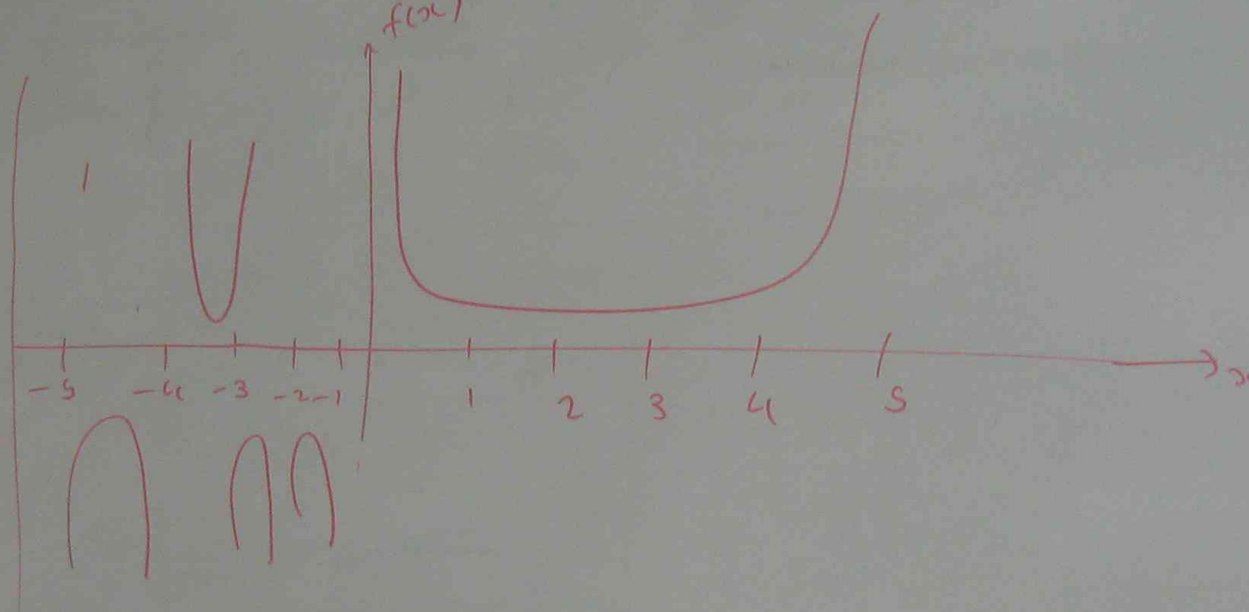
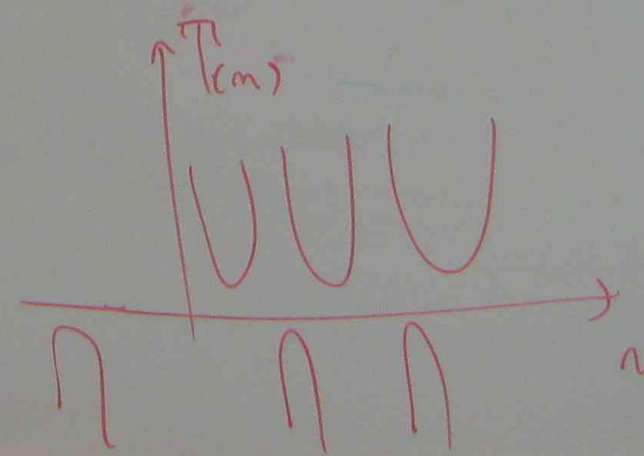
GAMMA FUNCTION



$\Gamma(n)$ - GAMMA FUNCTION OF VARIABLE (n)



$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x}\,dx$$

I $\boxed{\Gamma(n+1) = n\,\Gamma(n)}$ ← INTEGER

II $\boxed{\Gamma(n+1) = n!}$ ← FRACTION

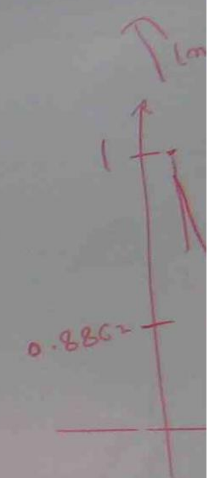
III $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

