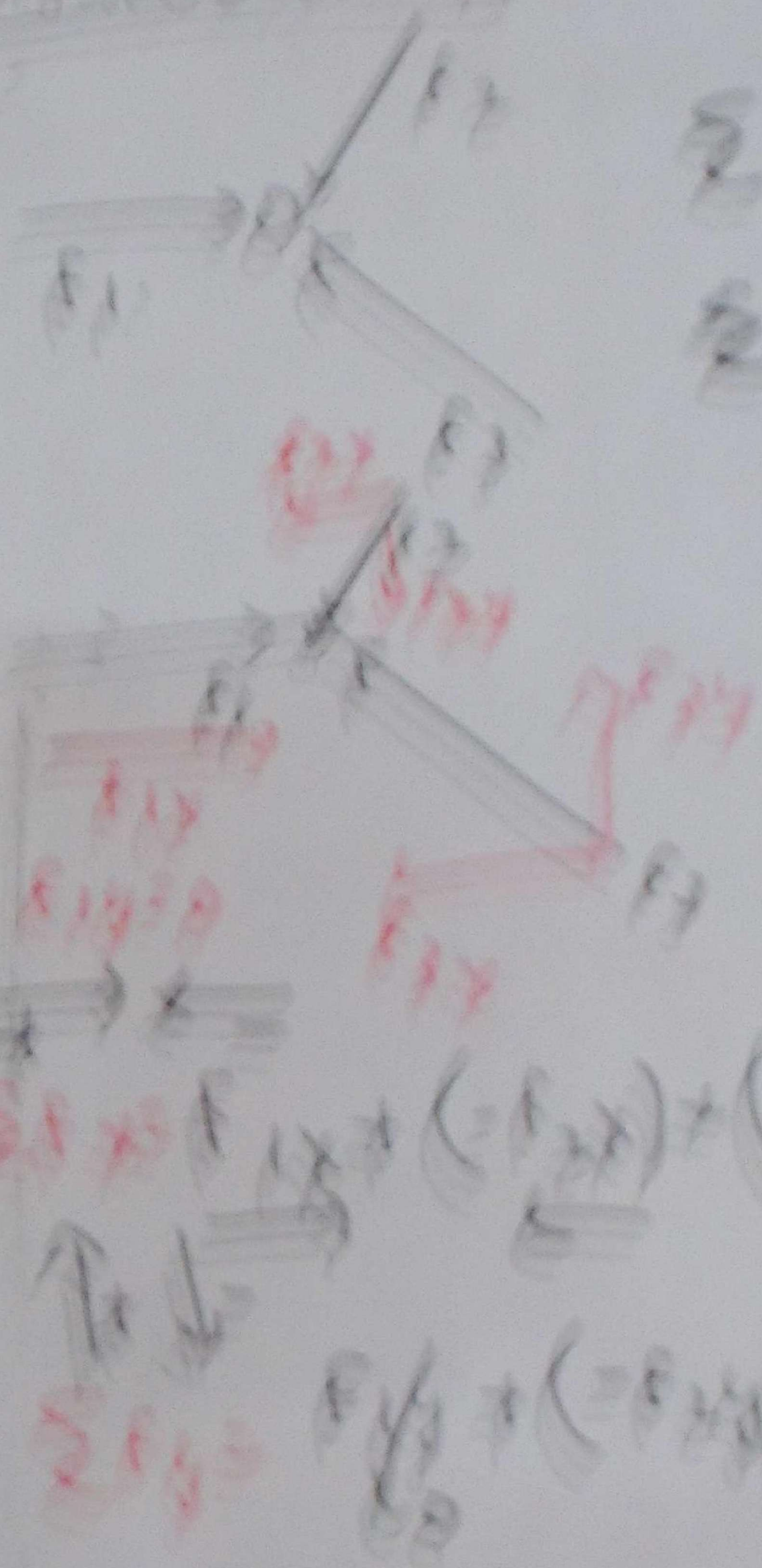
$$\boxed{I = I_c + A d^2}$$

$$= \frac{bh^3}{12} + (bh)\, d^2$$

$$= \frac{30 \times 20^3}{12} + (30 \times 20) \times 23^2$$

$$= 337400 \text{ mm}^4$$

EQUILIBRIUM



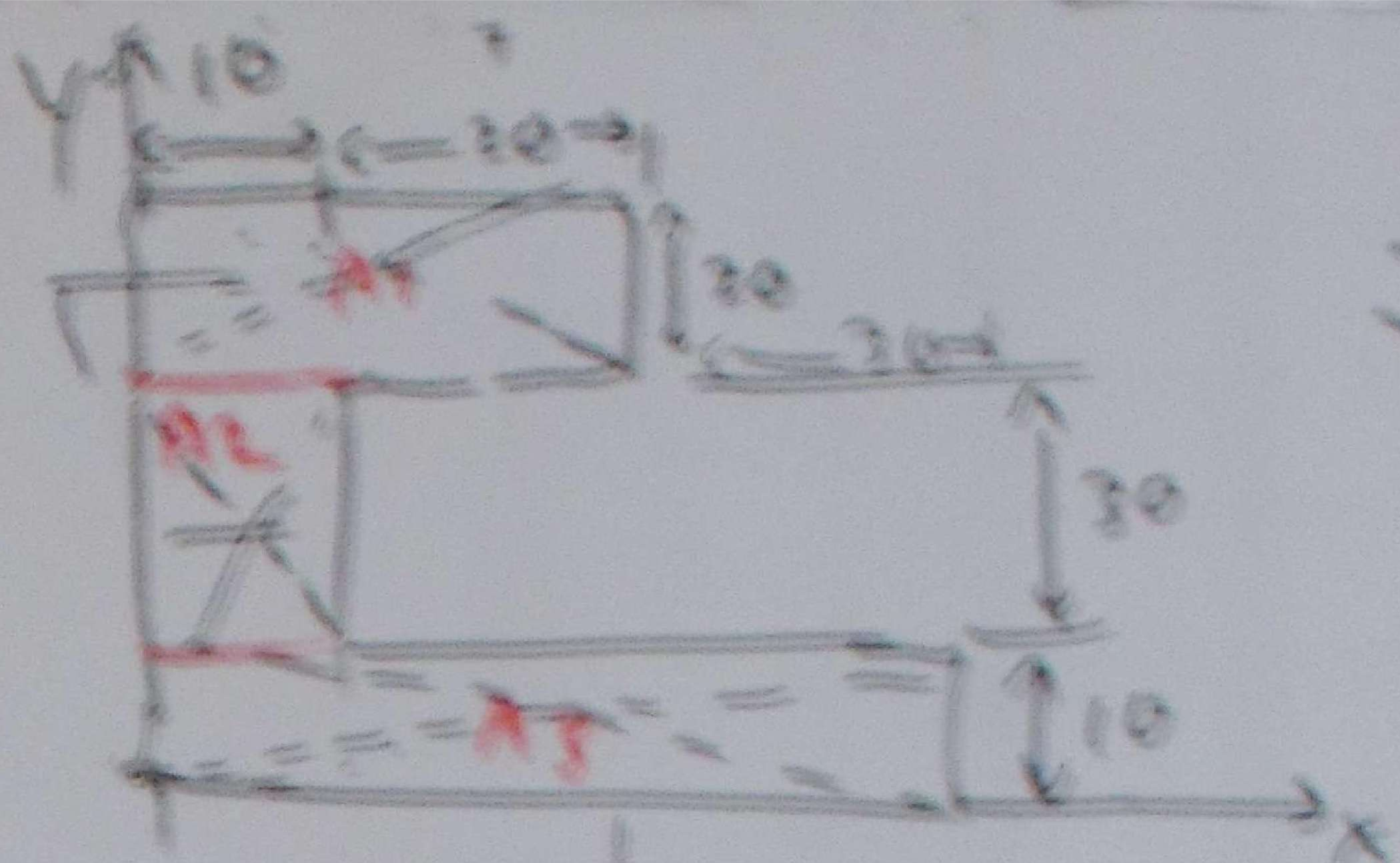
$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_x = F_{1x} + (-F_{2x}) + (-F_{3x}) = 0$

