CALCULATION OF HARMONIC FREQUENCIES

Pb A WAVE FORM HAS A PERIOD $T = 40$ ms. CALCULATE THE FREQUENCIES OF FUNDAMENTAL, SECOND, THIRD AND FOURTH HARMONICS

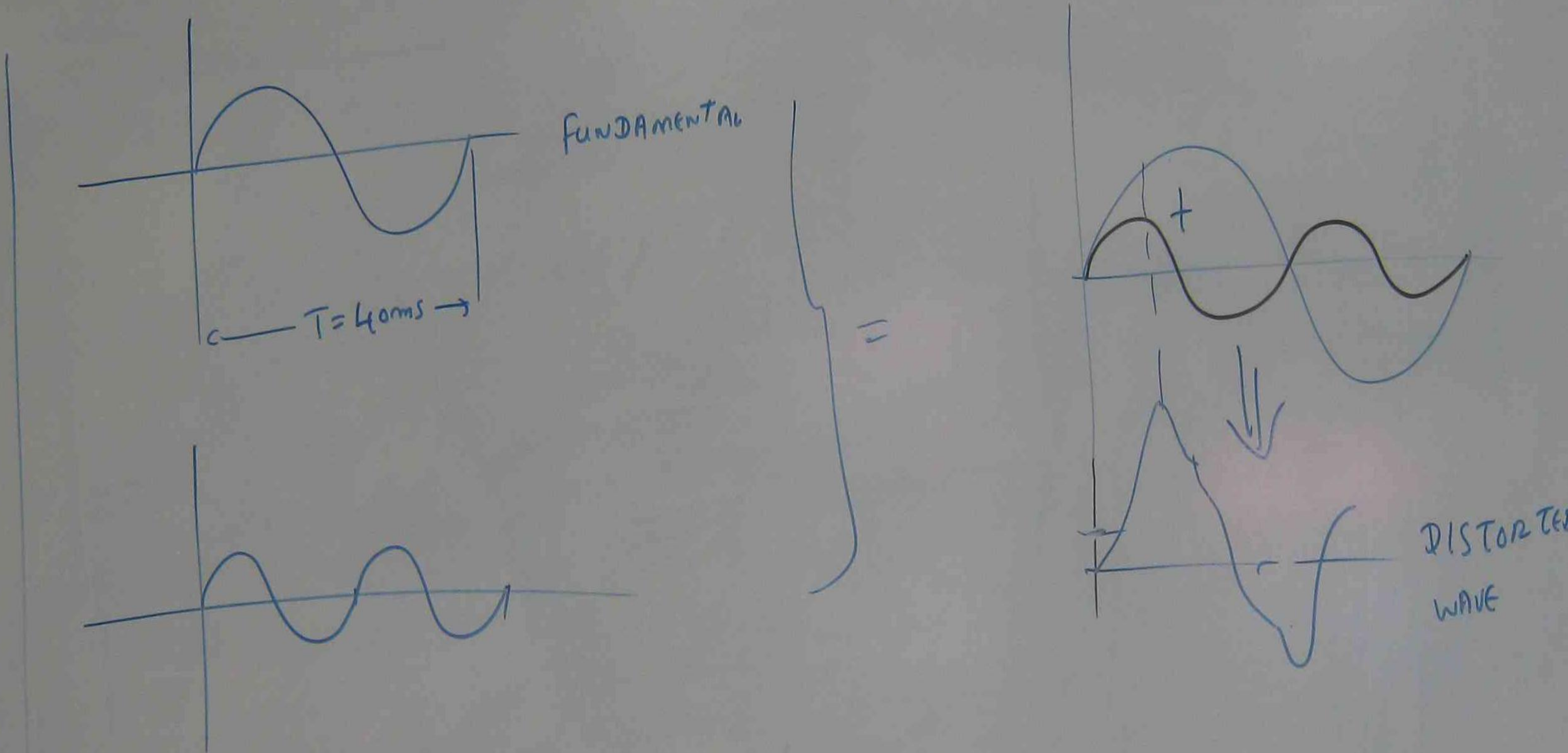
$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = \frac{10^{3}}{40} = 25 \text{ Hz}$$

