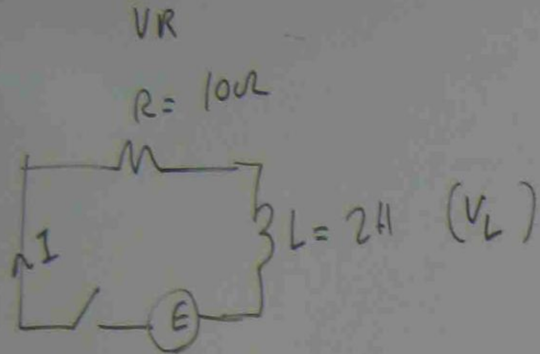


$$V_L = L\frac{dI}{dt}$$

$$V_R = I \times R$$

$$E = IR + L\frac{dI}{dt}$$

$$E = L\frac{dI}{dt} + IR$$

$$I(t) =$$

$$y' = \frac{dy}{dt}$$

$$y'' = \frac{d^2y}{dt^2}$$

$$\mathcal{L}y'' = s^2Y - s\,\underline{y(0)} - \underline{y'(0)}$$

INITIAL CONDITION of FUNCTION $y(t)$

INITIAL CONDITION OF DERIVATIVE $\frac{dy}{dt}$

$$\mathcal{L}y' = sY - y(0)$$

$$\mathcal{L}y = Y$$

Pb 14 A RESISTOR of $R = 10\Omega$, AN INDUCTOR of $L = 2H$ AND A VOLTAGE SOURCE of E VOLT ARE CONNECTED IN SERIES WITH A SWITCH "S"

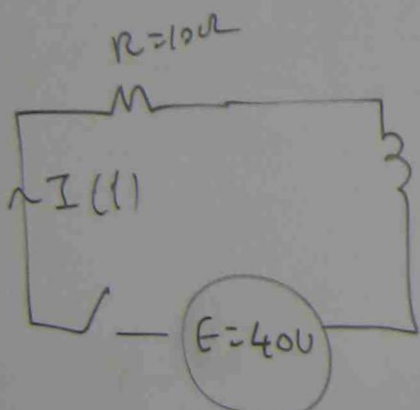
AT $t=0$ THE SWITCH IS CLOSED AND THE CURRENT $I = 0$

FIND I FOR $t>0$ IF (a) $E = 40$ VOLT

(b) $E = 20e^{-3t}$ VOLT

(c) $E = 50 \sin 5t$ VOLT

(a)



$$E = L\frac{dI}{dt} + IR$$

$$40 = 2\frac{dI}{dt} + I \times 10$$

$$20 = \frac{dI}{dt} + 5I$$

$$20 = I' + 5I$$

$$\mathcal{L}\,20 = \mathcal{L}\,I' + \mathcal{L}\,5I$$

$$\boxed{\mathcal{L}\,1 = \frac{1}{s}}$$

$$\frac{20}{s} = s\,I(s) - I(0) + 5\,I(s)$$

$I(0) = 0$ (SWITCH IS OFF)

$$\frac{20}{s} = s\,I(s) + 5\,I(s)$$

$$I(s)(s+5) = \frac{20}{s}$$

$$I(s) = \frac{20}{s(s+5)}$$

$$\frac{20}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$$

$$\frac{20}{s(s+5)} = \frac{A(s+5)+Bs}{s(s+5)}$$

$$A(s+5)+Bs = 20$$

$s=0 \Rightarrow$

$$A(0+5)+B(0) = 20$$

$$5A = 20 \rightarrow A = \frac{20}{5} = 4$$

$s - 5$

$$A(\cancel{-5+5})+B(-5) = 20$$

$\to 0$

$$-5B = 20$$

$$B = -4$$

$$\frac{20}{s(s+5)} = \frac{4}{s} + \frac{(-4)}{s+5} = \frac{4}{s} - \frac{4}{s+5}$$

$$\mathcal{L}^{-1} I(s) = \mathcal{L}^{-1}\left[\frac{4}{s} - \frac{4}{s+5}\right]$$

$$= \mathcal{L}^{-1}\frac{4}{s} - \mathcal{L}^{-1}\frac{4}{s+5}$$

$$i(t) = 4 - 4e^{-5t} \text{ Amp}$$

(b) $20e^{-3t} = 2\frac{dI}{dt} + 10I$

$$10e^{-3t} = \frac{dI}{dt} + 5I$$

$$10e^{-3t} = I' + 5I$$

$$\mathcal{L}10e^{-3t} = \mathcal{L}I' + \mathcal{L}5I$$

$$\boxed{\mathcal{L}e^{-at} = \frac{1}{s+a}}$$

$$\frac{10}{s+3} = s\,I(s) - \underset{0}{\cancel{I(0)}} + 5\,I(s)$$

$$\frac{10}{s+3} = s\,I(s) + 5\,I(s)$$

$$\frac{10}{s+3} = I(s)\left(s+5\right)$$

$$\therefore I(s) = \frac{10}{[s+3]\,[s+5]}$$

$$\frac{10}{(s+3)(s+5)} = \frac{A}{s+3} + \frac{B}{s+5}$$

$$= A(s+5) + B(s+3) = 10$$

If $s = -5 \Rightarrow A(\underset{0}{\cancel{-5+5}}) + B(-5+3) = 10$

$$B = -5$$

If $s = -3 \Rightarrow A(-3+5) + B(\underset{0}{\cancel{-3+3}}) = 10$

$$A = 5$$

$$I(s) = \frac{5}{s+3} + \frac{-5}{s+5}$$

$$I(t) = \mathcal{L}^{-1} I(s) = \mathcal{L}^{-1}\frac{5}{s+3} - \mathcal{L}^{-1}\frac{5}{s+5}$$