TABLE FOR GAMMA FUNCTIONS

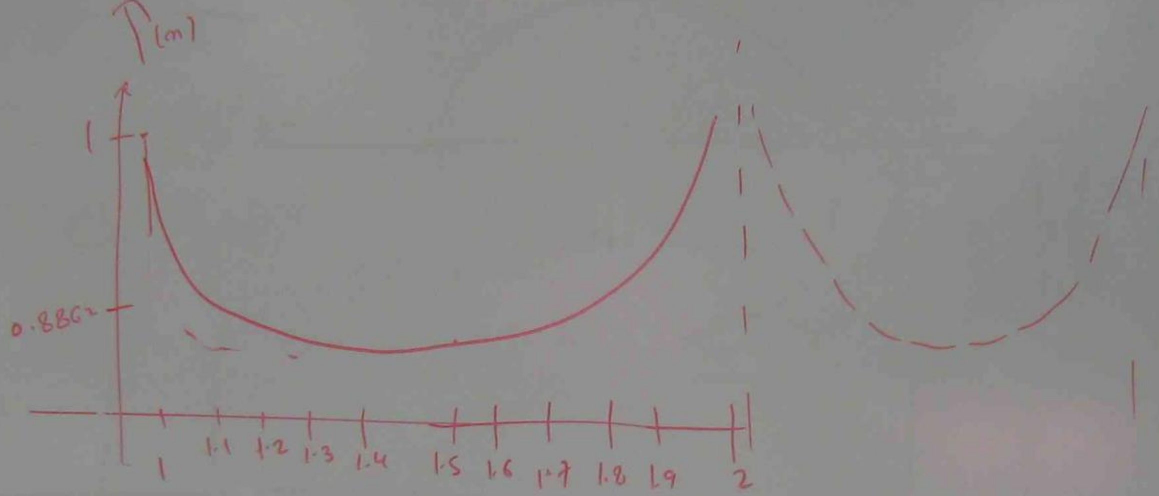
n	$\Gamma(n)$
1	1
1.1	0.9514
1.2	0.9182
1.3	0.8975
1.4	0.8873
1.5	0.8862
1.6	0.8935
1.7	0.9086

n	$\Gamma(n)$
1.8	0.9314
1.9	0.9612
2.00	1.00



GER

$! = 5\times4\times3\times2\times1$



n	$\Gamma(n)$
1.8	0.9314
1.9	0.9612
2.00	1.00

pb EVALUATE EACH OF THE FOLLOWINGS.

(a) $\dfrac{\Gamma(6)}{2\Gamma(3)}$ (b) $\dfrac{\Gamma(5/2)}{\Gamma(\frac{1}{2})}$ (c) $\dfrac{\Gamma(3)\,\Gamma(2.5)}{\Gamma(5.5)}$

(d) $\dfrac{6\,\Gamma(8/3)}{2\,\Gamma(2/3)}$

$$\boxed{\Gamma(n+1) = n!}$$

(a) $$\frac{\Gamma(6)}{2\,\Gamma(3)} = \frac{\Gamma(5+1)}{2\,\Gamma(2+1)} = \frac{5!}{2 \times 2!} = \frac{5\times4\times3\times2\times1}{2\times 2\times1}$$

$$= 30$$

$$\boxed{\Gamma(n+1) = n\,\Gamma(n)}$$

(b) $$\frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})} = \frac{\Gamma(\frac{3}{2}+1)}{\Gamma(\frac{1}{2})} = \frac{\frac{3}{2}\,\Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2})}$$

$$= \frac{3}{2}\,\frac{\Gamma(\frac{1}{2}+1)}{\Gamma(\frac{1}{2})} = \frac{\frac{3}{2}\times\frac{1}{2}\,\cancel{\Gamma(\frac{1}{2})}}{\cancel{\Gamma(\frac{1}{2})}}$$

$$= \frac{3}{2\times2} = \frac{3}{4}$$

(c) $\dfrac{\Gamma(3)\times\Gamma(2.5)}{\Gamma(5.5)} = \dfrac{\Gamma(3)\,\Gamma(\frac{5}{2})}{\Gamma(\frac{11}{2})}$