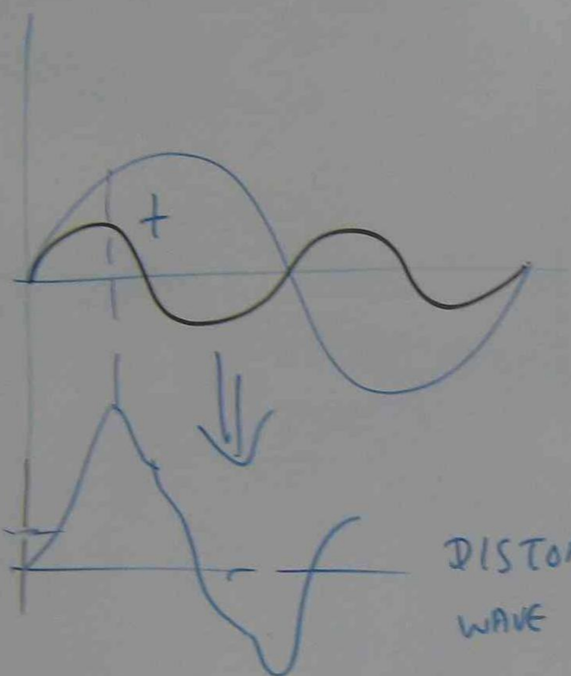
FUNDAMENTAL = 25 Hz

SECOND HARMONIC = $2f = 2 \times 25 = 50$ HZ

THIRD HARMONIC = $3f = 3 \times 25 = 75$ HZ

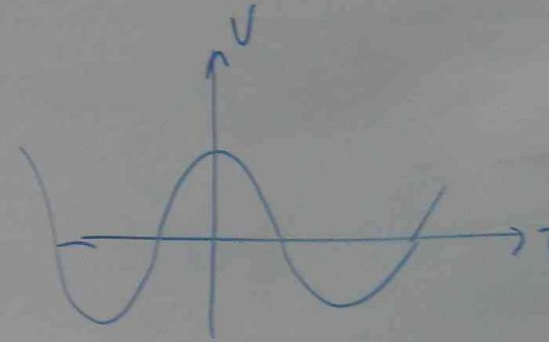
FOURTH HARMONIC = $4f = 4 \times 25 = 100$ HZ


FUNDAMENTAL
T=40ms
+
=
DISTORTED
WAVE



DISTORTED WAVE

(a)

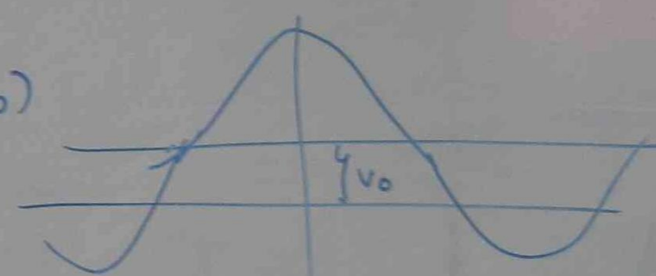


f_0 = FUNDAMEN
FREQUEN

EVEN SYMMETRY

$$V(t) = A \cos 2\pi f_0 t$$

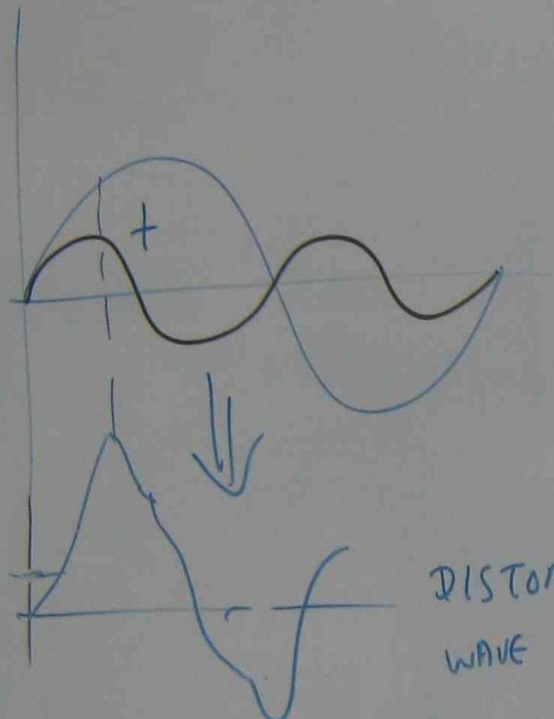
(b)



EVEN SYMMETRY + DC CONSTANT

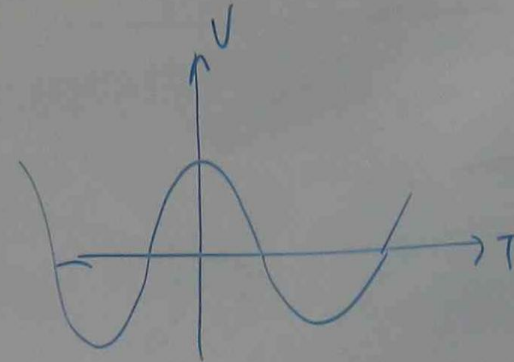
$$V(t) = V_0 + A \cos 2\pi f_0 t$$

f_0 = FUNDAMENTAL FREQUENCY



DISTORTED WAVE

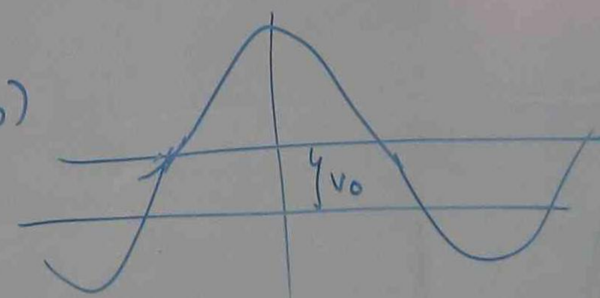
(a)



EVEN SYMMETRY

$$V(t) = A \cos 2\pi f_0 t$$

(b)