$\Sigma F_y = F_{1y} + (-F_{2y}) + (+F_{3y}) = 0$

Pb



$x_1 = 15$, $y_1 = 50$ $A_1 = 30\times 20$

$x_2 = 5$ $y_2 = 25$ $A_2 = 30\times 10$

$x_3 = 30$ $y_3 = 5$ $A_3 = 60\times 10$

$$\bar{X} = \frac{\sum AX}{\sum A} = \frac{A_1X_1 + A_2X_2 + A_3 \ldots}{A_1 + A_2 + A_3}$$

$$= \frac{600\times 15 + 300\times 5 \ldots}{600 + 300 + \ldots}$$

$= 19$ mm

$$\bar{Y} = \frac{\sum AY}{\sum A} = \frac{A_1Y_1 + A_2Y_2 + A_3 \ldots}{A_1 + A_2 + A_3}$$

$$= \frac{600\times 50 + 300\times 25 \ldots}{600 + 300 + 6 \ldots}$$

$= 27$ mm

BENDING STRESSES

MOMENT OF INERTIA

$$\boxed{f_b = \frac{My}{I}}$$

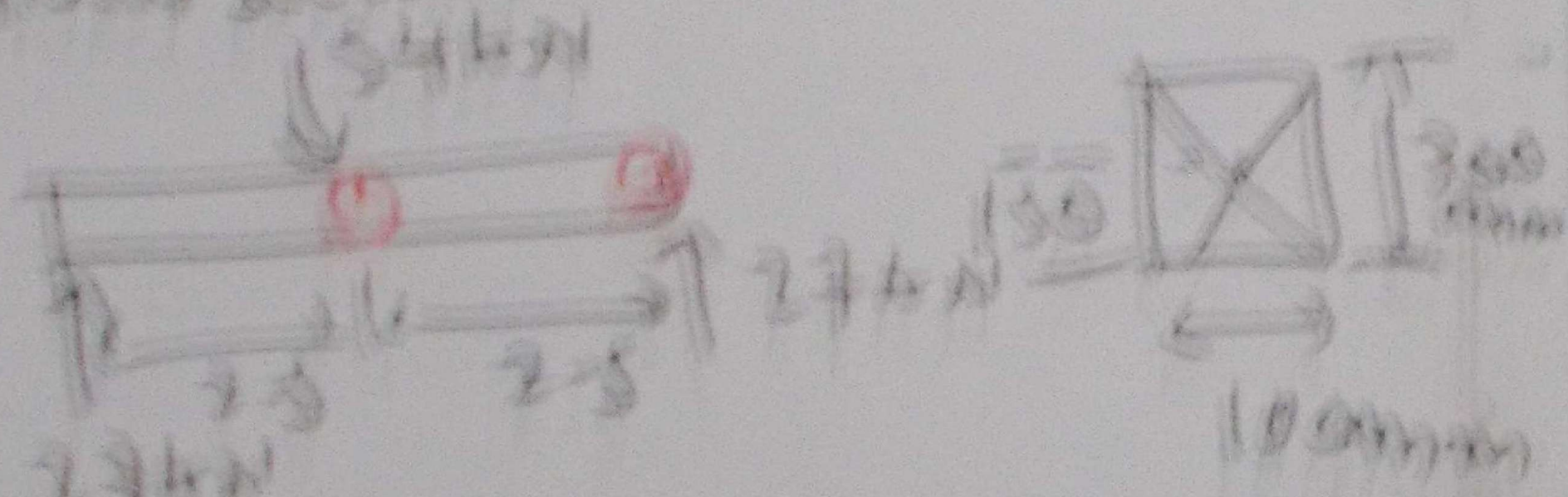
f_b = BENDING STRESS, MPa

M = BENDING MOMENT OF GIVEN CROSS SECTION

I = MOMENT OF INERTIA OF GIVEN SECTION

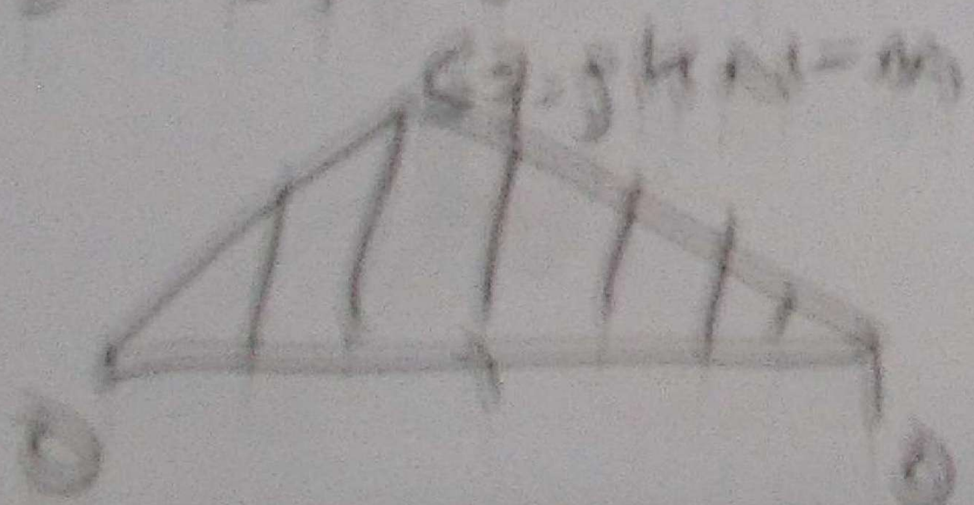
y = DISTANCE FROM NEUTRAL PLANE

Pb



$BM_1 = 27 \times 2.5 = 67.5$ kN-m

$BM_2 = 27 \times 5 = 54 \times 2.5 = 135 - 135 = 0$



M = 67.5

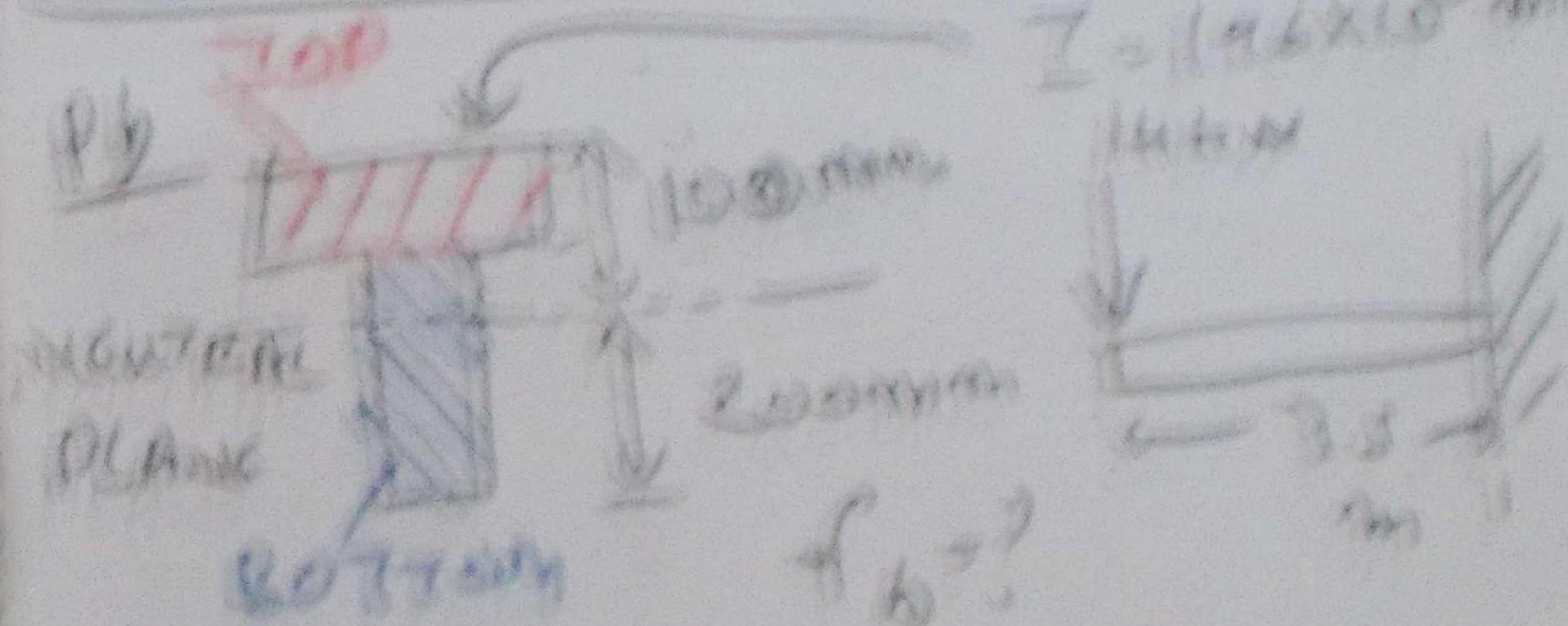
$y = 150\ mm$

$$I = \frac{bh^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6\ mm^4$$

$$f_b = \frac{My}{I} = \frac{67.5 \times 150}{225 \times 10^6} = 45\ MPa$$

$I = 196 \times 10^6\ mm^4$

Pb



$M = 14 \times 3.5 = 49\ kN\text{-}m$