$\Gamma(3) = \Gamma(2+1) = 2! = 2\times 1 = 2$

$\Gamma(5/2) = \Gamma(\frac{3}{2}+1) = \frac{3}{2}\Gamma(3/2)$

$= \frac{3}{2}\Gamma(\frac{1}{2}+1) = \frac{3}{2}\times\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\times\sqrt{\pi}$

$\Gamma(\frac{11}{2}) = \Gamma(\frac{9}{2}+1) = \frac{9}{2}\Gamma(9/2)$

$= \frac{9}{2}\times\Gamma(\frac{7}{2}+1) = \frac{9}{2}\times\frac{7}{2}\Gamma(\frac{7}{2})$

$= \frac{9}{2}\times\frac{7}{2}\Gamma(5/2+1) = \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\Gamma(5/2)$

$= \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\Gamma(3/2+1) = \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\Gamma(3/2)$

$= \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\Gamma(\frac{1}{2}+1)$

$= \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma(1/2) = \frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}$

$=$

$$\frac{(2)\times\left(\cancel{\frac{5}{4}}\times\cancel{\sqrt{\pi}}\right)}{\left(\frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\cancel{\frac{3}{2}}\times\cancel{\frac{1}{2}}\times\cancel{\sqrt{\pi}}\right)}$$

$$\frac{\cdot 2}{\frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}}$$

$$\frac{8\times 2}{315} = \frac{16}{315}$$

(d) $\dfrac{6\ \Gamma(8/3)}{2\ \Gamma(2/3)}$

$\Gamma\left(\frac{8}{3}\right) = \frac{8}{3}-1 = \frac{8-3}{3} = \frac{5}{3}$

$\frac{5}{3}+1$

$$\frac{6\ \Gamma\left(\frac{5}{3}+1\right)}{2\ \Gamma(2/3)}$$

$$\frac{6\times\frac{5}{3}\ \Gamma(5/3)}{2\ \Gamma\left(\frac{2}{3}\right)}$$

$\frac{5}{3}-1 = \frac{5-3}{3} = \frac{2}{3}$

$\therefore \frac{5}{3} = \frac{2}{3}+1$

$$\frac{6\times\frac{5}{3}\ \Gamma\left(\frac{2}{3}+1\right)}{2\ \Gamma(2/3)}$$

$$\frac{6\times\frac{5}{3}\times\frac{2}{3}\ \cancel{\Gamma\left(\frac{2}{3}\right)}}{2\ \cancel{\Gamma(2/3)}}$$

$$\frac{\frac{6\times 5\times 2}{9}}{2}$$

$$\frac{6\times 5\times 2}{2\times 9}$$

$= \frac{30}{9}$

BETA FUNCTION

$$B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx$$

$$B(m, n) = \frac{\Gamma(m) \times \Gamma(n)}{\Gamma(m+n)}$$

Pb $\int_0^1 x^4 (1-x)^3 \, dx$

COMPARE $\int_0^1 x^{m-1} (1-x)^{n-1} \, dx$

$m-1=4$, $n-1=3$

$m=1+4=5$, $n=1+3=4$

$\int_0^1 x^4 (1-x)^3 \, dx = B(m,n) = B(5,4)$

$= \frac{\Gamma(5) \times \Gamma(4)}{\Gamma(5+4)} = \frac{\Gamma(4+1) \times \Gamma(3+1)}{\Gamma(8+1)} = \frac{4! \times 3!}{8!}$

$= \frac{4! \times 3!}{\Gamma(9)} = \frac{4! \times}{\Gamma(8+1}$

! = FACTORIAL

$2! = 2 \times 1$

$3! = 3 \times 2 \times 1$

$$\frac{!\times 3!}{(8+1) = 8!} = \frac{4\times 3\times 2\times 1 \quad \times 3\times 2\times 1}{8\times 7\times 6\times 5\times 4\times 3\times 2\times 1} = \frac{1}{280}$$