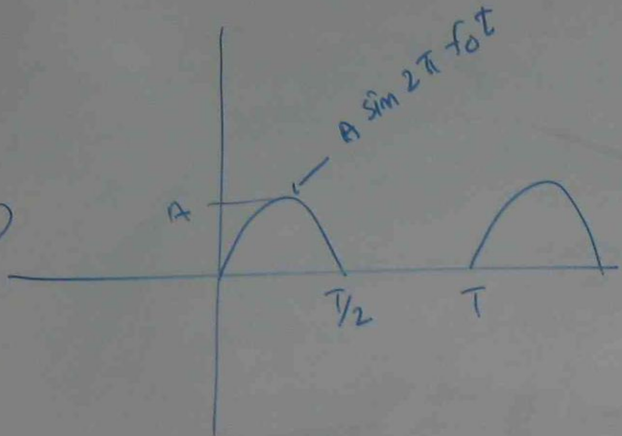


$$V(t) = V_0 + A \cos 2\pi f_0 t$$

EVEN SYMMETRY + DC CONSTANT

AL

(c)

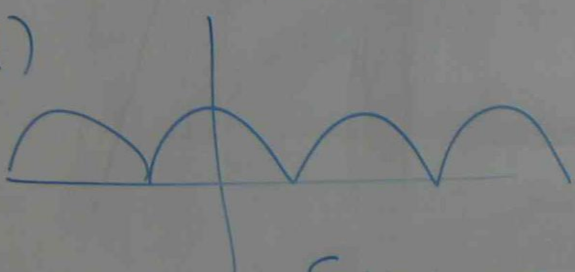


HALF WAVE RECTIFIER

$$v(t) = \frac{A}{\pi} + \frac{A}{2}\sin 2\pi f_0 t - \frac{2A}{3\pi}\cos 2\pi (2f_0)t - \frac{2A}{15\pi}\cos 2\pi (4f_0)t + \cdots$$

$$= \frac{A}{\pi} + \frac{A}{2}\sin 2\pi f_0 t + \sum_{n=2}^{\infty} \frac{A\left[1+(-1)^n\right]}{\pi(1-n^2)}\cos 2\pi (nf_0)t$$

d)

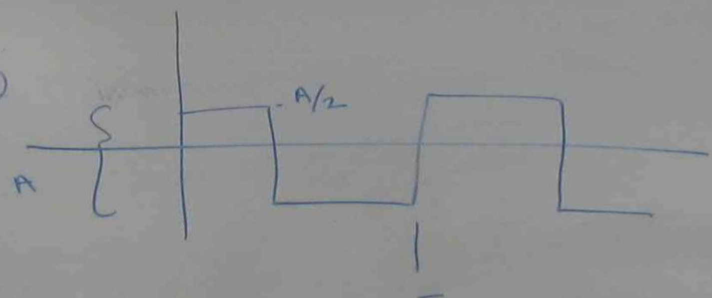


FULL WAVE RECTIFIER

$$v(t) = \frac{2A}{\pi} + \frac{4A}{3\pi}\cos 2\pi f_0 t - \frac{4A}{15\pi}\cos 2\pi (2f_0)t + \cdots$$

$$= \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A(-1)^n}{\pi\left[1-(2n)^2\right]}\cos 2\pi (nf_0)t$$

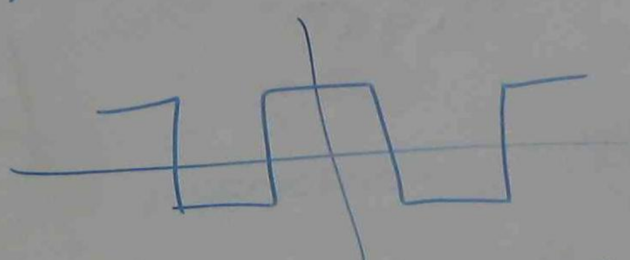
(e)



$$v(t) = \frac{2A}{\pi}\sin 2\pi f_0 t + \frac{2A}{3\pi}\sin 2\pi(3f_0)t + \cdots$$

$$= \sum_{\substack{n=1 \\ \text{ODD ONLY}}}^{\infty} \frac{2A}{n\pi}\sin 2\pi(nf_0)t$$

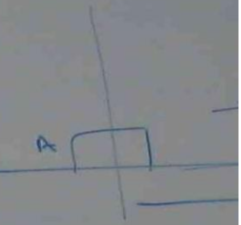
(f)



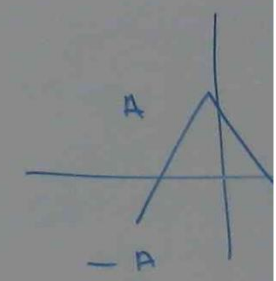
$$v(t) = \frac{2A}{\pi}\cos 2\pi f_0 t - \frac{2A}{3\pi}\cos 2\pi(3f_0)t + \frac{2A}{5\pi}\cos 2\pi(5f_0)t$$

$$= \sum_{n=1}^{\infty}\left(A\,\frac{\sin n\pi/2}{n\pi/2}\right)\cos 2\pi(nf_0)t$$

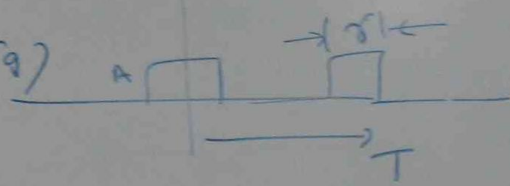
(g)