$$f_b = \frac{My}{I} = 49 \times 10^6\ N\text{-}mm$$

TOP $$f_{b_1} = \frac{49 \times 10^6 \times 100}{196 \times 10^6} = 25\ MPa$$

BOTTOM $$f_{b_2} = \frac{49 \times 10^6 \times 200}{196 \times 10^6} = 50\ MPa$$

BENDING STRESSES

MOMENT OF INERTIA

$$f_b = \frac{My}{I}$$

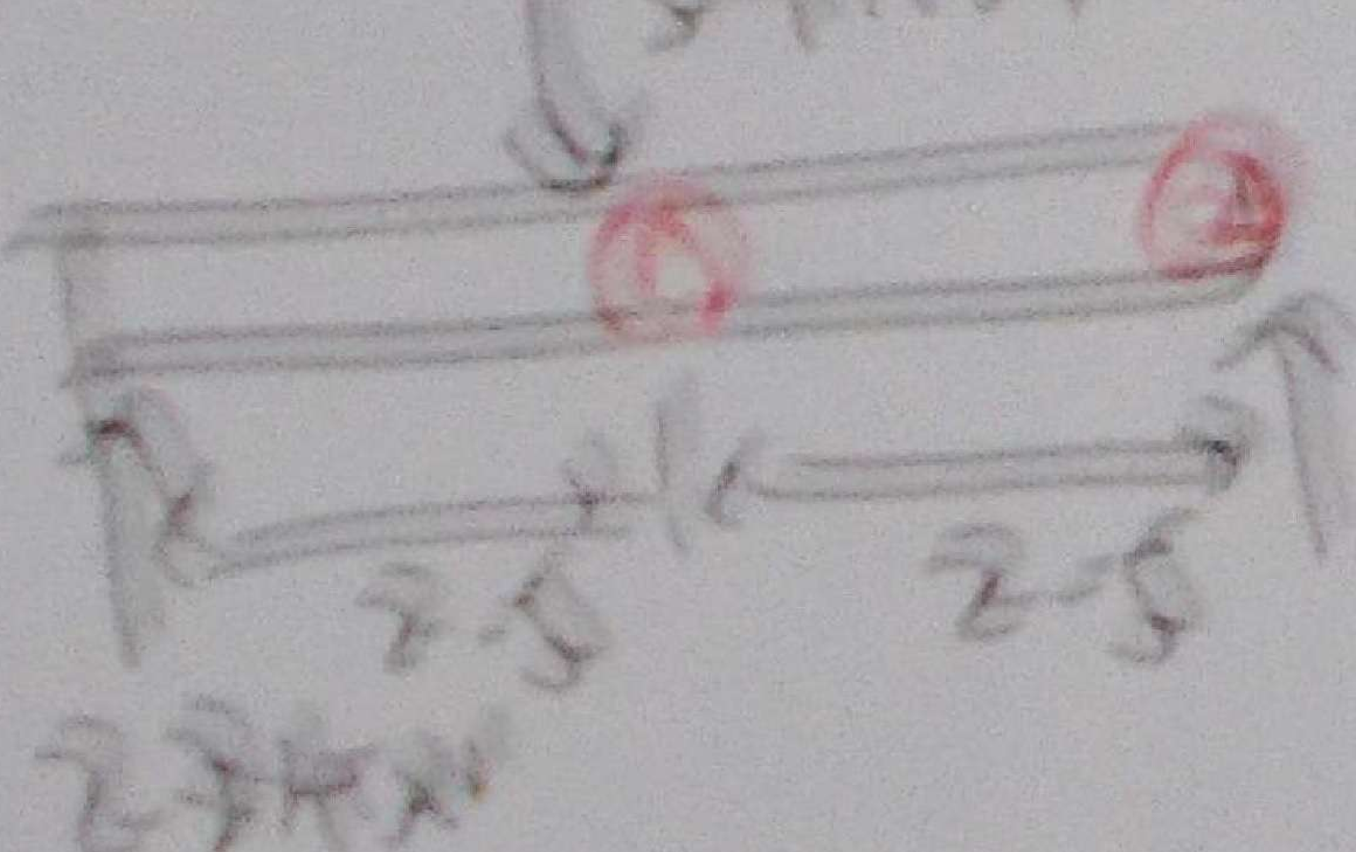
f_b = BENDING STRESS MPa

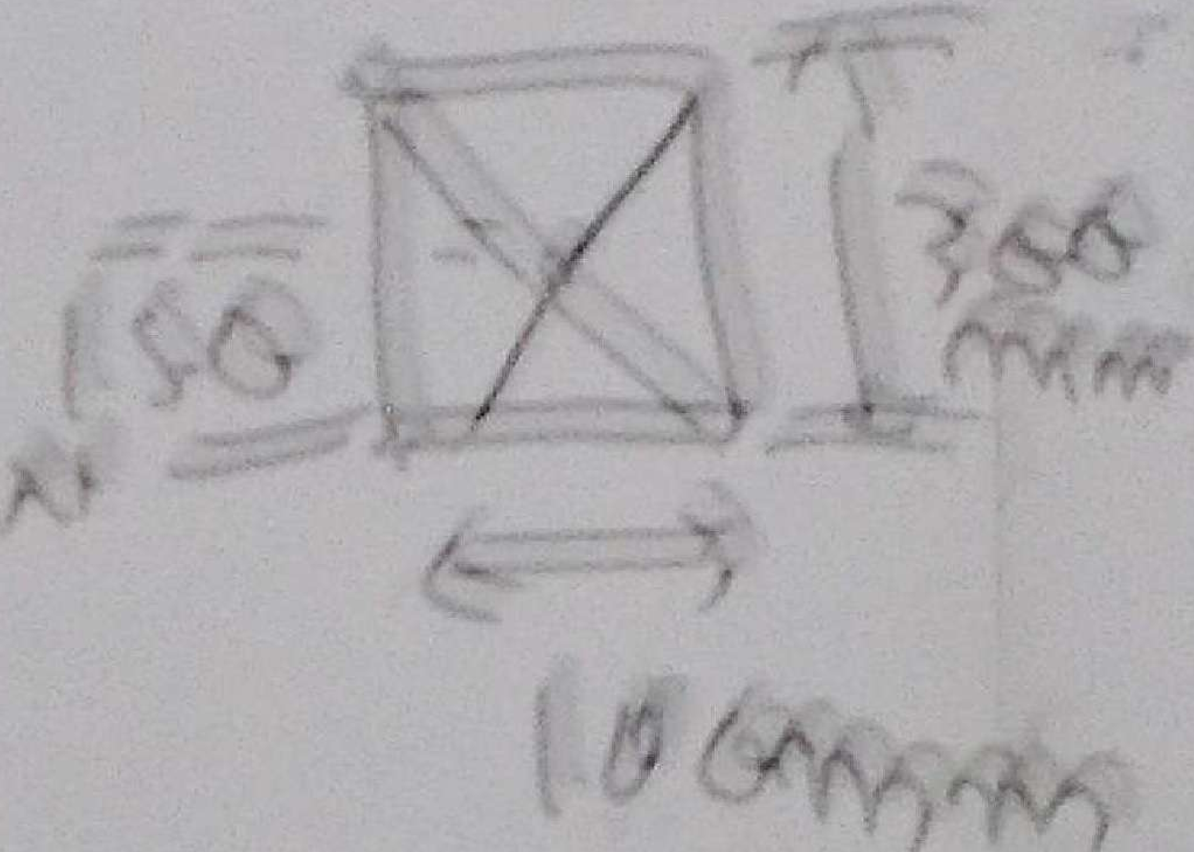
M = BENDING MOMENT OF GIVEN CROSS SECTION

I = MOMENT OF INERTIA OF GIVEN SECTION

y = DISTANCE FROM NEUTRAL PLANE

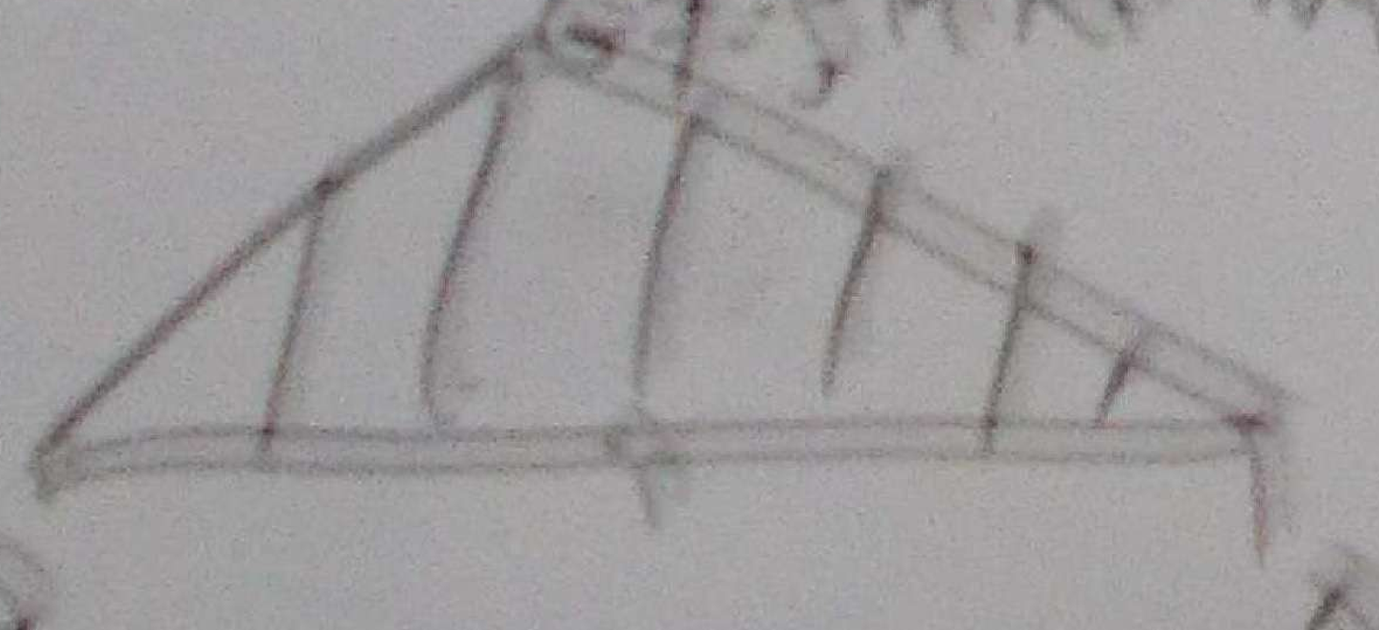
Ex





$+$ $BM_1 = 27 \times 2.5 = 67.5$ kN-m

$BM_2 = 27 \times 5 - 54 \times 2.5 = 135 - 135 = 0$



