APPLICATION OF GAMMA FUNCTION IN LAPLACE TRANSFORM

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(n+1) = n\,\Gamma(n)$$

$$\Gamma(3/2) = \Gamma\left(\tfrac{1}{2}+1\right) = \tfrac{1}{2}\,\Gamma\left(\tfrac{1}{2}\right) = \tfrac{1}{2} \times \sqrt{\pi}$$

$$\boxed{\mathcal{L}^{-1}\frac{n!}{s^{n+1}} = t^{n}}$$

n = INTEGER

$n+1$ = INTEGER

THEN WE CAN USE THIS EQUATION

FOR THE POWERS WITH NO INTEGER → APPLY GAMMA FUNCTION

$$\mathcal{L}^{-1}\frac{1}{s^{p}} = \frac{t^{p-1}}{\Gamma(p)}$$

p IS NOT INTEGER

pb 6 FIND $\mathcal{L}^{-1}$

$\mathcal{L}^{-1}$

$\mathcal{L}^{-1}$

$\mathcal{L}^{-1}$

→ APPLY GAMMA FUNCTION

pb 6 FIND $\mathcal{L}^{-1}\dfrac{4-5s}{s^{3/2}}$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}}-\mathcal{L}^{-1}\frac{5s}{s^{3/2}}$$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}}-\mathcal{L}^{-1}\frac{5}{s^{3/2}\,s^{-1}}$$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}}-\mathcal{L}^{-1}\frac{5}{s^{3/2-1}}$$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}}-\mathcal{L}^{-1}\frac{5}{s^{1/2}}$$

$$\boxed{\mathcal{L}^{-1}\frac{1}{s^{p}}=\frac{t^{p-1}}{\Gamma(p)}}$$

$$\mathcal{L}^{-1}\frac{4}{s^{3/2}}=4\,\mathcal{L}^{-1}\frac{1}{s^{3/2}}=4\,\frac{t^{3/2-1}}{\Gamma(3/2)}=\frac{4t^{1/2}}{\frac{1}{2}\sqrt{\pi}}=\frac{8t^{1/2}}{\sqrt{\pi}}$$

$$\mathcal{L}^{-1}\frac{5}{s^{1/2}}=5\,\mathcal{L}^{-1}\frac{1}{s^{1/2}}=5\times\frac{t^{1/2-1}}{\Gamma(1/2)}=\frac{5t^{-1/2}}{\sqrt{\pi}}$$

$$\Longleftrightarrow\ \frac{8t^{1/2}}{\sqrt{\pi}}-\frac{5t^{-1/2}}{\sqrt{\pi}}=\frac{1}{\sqrt{\pi}}\left(8t^{1/2}-5t^{-1/2}\right)$$

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LAPLACE TRANSFORM OF GAMMA FUNCTION

$$\mathcal{L}\, t^{p} = \frac{\Gamma(p+1)}{s^{p+1}} \Longleftrightarrow \mathcal{L}^{-1} \frac{\Gamma(p+1)}{s^{p+1}} = t^{p}$$

P = NOT INTEGER

Pb 10 FIND $\mathcal{L}^{-1} \dfrac{1}{s^{2}+2s}$

$$\mathcal{L}^{-1}\frac{1}{s^{2}+2s} = \mathcal{L}^{-1}\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$\frac{1}{s(s+2)} = \frac{A(s+2)+Bs}{s(s+2)}$$

$= t^{p}$

$A(s+2) + Bs = 1$

If $s = 0 \Rightarrow A(0+2) + B \times 0 = 1$

$2A + 0 = 1$

$A = 1/2$

If $s = 1$

$A(1+2) + B \times 1 = 1$

$A \times 3 + B = 1$

$\frac{1}{2} \times 3 + B = 1$

$B = 1 - \frac{3}{2} = -\frac{1}{2}$

$\therefore \frac{1}{s(s+2)} = \frac{1/2}{s} + \frac{(-1/2)}{s+2}$

$\mathcal{L}^{-1} \frac{1}{s(s+2)} = \mathcal{L}^{-1} \frac{1/2}{s} + \mathcal{L}^{-1} \frac{-1/2}{s+2}$

$$\boxed{\mathcal{L}^{-1} \frac{1}{s} = 1 \ , \quad \mathcal{L}^{-1} \frac{1}{s+a} = e^{-at}}$$

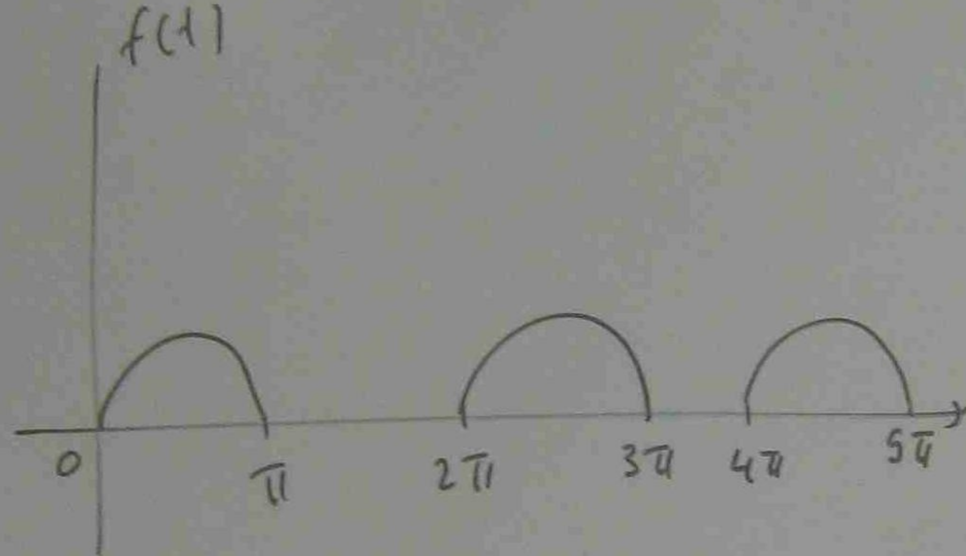
$\frac{1}{2} \mathcal{L}^{-1} \frac{1}{s} - \frac{1}{2} \mathcal{L}^{-1} \frac{1}{s+2}$

$\frac{1}{2} \times 1 - \frac{1}{2} \times e^{-2t}$

$\frac{1}{2} - \frac{1}{2} e^{-2t}$

$\frac{1}{2}\left(1 - e^{-2t}\right)$ ※

FUNCTION OF HALF WAVE RECTIFIED DC



$$F(s) = \frac{1}{(s^2+1)(1-e^{-\pi s})}$$