(h)



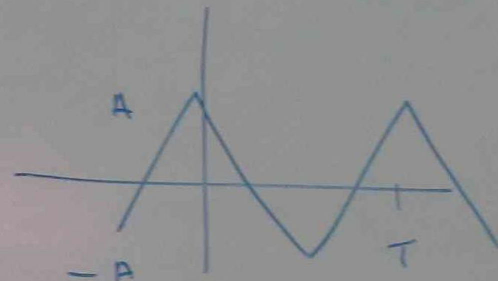
(g)



$$v(t) = \frac{A\tau}{T} + \sum_{n=1}^{\infty} \left(\frac{2A\tau}{T}\right)\left(\frac{\sin\frac{n\pi\tau}{T}}{\frac{n\pi\tau}{T}}\right)$$

$\cos 2\pi (nf_0)t$

(h)



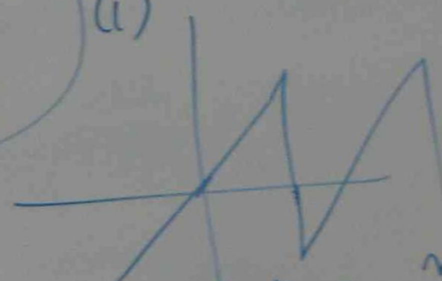
$$v(t) = \frac{8A}{\pi^2}\cos 2\pi f_0 t + \frac{8A}{9\pi^2}\cos 2\pi(3f_0)t + \cdots$$

$$= \sum_{n=\text{odd}}^{\infty} \frac{8A}{(n\pi)^2}\cos 2\pi(nf_0)t$$

$\frac{A}{5\pi}\cos 2\pi(5f_0)t$

$nf_0)t$

(i)



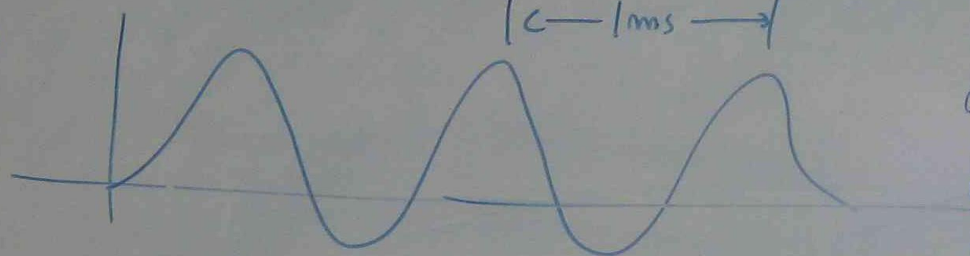
$$v(t) = \frac{2A}{\pi}\left[\sin 2\pi f_0 t - \frac{1}{2}\sin 2\pi(2f_0)t + \cdots\right]$$

$$v(t) = \sum_{n=1}^{\infty} (-1)^n \left(\frac{2A}{n\pi}\right)\sin 2\pi(nf_0)t$$

pb

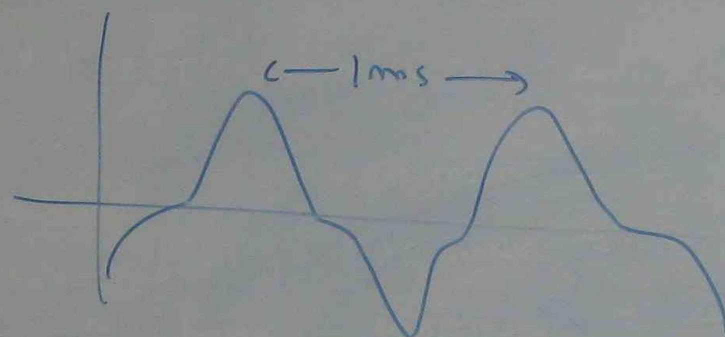
FOR EACH OF THE FOLLOWING WAVE FORMS, STATE THE FREQUENCIES OF THE FIRST THREE COMPONENTS PRESENT

(a)



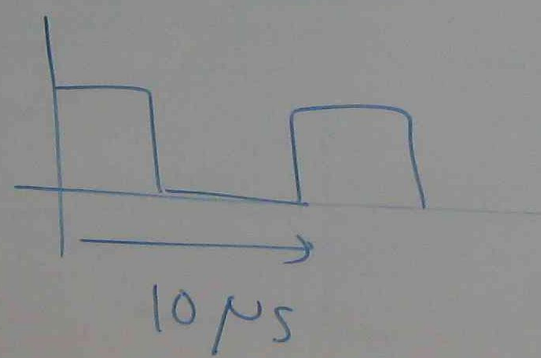
OUT PUT OF COMMON EMITTER AMPLIFIER

(b)