$M = 67.5$

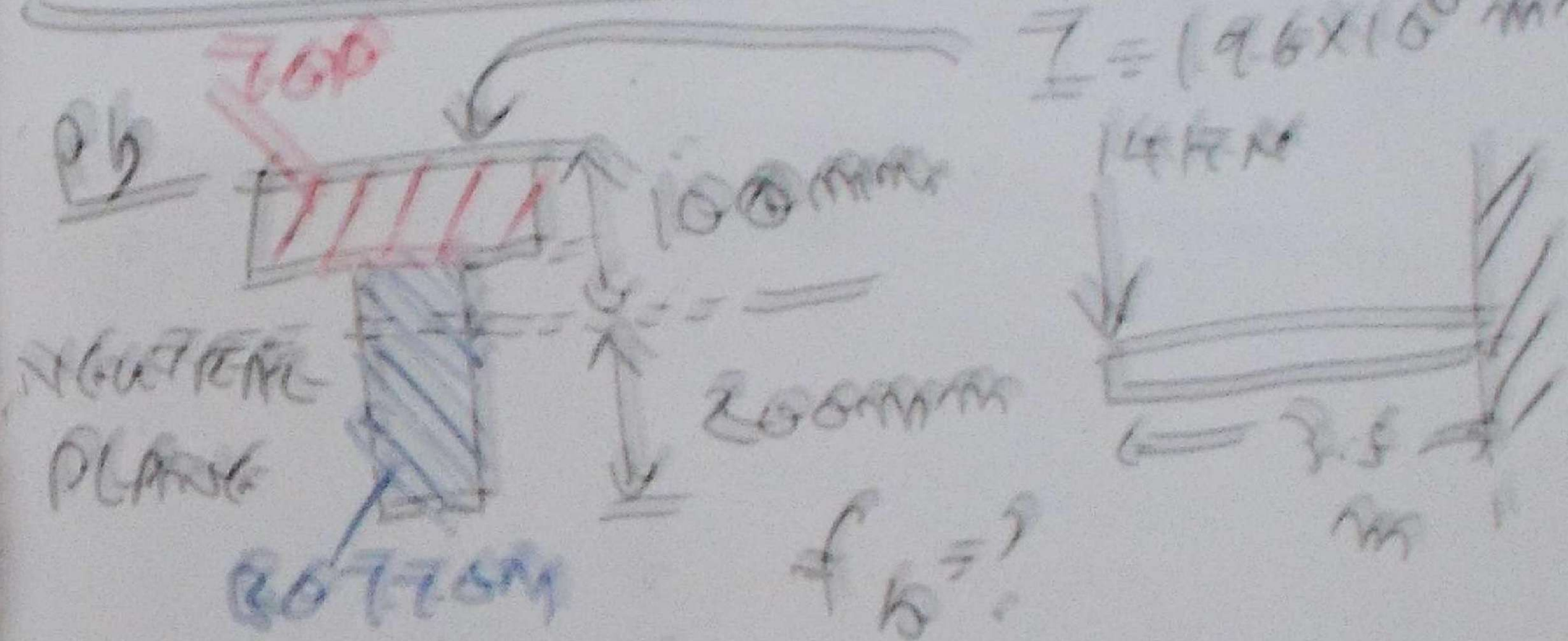
$y = 150\ mm$

$$I = \frac{bh^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6\ mm^4$$

$$f_b = \frac{My}{I} = \frac{67.5 \times 150}{225 \times 10^6} = 45\ MPa$$

$I = 196 \times 10^6\ mm^4$

Pb



$f_b = ?$

$M = 14 \times 3.5 = 49\ kN\text{-}m$

$$f_b = \frac{My}{I} = 49 \times 10^6\ N\text{-}mm$$

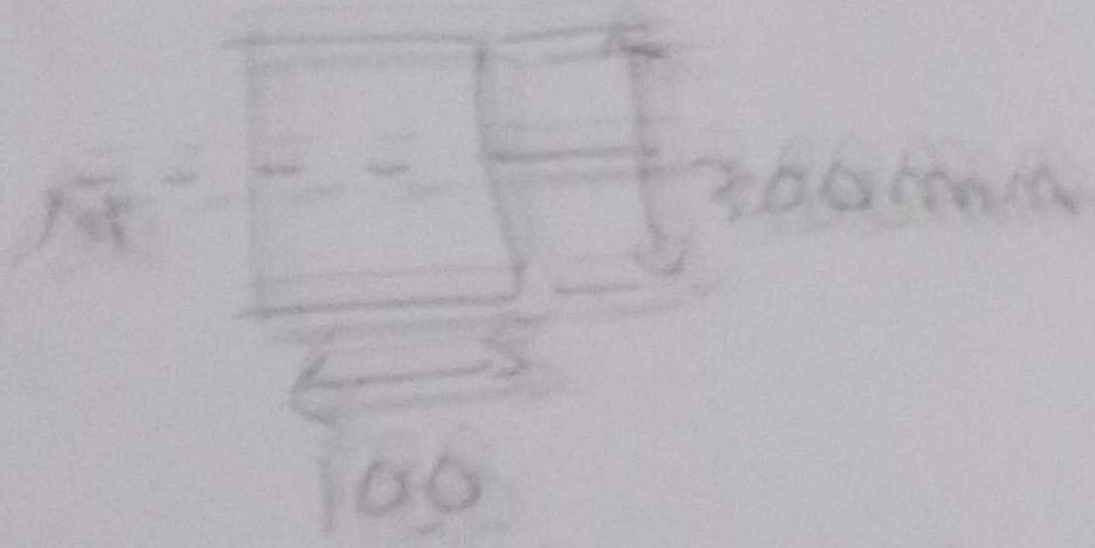
Top $$f_{b_1} = \frac{49 \times 10^6 \times 100}{196 \times 10^6} = 25\ MPa$$

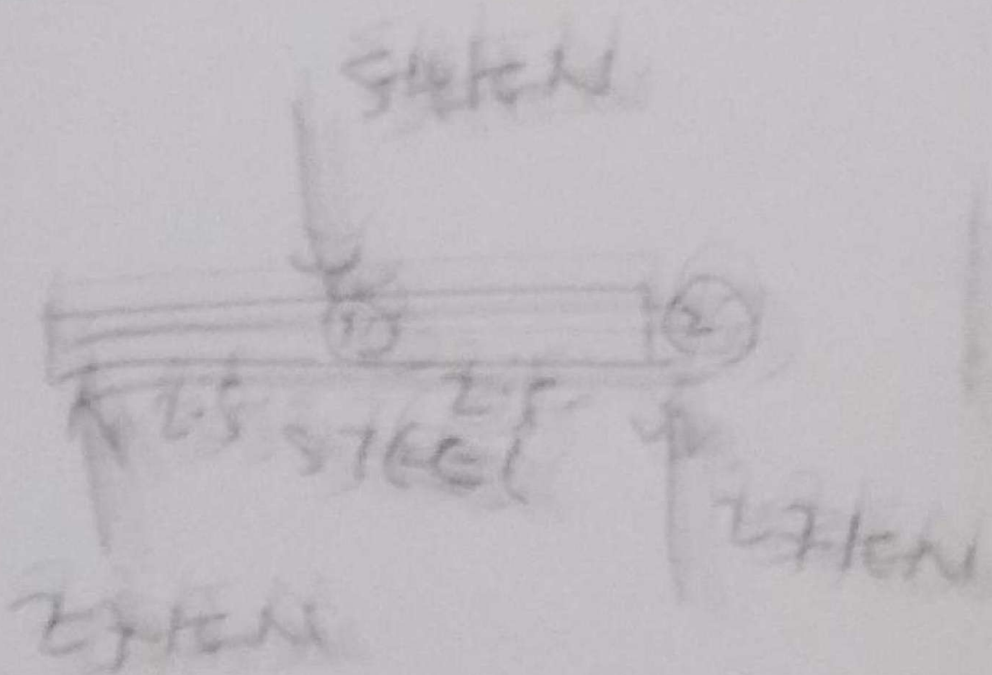
Bottom $$f_{b_2} = \frac{49 \times 10^6 \times 200}{196 \times 10^6} = 50\ MPa$$