APPLICATION OF BETA FUNCTION IN ANGULAR FUNCTIONS

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n)$$

Pb. EVALUATE $\int_0^{\pi/2} \sin^6\theta \, d\theta$

COMPARE $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

Pb. $\int_0^1 x^4 (1-x)^3 \, dx$

Compare $\int_0^1 x^{m-1} (1-x)^{n-1} \, dx$

$m-1 = 4, \quad n-1 = 3$

$m = 1+4 = 5, \quad n = 1+3 = 4$

$$\int_0^1 x^4 (1-x)^3 \, dx = B(m,n) = B(5,4)$$

$$= \frac{\Gamma(5) \times \Gamma(4)}{\Gamma(5+4)} = \frac{\Gamma(4+1) \times \Gamma(3+1)}{\Gamma(8+1)} = \frac{4! \times 3!}{8!}$$

$$= \frac{4! \times 3!}{\Gamma(9)} = \frac{4! \times 3!}{\Gamma(8+1) = 8!} = \frac{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{280}$$

$! =$ FACTORIAL

$2! = 2 \times 1$

$3! = 3 \times 2 \times 1$

APPLICATION OF BETA FUNCTION IN ANGULAR FUNCTIONS

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m,n)$$

Pb. EVALUATE $\int_0^{\pi/2} \sin^6\theta \, d\theta$

Compare $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

$$\frac{4! \times 3!}{\Gamma(8+1) = 8!} = \frac{4\times3\times2\times1 \quad \times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{1}{280}$$

APPLICATION OF BETA FUNCTION IN ANGULAR FUNCTIONS

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n)$$

Pb. EVALUATE $\int_0^{\pi/2} \sin^6\theta \, d\theta$

COMPARE $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

$3!$

$!$

$$\int_0^{\pi/2} \sin^6\theta \times \cos^0\theta \, d\theta$$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

COMPARE

$2m-1 = 6$

$m = \frac{1+6}{2} = \frac{7}{2}$

$2n-1 = 0$

$2n = 1$

$n = 1/2$

$B(m, n) = B(7/2, 3/2)$

$$= \frac{\Gamma(\frac{7}{2}) \times \Gamma(\frac{1}{2})}{\Gamma(\frac{7}{2} + \frac{1}{2})}$$

$$= \frac{\Gamma(7/2) \times \Gamma(1/2)}{\Gamma \frac{8}{2}}$$

$$\frac{\Gamma\left(\frac{7}{2}\right) \times \Gamma\left(3/2\right)}{\Gamma(5)}$$

$$\Gamma(9/2) = \Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right) = \frac{5}{2} \times \Gamma\left(\frac{3}{2}+1\right) = \frac{5}{2} \times \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{5}{2} \times \frac{3}{2}\Gamma\left(\frac{1}{2}+1\right) = \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{15}{8} \times \sqrt{\pi}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(4) = \Gamma(3+1) = 3! = \times 3 \times 2 \times 1 = 6$$

$$\frac{\frac{15}{8}\sqrt{\pi} \times \sqrt{\pi}}{6} \qquad \frac{15\pi}{48} = \frac{5\pi}{16}$$

$$\frac{\Gamma\left(\frac{7}{2}\right) \times \Gamma(3/2)}{\Gamma(5)}$$

$$\Gamma(9/2) = \Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right) = \frac{5}{2} \times \Gamma\left(\frac{3}{2}+1\right) = \frac{5}{2} \times \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{5}{2} \times \frac{3}{2}\Gamma\left(\frac{1}{2}+1\right) = \frac{5}{2} \times$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(4) = \Gamma(3+1) = 3! = \times 3 \times 2 \times 1 = 6$$

$$\frac{\frac{15}{8}\sqrt{\pi} \quad \times \sqrt{\pi}}{6} \qquad \frac{15\pi}{48} = \frac{5\pi}{16}$$

$\frac{3}{2}\Gamma(\frac{1}{2}+1) = \frac{3}{2}\times\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\sqrt{\pi}$

$= \frac{4\times3\times2\times1 \quad \times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{1}{280}$

$\Gamma(\frac{7}{2}+1) = \frac{9}{2}\times\frac{7}{2}\times\frac{7}{2}\Gamma(\frac{7}{2})$

$\frac{\times7\times7}{\times2\times2}\times\frac{5}{2}\Gamma\frac{5}{2}$

$\frac{1}{2}\times\frac{\frac{3}{4}\cancel{\sqrt{\pi}}\times2}{\frac{4\times7\times7\times5\times3}{2^6}\cancel{\sqrt{\pi}}}$