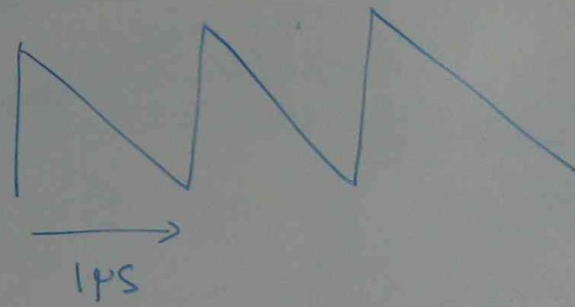
OUT PUT OF AN AMPLIFIER WITH CROSS OVER DISTORTION

(c)



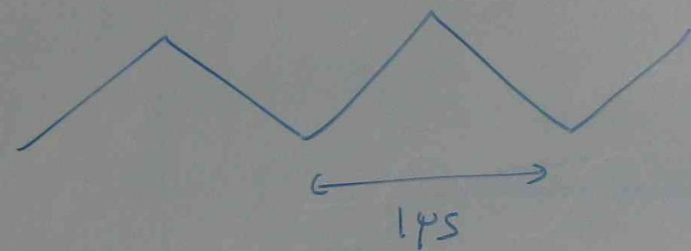
SQUARE WAVE

(d)



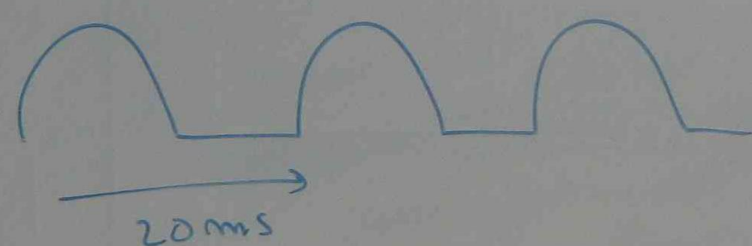
SAW TOOTH WAVE

(e)



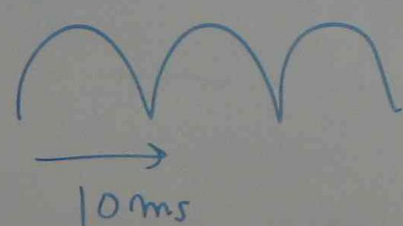
TRIANGULAR WAVE

(f)



OUTPUT OF A HALF WAVE RECTIFIER

(g)



OUTPUT OF A FULL WAVE RECTIFIER

(a) $f = \frac{1}{T} = \frac{1}{1 \times 10^{-3}} = 10^{3}$ Hz = 1 KHZ (FUNDAMENTAL)

2 KHZ (2nd HARMONIC)

3 KHZ (3rd HARMONIC)

(b)

(c) $f = \frac{1}{T} = \frac{1}{10 \times 10^{-6}} = 10^{5}$ HZ = 100 KHZ, 200 KHZ, 300 KHZ

(d) $f = \frac{1}{T} = \frac{1}{10^{-6}} = 10^{6}$ HZ = 1 mHZ, 2 mHZ, 3 mHZ

(e)

(f) $f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = \frac{10^{3}}{20}$ = 50 Hz, 100 Hz, 150 Hz,

(g) $f = \frac{1}{T} = \frac{1}{10 \times 10^{-3}} = \frac{10^{3}}{10}$ = 100 Hz, 200 Hz, 300 Hz

TRIGOMETRIC REPRESENTATION OF FOURIER SERIES

GENERALIZED VOLTAGE WAVE FORM

$$V(t) = V_{dc} + V_1 \sin(\omega t + \theta_1) + V_2 \sin(2\omega t + \theta_2) + V_3 \sin(3\omega t + \theta_3) + V_4 \sin(4\omega t + \theta_4) + \cdots$$

$V_{dc} = DC$, $V_1 \sin(\omega t + \theta_1)$: FUNDAMENTAL

$V_2 \sin(2\omega t + \theta_2) =$ 2nd HARMONIC

$V_3 \sin(3\omega t + \theta_3) =$ 3rd HARMONIC

$V_4 \sin(4\omega t + \theta_4) =$ 4th HARMONIC

$$\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$$

$$\sin(\omega t + \theta_1) = \sin\omega t \cos\theta_1 + \cos\omega t \sin\theta_1$$

$$\sin(2\omega t + \theta_2) = \sin 2\omega t \cos\theta_2 + \cos 2\omega t \sin\theta_2$$

$$\sin(3\omega t + \theta_3) = \sin 3\omega t \cos\theta_3 + \cos 3\omega t \sin\theta_3$$

$$V(t) = V_{dc} + V_1\left[\sin\omega t\cos\theta_1 + \cos\omega t \sin\theta_1\right] + V_2\left[\sin 2\omega t \cos\theta_2 + \cos 2\omega t \sin\theta_2\right] + \cdots$$

$$= V_{dc} + \underbrace{V_1\cos\theta_1}_{B_1} \sin\omega t + \underbrace{V_1 \sin\theta_1}_{A_1} \cos\omega t + \underbrace{V_2\cos\theta_2}_{B_2} \sin 2\omega t + \underbrace{V_2 \sin\theta_2}_{A_2} \cos 2\omega t + \cdots$$

$V(t) = V_d$

WHERE

B_1

A_1

$V(t) =$

A_n

B

$\sin(4\omega t+\theta_4)+ - -$

$$v(t)=V_{dc}+B_1\sin\omega t+A_1\cos\omega t+B_2\sin 2\omega t+A_2\cos 2\omega t+ -$$

