LAPLACE TRANSFORM

LOGARITHM

NUMERICAL

LOGARITHM

MULTIPLICATION $\longrightarrow$ ADDITION

$$\log_{10} MN = \log_{10} M + \log_{10} N$$

DIVISION $\longrightarrow$ SUBSTRACTION

$$\log_{10} \frac{M}{N} = \log_{10} M - \log_{10} N$$

ANTI LOG $\longrightarrow$ LOAG $\longrightarrow$ NUMBER

CALCULUS		LAPLACE $\mathcal{L}$
INTEGRATION	$\longrightarrow$	MULTIPLICATION
DIFFERENTIATION	$\longrightarrow$	DIVISION

ANTI LAPLACE = $\mathcal{L}^{-1}$

$$\mathcal{L}\, f(t) = F(s)$$

$$\mathcal{L}^{-1} F(s) = f(t)$$

LAPLACE TRANSFORM

INVERSE LAPLACE

INPUT FUNCTION $\longrightarrow$ $\boxed{\text{OP-AMP}}$ ——

$\boxed{\text{PHASE SHIFT OSCILLATOR}}$ ——

$A \sin \omega t$

$I(s)$

$H(s) \leftarrow$ SYSTEM FUNCTI

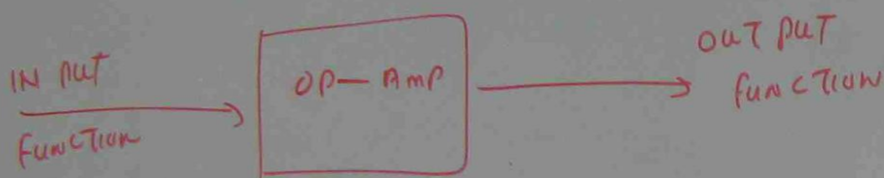
LAPLACE $\mathcal{L}$

MULTIPLICATION

DIVISION

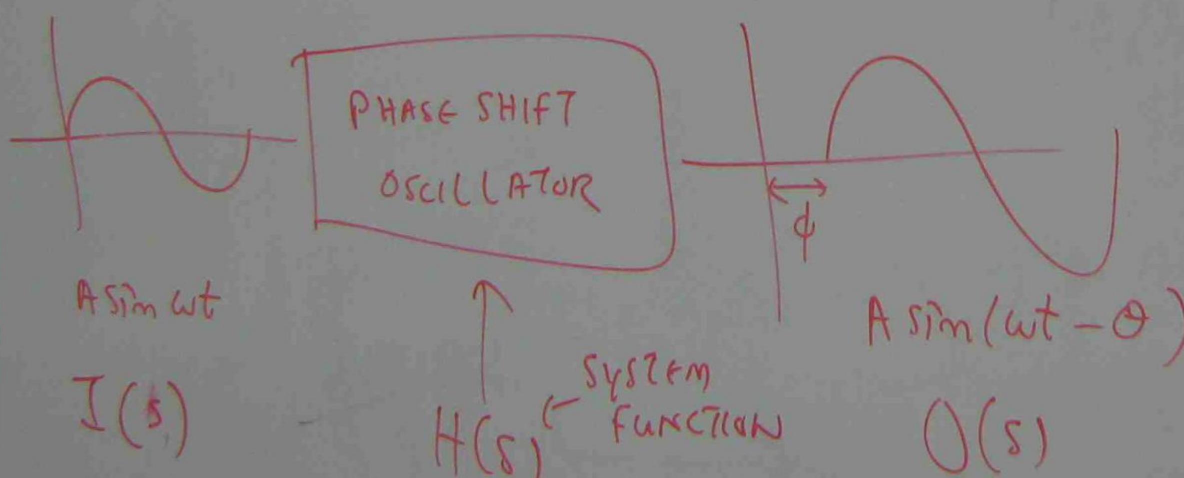
LAPLACE TRANSFORM OF TIME DOMAIN FUNCTION = FREQUENCY DOMAIN FUNCTION

INVERSE LAPLACE TRANSFORM OF FREQUENCY DOMAIN FUNCTION = TIME DOMAIN FUNCTION



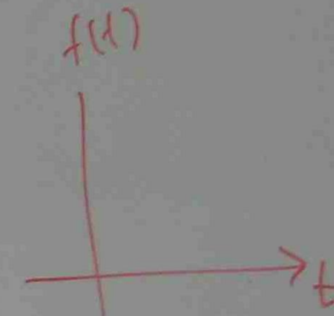
INPUT × SYSTEM FUNCTION = OUT PUT FUNCTION

$$I(s) \times H(s) = O(s)$$



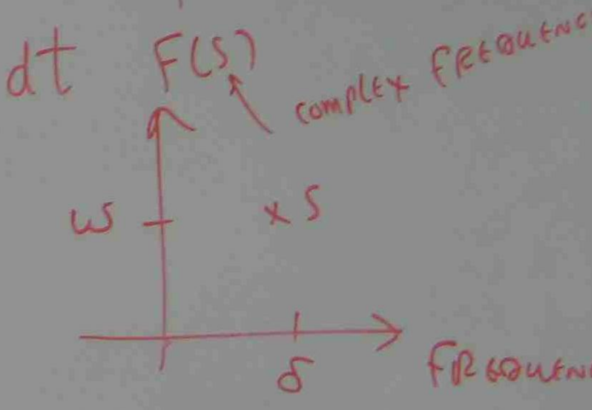
$$\boxed{\text{SYSTEM FUNCTION } H(s) = \frac{O(s)}{I(s)}}$$

LAPLACE TRANSFORM IS UTILIZED TO DETERMINE THE THE SYSTEM FUNCTION OF ELECTRONICS SYSTEMS. SO THAT DEPENDING ON THE KIND OF INPUT FUNCTION, THE OUTPUT FUNCTION CAN BE ESTIMATED.