DEFLECTIONS OF BEAMS

RADIUS OF CURVATURE (R)

$$R = \frac{EI}{M}$$

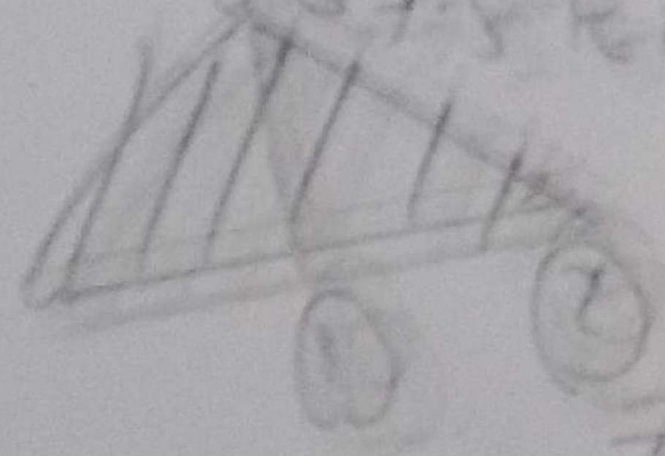




(R)? $E = 200{,}000$ mPa

① $BM_1 = 27 \times 2.5 = 67.5$

② $BM_2 = 27 \times 5 - 50 \times 2.5 = 0$



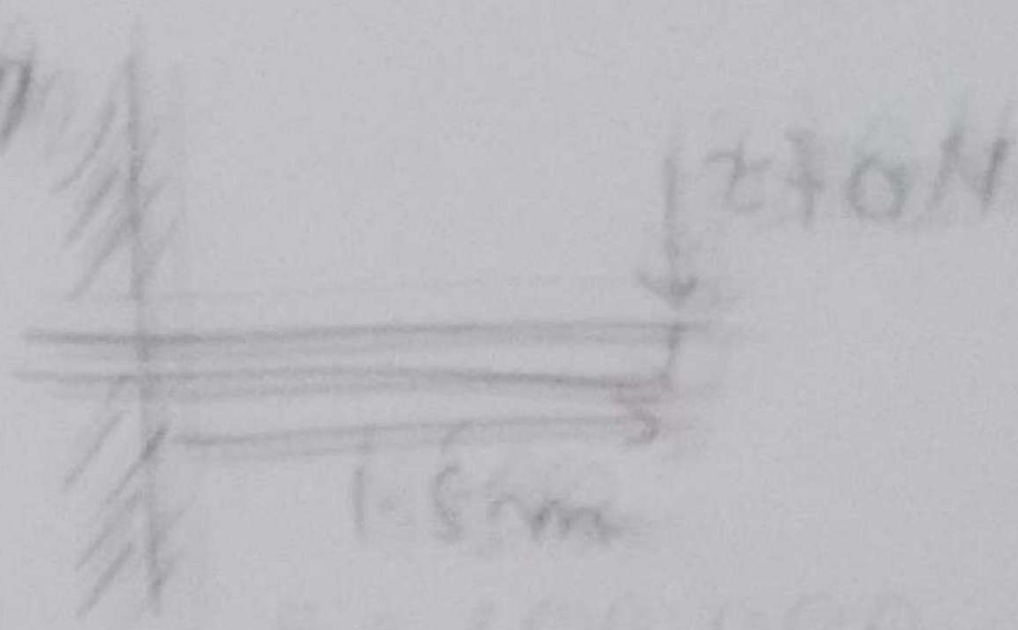
$M = 67.5$ kN-m

$= 67.5 \times 10^6$ N-mm

$$I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6 \text{ mm}^4$$

$$R = \frac{EI}{M} = \frac{200{,}000 \times 22.5 \times 10^6}{67.5 \times 10^6} = 680.1\ m$$

(b)



$E = 200{,}000$

$D = 80\,mm$

$$I = \frac{\pi D^4}{64} = \frac{3.1416 \times 80^4}{64}$$

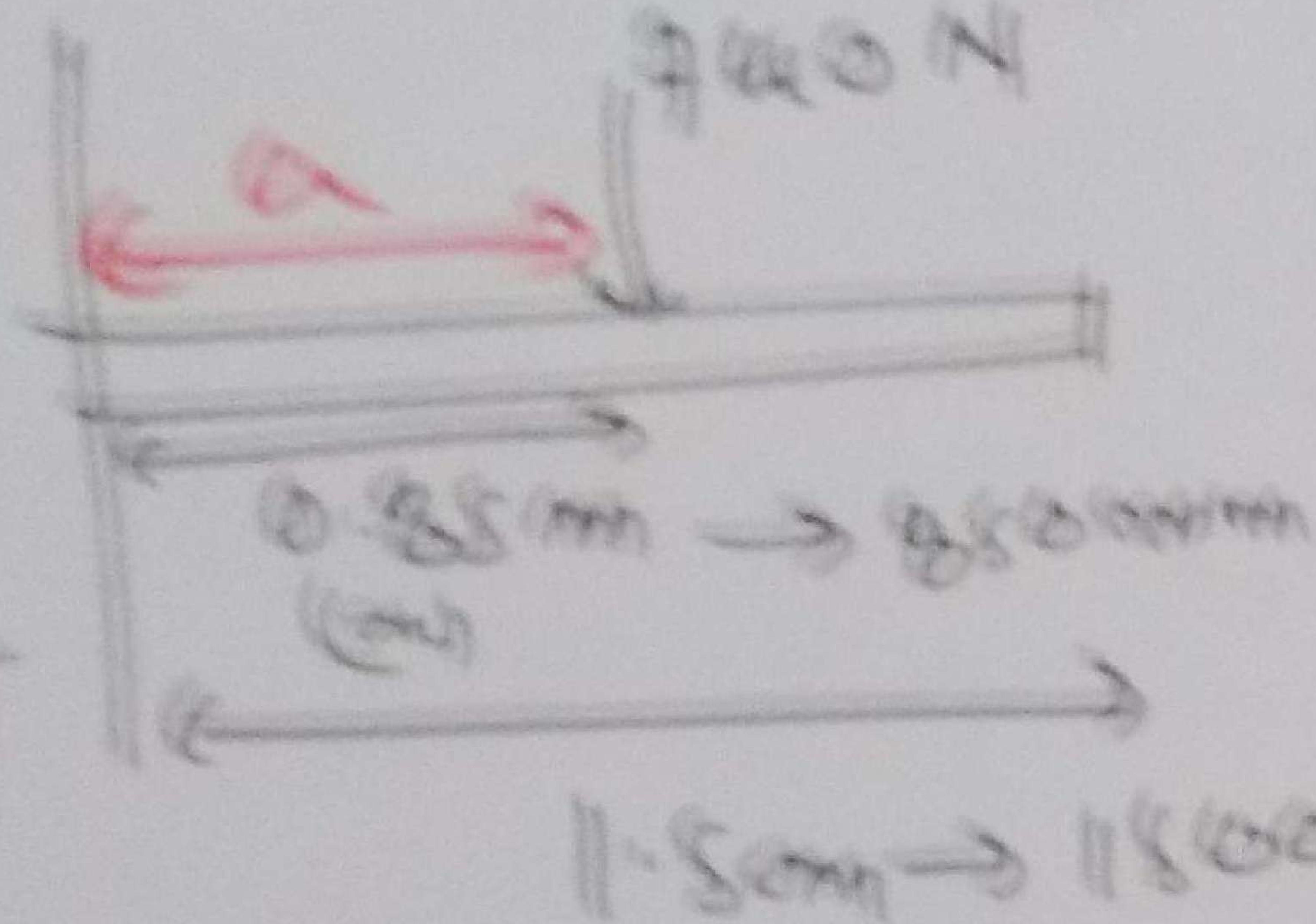
$$= 305.9 \times 10^3\ mm^4$$

$$Y = \frac{PL^3}{3EI}$$

$$= \frac{270 \times (1500)^3}{3 \times 200000 \times 305.9 \times 10^3}$$

$$= 4.45\ mm$$

(a)