WHERE

$B_1=V_1\cos\theta_1$

$A_1=V_1\sin\theta_1$

$B_2=V_2\cos\theta_2$

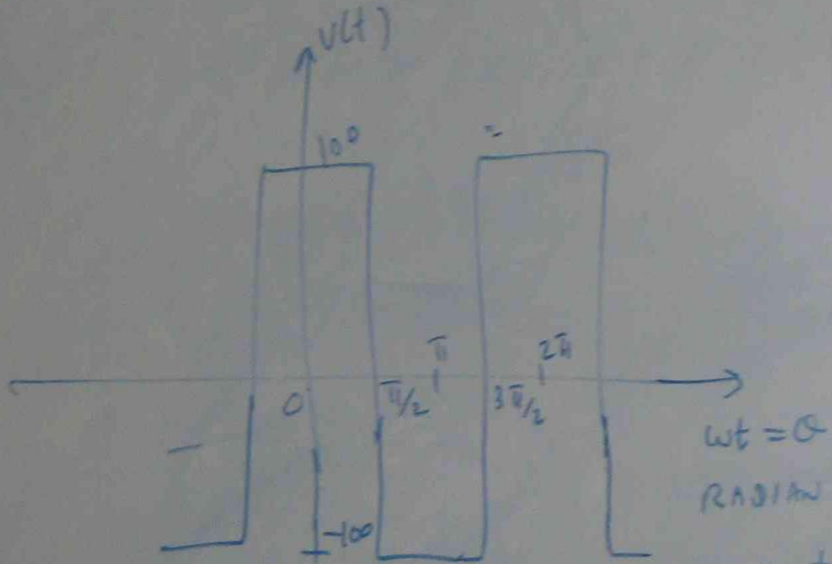
$A_2=V_2\sin\theta_2$

$$v(t)=\underbrace{V_{dc}+A_1\cos\omega t+A_2\cos 2\omega t+ - - -}_{\text{EVEN}}+\underbrace{B_1\sin\omega t+B_2\sin 2\omega t+ - - -}_{\text{ODD}}$$

$$A_n=\frac{1}{\pi}\int_0^{2\pi} v(t)\cos(n\theta)\,d\theta \qquad n=1,2,3 - - -$$

$$B_n=\frac{1}{\pi}\int_0^{2\pi} v(t)\sin(n\theta)\,d\theta \qquad n=1,2,3 - -$$

P6 FIND THE FIRST FOUR TERMS IN THE GIVEN TRIGOMETRIC FOURIER SERIES FOR THE SQUARE WAVE



$0 \rightarrow \pi/2 \Rightarrow v(t) = 100 \text{ V}$

$\pi/2 \rightarrow 3\pi/2 \Rightarrow v(t) = -100$

$3\pi/2 \rightarrow 2\pi \Rightarrow v(t) = +100$

$$V(t) = A_1 \cos\omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t$$

WHERE $A_m = \frac{1}{\pi} \int_0^{2\pi} v(t) \cos(m\theta)\, d\theta$

$$A_m = \frac{1}{\pi} \int_0^{\pi/2} 100 \cos(m\theta)\, d\theta + \frac{1}{\pi} \int_{\pi/2}^{3\pi/2} (-100 \ldots$$

$$\frac{100}{\pi} \Bigg[\underbrace{\int_0^{\pi/2} \cos(m\theta)\, d\theta}_{I_1} - \underbrace{\int_{\pi/2}^{3\pi/2} \cos(m\theta \ldots}_{I_2 \ldots}$$

GIVEN TRIGONOMETRIC

$0 \longrightarrow \pi/2 \Longrightarrow v(t) = 100 \text{ V}$

$\pi/2 \longrightarrow 3\pi/2 \Longrightarrow v(t) = -100$

$3\pi/2 \longrightarrow 2\pi \Longrightarrow v(t) = +100$

$$A_n = \frac{1}{\pi}\int_0^{\pi/2} 100\cos(n\theta)\,d\theta + \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} (-100)\cos(n\theta)\,d\theta + \frac{1}{\pi}\int_{3\pi/2}^{2\pi} 100\cos(n\theta)\,d\theta$$

$$\frac{100}{\pi}\left[\underbrace{\int_0^{\pi/2}\cos(n\theta)\,d\theta}_{I_1} - \underbrace{\int_{\pi/2}^{3\pi/2}\cos(n\theta)\,d\theta}_{I_2} + \underbrace{\int_{3\pi/2}^{2\pi}\cos n\theta\,d\theta}_{I_3}\right]$$