LAPLACE CONVERSION FROM INTEGRATION

$$\mathcal{L} f(t) = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

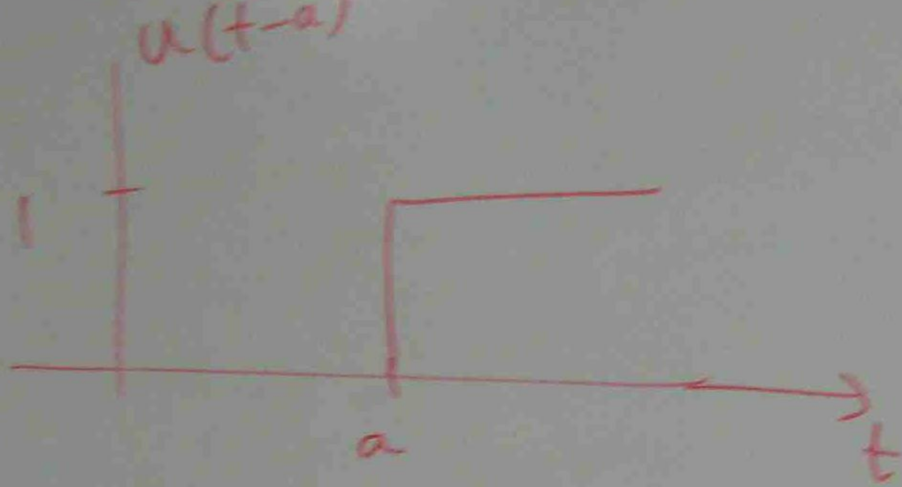


LAPLACE TRANSFORM TABLE

	$f(t)$	$\mathcal{L} f(t) = F(s)$	
$\mathcal{L}$	1	$\frac{1}{s}$	$s > 0$
$\mathcal{L}$	t^n $n = 1, 2, 3, 4$	$\frac{n!}{s^{n+1}}$	
$\mathcal{L}$	t^p	$\frac{P(p+1)}{s^{p+1}}$	$s > 0$
$\mathcal{L}$	e^{at}	$\frac{1}{s-a}$	$s > a$
$\mathcal{L}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$s > 0$
$\mathcal{L}$	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$s > 0$
$\mathcal{L}$	$\cosh at$	$\frac{a}{s^2-a^2}$	

$\mathcal{L}^{-1} F(s)$	$= f(t)$
$\mathcal{L}^{-1} \frac{1}{s}$	1
$\mathcal{L}^{-1} \frac{n!}{s^{n+1}}$	t^n
$\mathcal{L}^{-1} \frac{P(p+1)}{s^{p+1}}$	t^p
$\mathcal{L}^{-1} \frac{1}{s-a}$	e^{at}
$\mathcal{L}^{-1} \frac{s}{s^2+\omega^2}$	$\cos \omega t$
$\mathcal{L}^{-1} \frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
$\mathcal{L}^{-1} \frac{a}{s^2-a^2}$	$\cosh at$

THE UNIT STEP FUNCTION



$$\mathcal{L}\, u(t-a) = \frac{e^{-as}}{s}$$

$$\mathcal{L}^{-1} \frac{e^{-as}}{s} = u(t-a)$$

pb 1

PROVE THAT $\mathcal{L}\, e^{at} = \dfrac{1}{s-a}$ IF $s > a$

$$\boxed{\mathcal{L}\, f(t) = F(s) = \int_0^\infty e^{-st} f(t)\,dt}$$

$$\mathcal{L}\, e^{at} = \int_0^\infty e^{-st}\, e^{at}\, dt$$

$$= \int_0^\infty e^{-st+at}\, dt$$

$$\int_0^{\infty} e^{-(s-a)t}\, dt$$

$$\int e^{u}\, du = e^{u} + c$$

$$d-(s-a)t = -(s-a)\, dt$$

$$dt = \frac{d\,(-(s-a)\,t}{-(s-a)}$$

$$\int_0^{\infty} \frac{e^{-(s-a)t}\, d-(s-a)\,t}{-(s-a)} = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^{\infty}$$