$\frac{3\times2^6\times2}{2\times4\times4\times7\times7\times5\times3}$

$\Gamma\frac{1}{2}$

$= \frac{8}{315}$

APPLICATION OF BETA FUNCTION IN ANGULAR FUNCTIONS

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2}B(m,n)$$

Pb. EVALUATE $\int_0^{\pi/2} \sin^6\theta \, d\theta$

COMPARE $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

$$\int_0^{\pi/2} \sin^6\theta \times \cos^0\theta \, d\theta$$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

COMPARE

$2m-1 = 6$

$m = \frac{1+6}{2} = \frac{7}{2}$

$2n-1 = 0$

$2n = 1$

$n = 1/2$

$$B(m,n) = \frac{1}{2} B\left(7/2, 3/2\right)$$

$$= \frac{1}{2} \frac{\Gamma\left(\frac{7}{2}\right) \times \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{7}{2} + \frac{1}{2}\right)}$$

$$= \frac{\Gamma(7/2) \times \Gamma(1/2)}{\Gamma 8}$$

$$\frac{1}{2} \frac{\Gamma\left(\frac{7}{2}\right) \times \Gamma(3/2)}{\Gamma(5)}$$

$\Gamma(7/2) = \Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma(4) = \Gamma(3+1) =$

$$\frac{1}{2} \frac{\frac{15}{8}}{}$$

$$\frac{7}{2}-1=\frac{7-2}{2}=\frac{5}{2} \quad \therefore \frac{7}{2}=\frac{5}{2}+1$$

$$\frac{1}{2}\,\frac{\Gamma(\frac{7}{2})\times\Gamma(3/2)}{\Gamma(5)}$$

$$\Gamma(7/2)=\Gamma\left(\frac{5}{2}+1\right)=\frac{5}{2}\Gamma\left(\frac{5}{2}\right)=\frac{5}{2}\times\Gamma\left(\frac{3}{2}+1\right)=\frac{5}{2}\times\frac{3}{2}\Gamma\left(\frac{3}{2}\right)=\frac{5}{2}\times\frac{3}{2}\Gamma\left(\frac{1}{2}+1\right)=\frac{5}{2}$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$$\Gamma(4)=\Gamma(3+1)=3!=\times 3\times 2\times 1=6$$

$$\frac{1}{2}\,\frac{\frac{15}{8}\sqrt{\pi}\times\sqrt{\pi}}{6} \qquad \frac{15\pi}{96}=\frac{5\pi}{32}$$

Ph

$$\int_0^{\pi/2} \sin^4\theta \cos^5\theta \, d\theta$$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n) = \frac{1}{2} \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$\int_0^{\pi/2} \sin^4\theta \cos^5\theta \, d\theta$$

$2m-1 = 4$, $2n-1 = 5$

$2m = 5$, $2n = 6$

$m = 5/2$, $n = 3$

$$= \frac{1}{2} B(5/2, 3) = \frac{1}{2} \frac{\Gamma(5/2)\Gamma(3)}{\Gamma(5/2+3)}$$

$$= \frac{1}{2} \frac{\Gamma(5/2)\Gamma(3)}{\Gamma\left(\frac{11}{2}\right)}$$

$$\Gamma(3) = \Gamma(2+1) = 2! = 2\times 1 = 2$$

$$\Gamma(5/2) = \Gamma(3/2+1) = \frac{3}{2}\Gamma(3/2) = \frac{3}{2}\Gamma\left(\frac{1}{2}+1\right) = \frac{3}{2}\times\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{\pi}$$

$$\boxed{\frac{5}{2}-1 = \frac{3}{2} \Rightarrow \frac{3}{2}+1}$$

$$\Gamma\left(\frac{11}{2}\right) = \Gamma\left(\frac{9}{2}+1\right) = \frac{9}{2}\Gamma\left(\frac{9}{2}\right) = \frac{9}{2}\Gamma\left(\frac{7}{2}+1\right) = \frac{9}{2}\times\frac{7}{2}\times\frac{7}{2}\Gamma\left(\frac{7}{2}\right)$$