$\sin\theta$

$\cos\theta$

$$I_1 = \int_0^{\pi/2} \cos n\theta \, d\theta \qquad I_3 = \int_{3\pi/2}^{2\pi} \cos n\theta \, d\theta$$

$$I_2 = \int_{\pi/2}^{3\pi/2} \cos n\theta \, d\theta$$

$$\int^{2\pi} 100 \cos(n\theta) d\theta$$

$\sin\theta$	$\dfrac{d\sin\theta}{d\theta} = \cos\theta$ $\int \sin\theta \, d\theta = -\cos\theta$
$\cos\theta$	$\dfrac{d\cos\theta}{d\theta} = -\sin\theta$ $\int \cos\theta \, d\theta = \sin\theta$

$$\int \cos\theta \, d\theta = \sin\theta$$

$$\int \cos 2\theta \, d2\theta = \sin 2\theta$$

$$\int \cos n\theta \, dn\theta = \sin n\theta$$

$$I_1 = \int_0^{\pi/2} \cos n\theta \, d\theta = \int_0^{\pi/2} \frac{\cos n\theta \, dn\theta}{n}$$

$$= \frac{1}{n} \int_0^{\pi/2} \cos n\theta \, dn\theta = \frac{1}{n} \left[\sin n\theta \right]_0^{\pi/2}$$

$$= \frac{1}{n} \left[\sin n\tfrac{\pi}{2} - \sin 0 \right]$$

$$= \frac{1}{n} \sin n\tfrac{\pi}{2}$$

$$I_2 = \int_{\pi/2}^{3\pi/2} \cos n\theta \, d\theta = \int_{\pi/2}^{3\pi/2} \frac{\cos n\theta \, dn\theta}{n} = \frac{1}{n}\Big[\sin n\theta\Big]_{\pi/2}^{3\pi/2} = \frac{1}{n}\left[\sin \frac{3n\pi}{2} - \sin n\frac{\pi}{2}\right]$$

$$I_3 = \int_{3\pi/2}^{2\pi} \cos n\theta \, d\theta = \int_{3\pi/2}^{2\pi} \frac{\cos n\theta \, dn\theta}{n} = \frac{1}{n}\Big[\sin n\theta\Big]_{3\pi/2}^{2\pi} = \frac{1}{n}\left[\sin 2n\pi - \sin \frac{3n\pi}{2}\right]$$

$$A_n = \frac{100}{\pi}\left[\underbrace{\frac{1}{n}\sin\frac{n\pi}{2}}_{I_1} + \underbrace{\frac{1}{n}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]}_{I_2} + \underbrace{\frac{1}{n}\left[\sin 2n\pi - \sin\frac{3n\pi}{2}\right]}_{I_3}\right]$$

$$= \frac{100}{\pi}\left[\cancel{\frac{1}{n}\sin\frac{n\pi}{2}} + \cancel{\frac{1}{n}\sin\frac{3n\pi}{2}} - \cancel{\frac{1}{n}\sin\frac{n\pi}{2}} + \frac{1}{n}\sin 2n\pi - \cancel{\frac{1}{n}\sin\frac{3n\pi}{2}}\right]$$

$$A_n = \frac{100}{\pi} \times \frac{1}{n} \sin 2n\pi$$

$$A_1 = \frac{100}{\pi} \times \frac{1}{1} \sin 2 \times 1 \pi$$

$$= \frac{100}{\pi} \sin 2\pi = 0$$

$$A_2 = \frac{100}{\pi} \times \frac{1}{2} \sin 2 \times 2\pi$$

$$= \frac{100}{\pi} \times \frac{1}{2} \sin 4\pi = 0$$

$$A_3 = \frac{100}{\pi} \times \frac{1}{3} \sin 2 \times 3\pi = 0$$

$=$

$$I_2 = \int_{\pi/2}^{3\pi/2} \cos n\theta \, d\theta = \int_{\pi/2}^{3\pi/2} \frac{\cos n\theta \, dn\theta}{n} = \frac{1}{n}\Big[\sin n\theta\Big]_{\pi/2}^{3\pi/2} = \frac{1}{n}\left[\sin \frac{3n\pi}{2} - \sin \frac{n\pi}{2}\right]$$

$$I_3 = \int_{3\pi/2}^{2\pi} \cos n\theta \, d\theta = \int_{3\pi/2}^{2\pi} \frac{\cos n\theta \, dn\theta}{n} = \frac{1}{n}\Big[\sin n\theta\Big]_{3\pi/2}^{2\pi} = \frac{1}{n}\left[\sin 2n\pi - \sin \frac{3n\pi}{2}\right]$$

$$A_n = \frac{100}{\pi}\left[\underbrace{\frac{1}{n}\sin\frac{n\pi}{2}}_{I_1} - \underbrace{\frac{1}{n}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]}_{I_2} + \underbrace{\frac{1}{n}\left[\sin 2n\pi - \sin\frac{3n\pi}{2}\right]}_{I_3}\right]$$

$$= \frac{100}{\pi}\left[\frac{1}{n}\sin\frac{n\pi}{2} - \frac{1}{n}\sin\frac{3n\pi}{2} + \frac{1}{n}\sin\frac{n\pi}{2} + \frac{1}{n}\sin 2n\pi - \frac{1}{n}\sin\frac{3n\pi}{2}\right]$$

$$\frac{100}{\pi}\left[\frac{2}{n}\sin\frac{n\pi}{2} + \frac{1}{n}\sin 2n\pi - \frac{2}{n}\sin\frac{3n\pi}{2}\right]$$

$$A_1 = \frac{100}{\pi}\left[\frac{2}{1}\sin\pi/2 + \frac{1}{1}\sin 2\pi - \frac{2}{1}\sin\frac{3\pi}{2}\right]$$