$$= 5e^{-3t} - 5e^{-5t}$$

$$I(t) = 5\left(e^{-3t} - e^{-5t}\right)$$

(c) $50 \sin 5t = 2\dfrac{dI}{dt} + 10I$

$$25 \sin 5t = \frac{dI}{dt} + 5I$$

$$\mathcal{L}\, 25 \sin 5t = \mathcal{L}\, I' + \mathcal{L}\, 5I$$

$$25 \quad \frac{5}{s^2 + 5^2} = s\, I(s) - I(0) + 5\, I(s)$$

$$\frac{125}{s^2 + 25} = s\, I(s) + 5\, I(s)$$

$$\frac{125}{s^2 + 25} = I(s)\,(s + 5)$$

$$I(s) = \frac{125}{(s^2 + 25)(s + 5)}$$

$$\frac{125}{(s^2+25)(s+5)} = \frac{As+B}{s^2+25} + \frac{C}{s+5}$$

$$\frac{(As+B)(s+5) + C(s^2+25)}{(s^2+25)(s+5)}$$

$$(As+B)(s+5) + C(s^2+25) = 125$$

$s = -5$

$$(A(-5)+B)\underbrace{(-5+5)}_{\to 0} + C((-5)^2+25) = 125$$

$$C \times 50 = 125$$

$$C = 5/2$$

$s = 0$

$$(A \times 0 + B)(0+5) + C(0^2+25) = 125$$

$$5B + 25C = 125$$

$$5 \times B + 25 \times 5/2 = 125 \longrightarrow B = \frac{25}{2}$$

$i(t) =$

$$\frac{}{s+5)} = \frac{As+B}{s^2+25} + \frac{C}{s+5}$$

$$\frac{(As+B)(s+5) + C(s^2+25)}{(s^2+25)(s+5)}$$

$(s+5) + C(s^2+25) = 125$

$)(-5+5) + C((-5)^2+25) = 125$

0

$C \times 50 = 125$

$C = 5/2$

$)(0+5) + C(0^2+25) = 125$

$5B + 25C = 125$

$5 \times B + 25 \times 5/2 = 125 \longrightarrow B = \frac{25}{2}$

$s = 1$

$(A \times 1 + B)(1+5) + C(1^2+25) = 125$

$(A + \frac{25}{2})(6) + \frac{5}{2}(26) = 125$

$A = -\frac{5}{2}$

$$= \frac{-\frac{5}{2}s + \frac{25}{2}}{s^2+25} + \frac{\frac{5}{2}}{s+5}$$

$$= \frac{-5/2\, s}{s^2+25} + \frac{\frac{25}{2}}{s^2+25} + \frac{\frac{5}{2}}{s+5}$$

$$i(t) = \mathcal{L}^{-1}\frac{-5/2\, s}{s^2+25} + \mathcal{L}^{-1}\frac{25/2}{s^2+25} + \mathcal{L}^{-1}\frac{5/2}{s+5}$$

$$= -\frac{5}{2}\cos 5t + \frac{25}{2}\sin 5t + \frac{5}{2}e^{-5t}$$

$$\frac{\quad}{(s+5)} = \frac{As+B}{s^2+25} + \frac{C}{s+5}$$

$$\frac{(As+B)(s+5) + C(s^2+25)}{(s^2+25)(s+5)}$$

$$(s+5) + C(s^2+25) = 125$$

$$(-5+5) + C((-5)^2+25) = 125$$

$$0$$

$$C \times 50 = 125$$

$$C = 5/2$$

$$(0+5) + C(0^2+25) = 125$$

$$5B + 25C = 125$$

$$5 \times B + 25 \times 5/2 = 125 \longrightarrow B = \frac{25}{2}$$

$$s = 1$$

$$(A \times 1 + B)(1+5) + C(1^2+25) = 125$$

$$\left(A + \frac{25}{2}\right)(6) + \frac{5}{2}(26) = 125$$

$$A = -\frac{5}{2}$$

$$= \frac{-\frac{5}{2}s + \frac{25}{2}}{s^2+25} + \frac{\frac{5}{2}}{s+5}$$

$$= \frac{-5/2\, s}{s^2+25} + \frac{\frac{25}{2}}{s^2+25} + \frac{\frac{5}{2}}{s+5}$$