$I = 306.8 \times 10^{3}\ mm^{4}$

$E = 200,000\ MPa$

$y = ?$

$$y = \frac{F a^2 (3L - a)}{6 E I}$$

$$= \frac{740 \times 850^2 (3 \times 1500 - 850)}{6 \times 200,000 \times 306.8 \times 10^{3}}$$

$= 5.3\ mm$

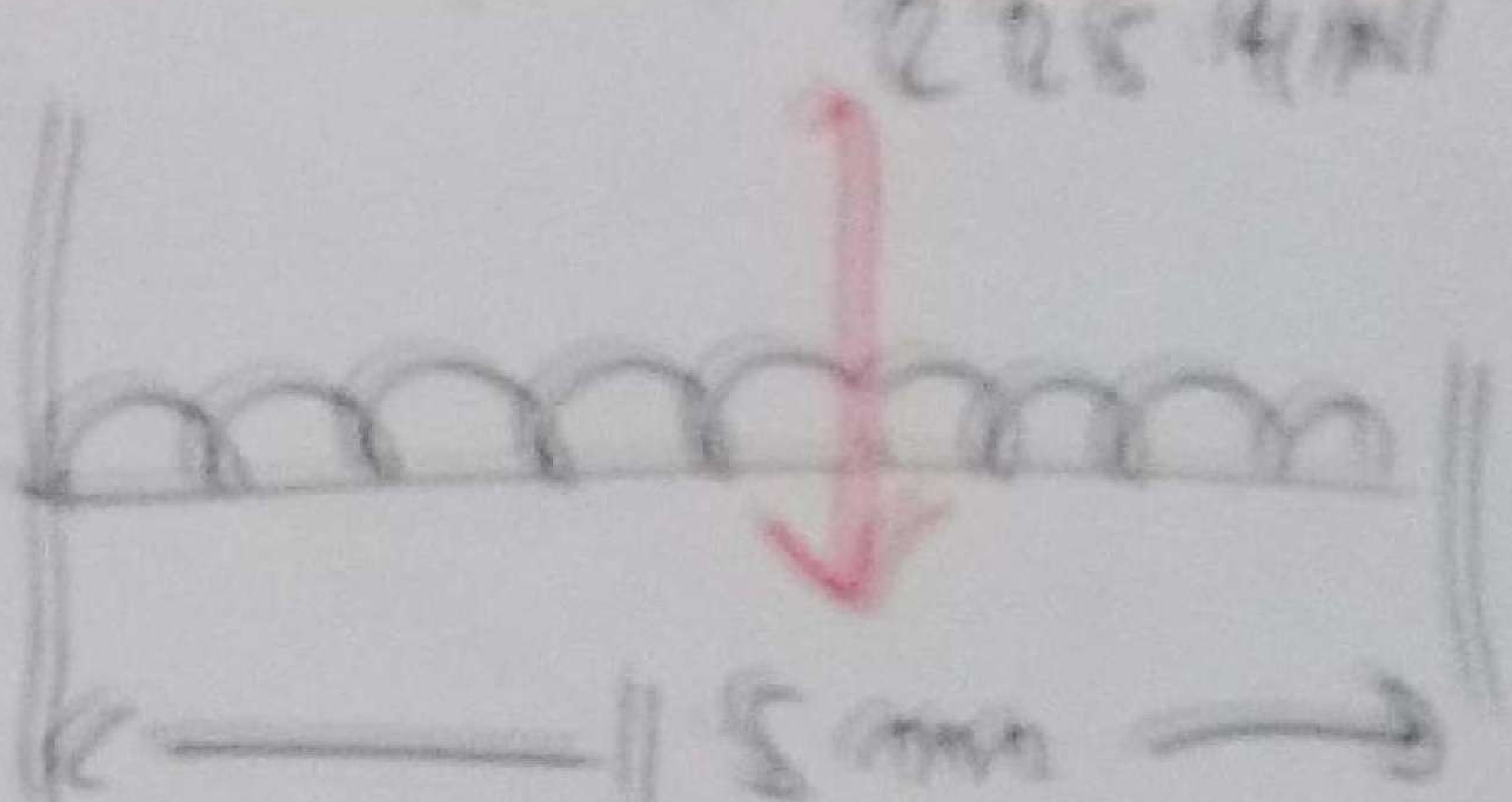
[illegible] MASS = 7800 kg/m³

DIAMETER = 0.05 m

$F = MASS \times g = volume \times Density \times g$

$$= \frac{\pi}{4}(0.05)^2 \times 1500 \times 7800 \times 9.81$$

$= 225.4\ N$

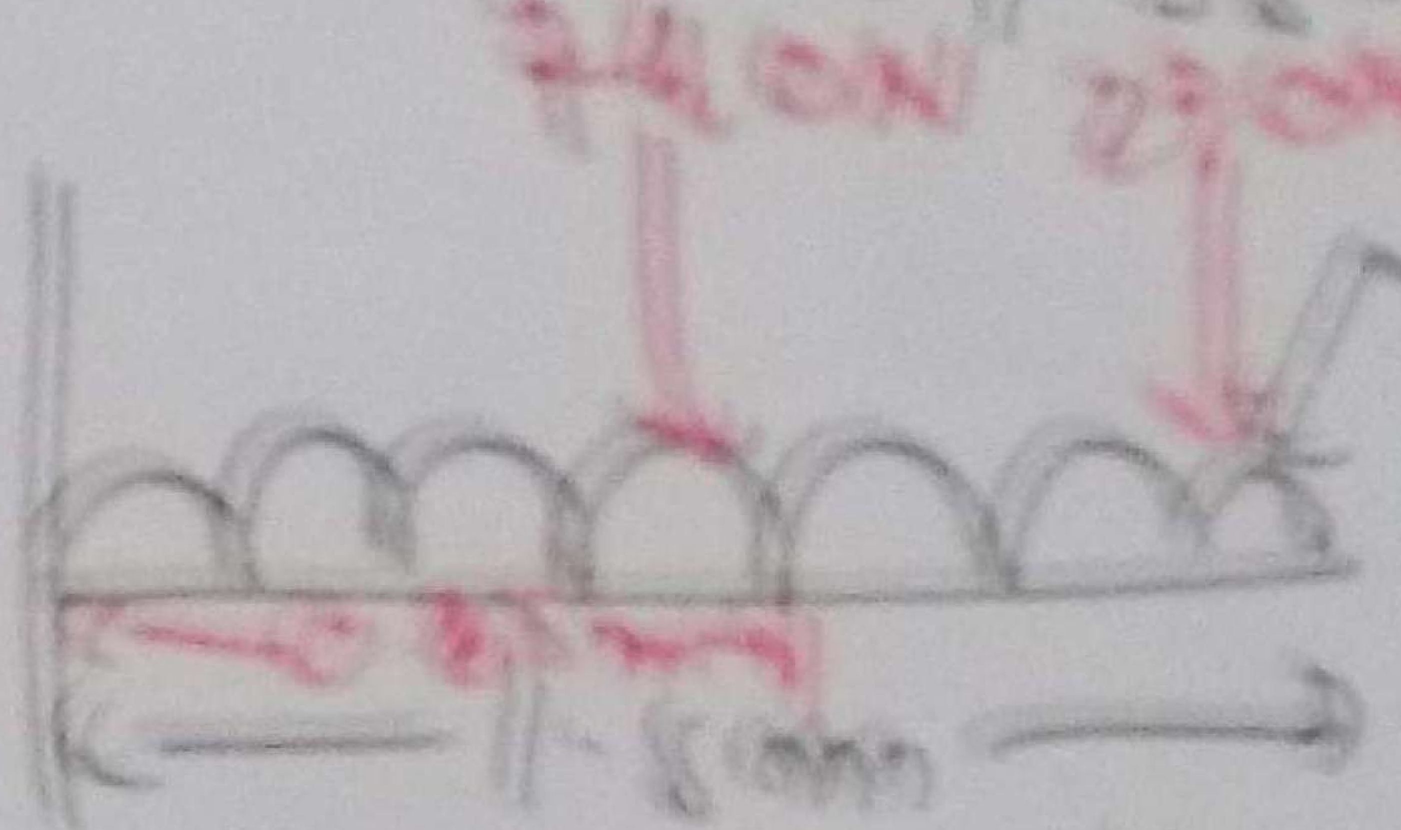


$y = ?$

$$y = \frac{FL^3}{8EI} = \frac{225.4 \times (1500)^3}{8 \times 200{,}000 \times 305.8 \times 10^3}$$

$= 11.55$ mm

P4



7800 kg/m^3

$d = 0.05$ m

$y_T = y_1 + y_2 + y_3 = 11.55 + 5.3 + 4.95 = 21.8$ mm