$$= \frac{100}{\pi}\left[2 + 0 - 2\times(-1)\right]$$

$$= \frac{100}{\pi}\left[2 + 2\right] = \frac{400}{\pi}$$

$$A_2 = \frac{100}{\pi}\left[\frac{2}{2}\sin 2\frac{\pi}{2} + \frac{1}{2}\sin 2\times 2\pi - \frac{2}{2}\sin\frac{3\times 2\pi}{2}\right. \to 0$$

$$A_3 = \frac{100}{\pi}\left[\frac{2}{3}\sin\frac{3\pi}{2} + \frac{1}{3}\underbrace{\sin 6\pi}_{0} - \frac{2}{3}\underbrace{\sin\frac{6\pi}{2}}_{0}\right]$$

$$= \frac{100}{\pi}\left[-\frac{2}{3}\right] = -\frac{200}{3\pi}$$

Rms VALUE OF COMPLEX WAVE FORM

$$V(t) = V_{dc} + V_1 \sin(\omega t + \theta_1) + V_2 \sin(2\omega t + \theta_2) + V_3 \sin(3\omega t + \theta_3) + V_4 \sin(4\omega t + \theta_4)$$

$$V_{rms} = \sqrt{V_{dc}^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2} + \frac{V_4^2}{2}}$$

$$V_1^2 = A_1^2 + B_1^2$$

$$V_2^2 = A_2^2 + B_2^2$$

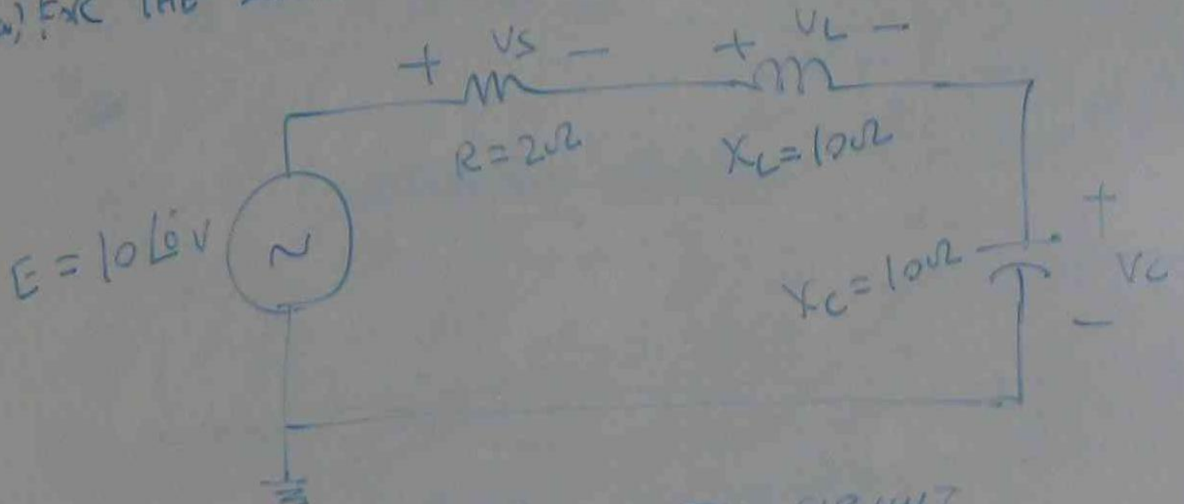
$$V(t) = V_{dc} + A_1 \cos\omega t + A_2 \cos 2\omega t + \cdots B_1 \sin\omega t + B_2 \sin 2\omega t + \cdots$$

$$V_{rms} = \sqrt{V_{dc}^2 + \frac{A_1^2}{2} + \frac{A_2^2}{2} + \cdots \frac{B_1^2}{2} + \frac{B_2^2}{2}}$$

SERIES RESONANCE CALCULATIONS

Pb.

(a) FOR THE SERIES RESONANT CIRCUIT OF GIVEN FIGURE, FIND I, V_R, V_L AND V_C



(b) WHAT IS THE Q OF THE CIRCUIT?

(C) IF THE RESONANT FREQUENCY IS 5000 HZ, FIND BANDWIDTH

(d) WHAT IS THE POWER DISSIPATED IN THE CIRCUIT AT THE HALF POWER FREQUENCY.

(a) $Z = R +$

$= 2$

$= 2$

$I = \frac{V}{Z}$

$V_R = I \times R =$

$V_L = I \times$

I, V_R, V_L AND V_C

(b)

(a) $Z = R + jX_L - jX_C$

$= 2 + j10 - j10$

$= 2\ \Omega$

(c)

DWIDTH

AT

$I = \dfrac{V}{Z} = \dfrac{10\angle 0}{2} = 5\angle 0$ Amp.

$V_R = I \times R = 5\angle 0 \times 2 = 10\angle 0^\circ\ V$

$V_L = I \times X_L = 5\angle 0 \times 10\angle 90^\circ$

$= 50\angle 90^\circ\ V$

POWER

$V_C = I\, X_C = 5\angle 0 \times 10\angle -90$

$= 50\angle -90^\circ\ V$