$$i(t) = \mathcal{L}^{-1}\frac{-5/2\, s}{s^2+25} + \mathcal{L}^{-1}\frac{5/2 \times 5}{s^2+25} + \mathcal{L}^{-1}\frac{5/2}{s+5}$$

$$= -\frac{5}{2}\cos 5t + 5/2 \sin 5t + \frac{5}{2} e^{-5t}$$

pb 12

SOLVE $y''(t) + y(t) = 1$

GIVEN $y(0) = 1$, $y'(0) = 0$

$$y''(t) + y(t) = 1$$

$$\mathcal{L}\, y''(t) + \mathcal{L}\, y(t) = \mathcal{L}\, 1$$

$$\left[s^2 Y(s) - s\, y(0) - y'(0) \right] + Y(s) = \frac{1}{s}$$

$$\left[s^2 Y(s) - s \times 1 - 0 \right] + Y(s) = \frac{1}{s}$$

$$s^2 Y(s) - s + Y(s) = \frac{1}{s}$$

$$s^2 Y(s) + Y(s) = \frac{1}{s} + s$$

pb 12

SOLVE $y''(t) + y(t) = 1$

GIVEN $y(0) = 1$, $y'(0) = 0$

$$y''(t) + y(t) = 1$$

$$\mathcal{L}\, y''(t) + \mathcal{L}\, y(t) = \mathcal{L}\, 1$$

$$\left[s^2 Y(s) - s\, y(0) - y'(0)\right] + Y(s) = \frac{1}{s}$$

$$\left[s^2 Y(s) - s \times 1 - 0\right] + Y(s) = \frac{1}{s}$$

$$s^2 Y(s) - s + Y(s) = \frac{1}{s}$$

$$s^2 Y(s) + Y(s) = \frac{1}{s} + s$$

$$Y(s)\left[s^2 + 1\right] = \frac{1 + s^2}{s}$$

$$Y(s) = \frac{1}{s}$$

$$y(t) = \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \frac{1}{s}$$

$$= 1$$

pb 13 SOLVE $y'' - 3y' + 2y = 2e^{-t}$

WHERE $y(0) = 2$, $y'(0) = -1$

$$\mathcal{L}^{-1} y'' - \mathcal{L}\, 3y' + \mathcal{L}\, 2y = \mathcal{L}\, 2e^{-t}$$

$$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] - 3\left[s\,Y(s) - y(0)\right] + 2Y(s) = \frac{2}{s+1}$$

$$\left[s^2 Y(s) - s(2) - (-1)\right] - 3\left[s\,Y(s) - 2\right] + 2Y(s) = \frac{2}{s+1}$$

$$s^2 Y(s) - 2s + 1 - 3s\,Y(s) + 6 + 2Y(s) = \frac{2}{s+1}$$

$$s^2 Y(s) - 2s - 3s\,Y(s) + 7 + 2Y(s) = \frac{2}{s+1}$$

$$s^2 Y(s) - 3s\,Y(s) + 2Y(s) = \frac{2}{s+1} + 2s - 7$$

$$Y(s)\left[s^2 - 3s + 2\right] = \frac{2}{s+1} + 2s - 7$$

$$Y(s)\left[s^2 - 3s + 2\right] = \frac{2 + (2s-7)(s+1)}{s+1} = \frac{2s^2 - 5s - 5}{s+1}$$

$$Y(s) = \frac{2s^2 - 5s - 5}{(s^2 - 3s + 2)(s+1)}$$

$s^2 - 3s + 2 = $ $s - 2$, $s - 1$

$y'' ... + 2y = 2e^{-t}$

$y'(0) = -1$

$= \mathcal{L}\, 2e^{-t}$

$$\frac{2s^2 - 5s - 5}{[s-2][s-1][s+1]} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{s+1}$$

$$\frac{2s^2 - 5s - 5}{(s-2)(s-1)(s+1)} = \frac{A(s-1)(s+1) + B(s-2)(s+1) + C(s-2)(s-1)}{(s-2)(s-1)(s+1)}$$

$2s^2 - 5s - 5 = A(s-1)(s+1) + B(s-2)(s+1) + C(s-2)(s-1)$

$s = 1 \Rightarrow 2(1)^2 - 5(1) - 5 = A(1-1)(1+1) + B(1-2)(1+1) + C(1-2)(1-1)$

$-5-5 = B(-1)(2) \Longrightarrow -8 = -2B \Rightarrow B = 4$

$s = -1 \Rightarrow 2(-1)^2 - 5(-1) - 5 = A(-1-1)(-1+1) + B(-1-2)(-1+1) + C(-1-2)(-1-1)$

$C = 1/3$

$s = 2 \quad 2(2)^2 - 5(2) - 5 = A(2-1)(2+1) + B(2-2)(2+1) + C(2-2)(2-1)$

$A = -7/3$

$$\frac{-7/3}{s-2} + \frac{4}{s-1} + \frac{1/3}{s+1} \Longrightarrow -\frac{7}{3}e^{2t} + 4e^{t} + \frac{1}{3}e^{-t}$$