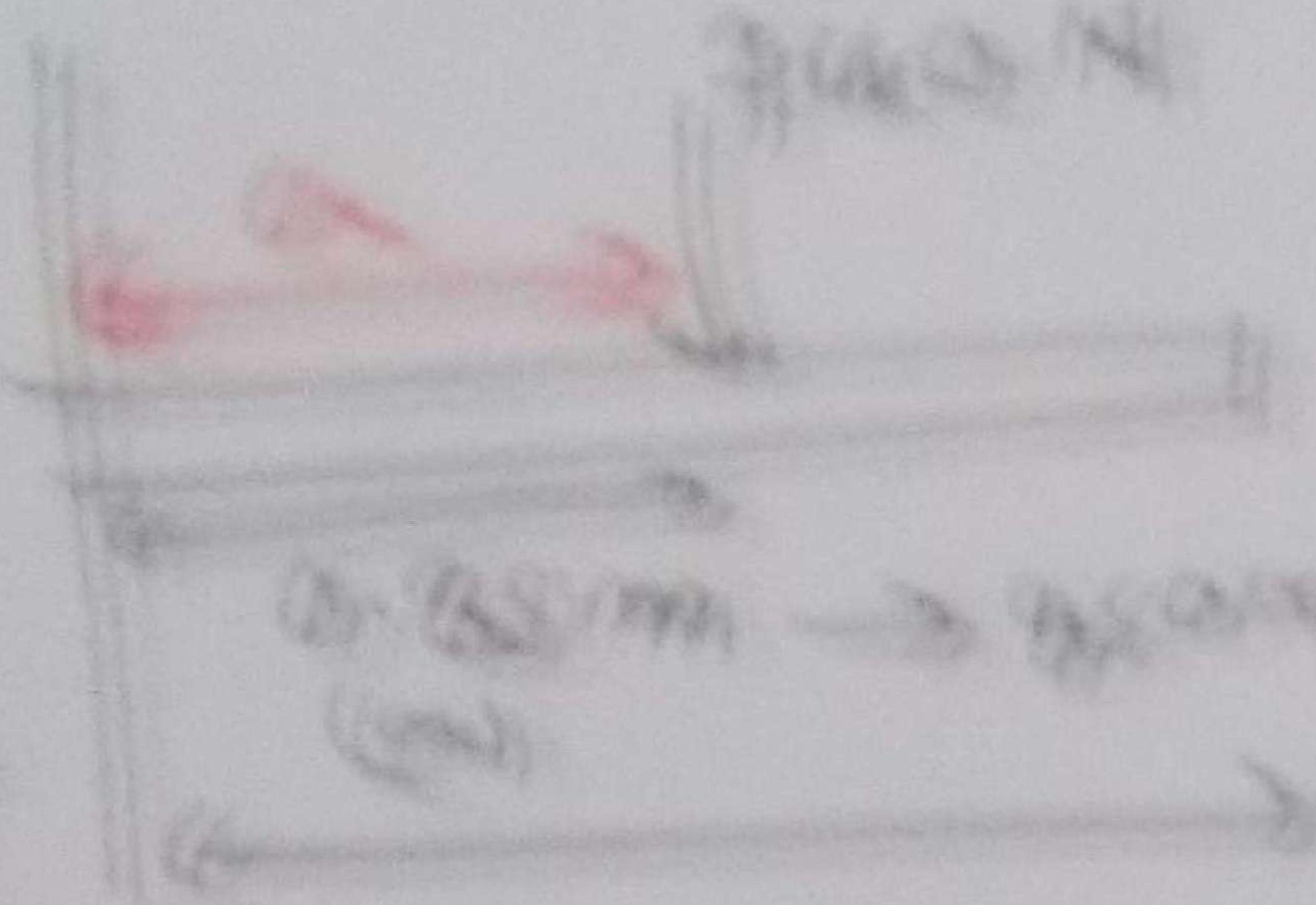
DEFLECTION DUE TO OWN WEIGHT $y_1 = 11.55$ mm

DEFLECTION DUE TO 740N $y_2 = 5.3$ mm

DEFLECTION DUE TO 270N $y_3 = \frac{Fa^2(3L - a)}{6EI}$

$$y_3 = \frac{270 \times 1500^2 (3 \times 1500 - 1500)}{8 \times 200{,}000 \times 305.8 \times 10^3} = 4.95 \text{ mm}$$

1b)



11.5 m → 1150 mm

$I = 306.8 \times 10^{3}\ mm^4$

$E = 200{,}000\ N/mm$

$y = ?$

$$y = \frac{F a^2 (3L - a)}{6EI}$$

$$= \frac{9140 \times 850^2 (3 \times 1150 - 850)}{6 \times 200{,}000 \times 306.8 \times 10^3}$$

$= 5.3\ mm$

2b) MASS $= 7800\ kg/m^3$

DIAMETER $= 0.03\ m$

$F = MASS \times g = volume \times density \times g$

$= \frac{\pi}{4}(0.03)^2 \times 1150 \times 7800 \times 9$

$= 625.4\ N$