$$= \frac{e^{-(s-a)\infty}}{-(s-a)} - \frac{e^{-(s-a)\times 0}}{-(s-a)}$$

$$= \frac{\cancel{e^{-\infty}}}{-(s-a)} + \frac{e^{0}}{s-a}$$

$\to 0$

$$= \frac{1}{s-a}$$

Pb(2) FIND THE LAPLACE TRANSFORM OF THE FOLLOWINGS

(a) $3e^{-4t}$ (b) $3t^2$ (c) $4\cos 5t$

(d) $\sin 2t$ (e) $\frac{-3}{\sqrt{t}}$

(a) $\mathcal{L}\, 3e^{-4t} = 3\,\mathcal{L}\, e^{-4t}$

$$\boxed{\mathcal{L}\, e^{at} = \frac{1}{s-a}}$$

$$= 3 \times \frac{1}{s-(-4)}$$

$$= \frac{3}{s+4}$$

(b) $\mathcal{L}\, 3t^2 = 3\,\mathcal{L}\, t^2$

$$\boxed{\mathcal{L}\, t^n = \frac{n!}{s^{n+1}}}$$

$$= 3 \times \frac{2!}{s^{2+1}} = \frac{3 \times 2 \times 1}{s^3} = \frac{6}{s^3}$$

(c) $\mathcal{L}\, 4$

$\mathcal{L}\cos w$

(a) $\mathcal{L}\, 3e^{-4t} = 3\,\mathcal{L}\, e^{-4t}$

$$\boxed{\mathcal{L}\, e^{at} = \frac{1}{s-a}}$$

$$= 3 \times \frac{1}{s-(-4)}$$

$$= \frac{3}{s+4}$$

(b) $\mathcal{L}\, 3t^2 = 3\,\mathcal{L}\, t^2$

$$\boxed{\mathcal{L}\, t^n = \frac{n!}{s^{n+1}}}$$

$$= 3 \times \frac{2!}{s^{2+1}} = \frac{3 \times 2 \times 1}{s^3} = \frac{6}{s^3}$$

(c) $\mathcal{L}\, 4\cos 5t = 4\,\mathcal{L}\cos 5t$

$$\boxed{\mathcal{L}\cos\omega t = \frac{s}{s^2+\omega^2}}$$

$$= 4 \times \frac{s}{s^2+5^2}$$

$$= \frac{4s}{s^2+25}$$

(d) $\mathcal{L}\sin 2t =$

$$\boxed{\mathcal{L}\sin\omega t = \frac{\omega}{s^2+\omega^2}}$$

$$= \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

(c) $\mathcal{L}\, -\frac{3}{\sqrt{t}} = -3\,\mathcal{L}\,\frac{1}{t^{1/2}} = -3\,\mathcal{L}\, t^{-\frac{1}{2}}$

$\mathcal{L}\, t^{n} = \frac{n!}{s^{n+1}} \longrightarrow \mathcal{L}\, t^{-\frac{1}{2}} = \frac{-\frac{1}{2}!}{s^{-\frac{1}{2}+1}}$? $\times$

$$\boxed{\mathcal{L}\, t^{p} = \frac{p(p+1)}{s^{p+1}}}$$

$-3\,\mathcal{L}\, t^{-1/2} = -3\times\frac{-\frac{1}{2}\left(-\frac{1}{2}+1\right)}{s^{-1/2+1}} = -3\,\frac{-\frac{1}{2}\left(\frac{1}{2}\right)}{s^{1/2}}$

$= \frac{+\,3/4}{s^{1/2}} = \frac{3/4}{s^{1/2}}$

$\frac{2}{s^{2}+4}$

pb 3

FIND LAPLACE TRANSFORM of $5 \sin 2t - 3 \cos 2t$

$$\mathcal{L} \sin \omega t = \frac{\omega}{s^2 + \omega^2}, \quad \mathcal{L} \cos \omega t = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}(5 \sin 2t - 3 \cos 2t) = \mathcal{L}\, 5 \sin 2t - \mathcal{L}\, 3 \cos 2t$$

$$= 5\, \mathcal{L} \sin 2t - 3\, \mathcal{L} \cos 2t$$

$$= 5 \times \frac{2}{s^2 + 2^2} - 3 \times \frac{s}{s^2 + 2^2}$$

$$= \frac{10}{s^2 + 4} - \frac{3s}{s^2 + 4}$$

$$= \frac{(10 - 3s)}{s^2 + 4}$$

$$\boxed{\frac{A}{C} - \frac{B}{C} \qquad \frac{A - B}{C}}$$

pb 4 FIND $\mathcal{L}(\sin t \cos t)$

$$\boxed{\sin 2A = 2 \sin A \cos A}$$

$$\frac{d}{dx} u.v = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\int u\,v\,dx =$$

$\sin 2t = 2 \sin t \cos t$

$\therefore \sin t \cos t = \dfrac{\sin 2t}{2}$

$$\mathcal{L} \sin t \cos t = \mathcal{L} \frac{\sin 2t}{2}$$

$$= \frac{1}{2} \mathcal{L} \sin 2t$$

$$\boxed{\mathcal{L} \sin \omega t = \frac{\omega}{s^2 + \omega^2}}$$

$$\frac{1}{2} \mathcal{L} \sin 2t = \frac{1}{2} \frac{2}{s^2 + 2^2}$$

$$= \frac{1}{s^2 + 4}$$