$$= \frac{9\times 7\times 7}{2\times 2\times 2}\times\Gamma\left(\frac{5}{2}+1\right) = \frac{9\times 7\times 7}{2\times 2\times 2}\times\frac{5}{2}\Gamma\frac{5}{2}$$

$$= \frac{9\times 7\times 7\times 5}{2\times 2\times 2\times 2}\Gamma\left(\frac{3}{2}+1\right)$$

$$= \frac{9\times 7\times 7\times 5}{2\times 2\times 2\times 2}\times\frac{3}{2}\Gamma\frac{3}{2}$$

$$= \frac{9\times 7\times 7\times 5\times 3}{2\times 2\times 2\times 2\times 2}\Gamma\left(\frac{1}{2}+1\right)$$

$$= \frac{9\times 7\times 7\times 5\times 3}{2\times 2\times 2\times 2\times 2}\times\frac{1}{2}\Gamma\frac{1}{2}$$

$$= \frac{9\times 7\times 7\times 5\times 3}{2^6}\times\sqrt{\pi} =$$

$$\frac{1}{2}\times\frac{\frac{3}{4}\cancel{\sqrt{\pi}}\times 2}{\frac{9\times 7\times 7\times 5\times 3}{2^6}\times\cancel{\sqrt{\pi}}}$$

$$\frac{3\times 2^6\times 2}{2\times 4\times 9\times 7\times 7\times 5\times 3}$$

$$= \frac{8}{315}$$

Pb $\int_0^{\pi/2} \sin^4\theta \cos^5\theta \, d\theta$

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n) = \frac{1}{2} \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$\int_0^{\pi/2} \sin^4\theta \cos^5\theta \, d\theta$$

$2m-1 = 4$, $2n-1 = 5$

$2m = 5$, $2n = 6$

$m = 5/2$, $n = 3$

$$= \frac{1}{2} B(5/2, 3) = \frac{1}{2} \frac{\Gamma(5/2)\Gamma(3)}{\Gamma(5/2+3)}$$

$$= \frac{1}{2} \frac{\Gamma(5/2)\Gamma(3)}{\Gamma(\frac{11}{2})}$$

$\Gamma(3) = \Gamma(2+1) = 2! = 2\times 1 = 2$

$\Gamma(5/2) = \Gamma(3/2+1) = \frac{3}{2}\Gamma(3/2) = \frac{3}{2}\Gamma(\frac{1}{2}+1) = \frac{3}{2}\times\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\sqrt{\pi}$

$\boxed{\frac{5}{2}-1 = \frac{3}{2} \Rightarrow \frac{3}{2}+1}$

$\Gamma(\frac{11}{2}) = \Gamma(\frac{9}{2}+1) = \frac{9}{2}\Gamma(\frac{9}{2}) = \frac{9}{2}\Gamma(\frac{7}{2}+1) = \frac{9}{2}\times\frac{7}{2}\Gamma(7/2)$

$= \frac{9\times 7\times}{2\times 2\times}\times\Gamma(\frac{5}{2}+1) = \frac{9\times 7\times}{2\times 2\times 2}\times\frac{5}{2}\Gamma\frac{5}{2}$

$= \frac{9\times 7\times \ \times 5}{2\times 2\times \ 2}\Gamma(\frac{3}{2}+1)$

$= \frac{9\times 7\times \ 5}{2\times 2\times \ 2}\times\frac{3}{2}\Gamma\frac{3}{2}$

$= \frac{9\times 7\times \ 5\times 3}{2\times 2\times \ 2\times 2}\Gamma(\frac{1}{2}+1)$

$= \frac{9\times 7\times \ 5\times 3}{2\times 2\times \ \times 2\times 2}\times\frac{1}{2}\Gamma\frac{1}{2}$

$= \frac{9\times 7\times \ \times 5\times 3}{2^5}\times\sqrt{\pi} =$

$\frac{1}{2}\times\frac{\frac{3}{4}\cancel{\sqrt{\pi}}\times 2}{\frac{9\times 7\times \ 5\times 3}{2^5}\times\cancel{\sqrt{\pi}}}$

$\frac{3\times 2^5\times 2}{2\times 4\times 9\times 7\times 5\times 3}$

$= \frac{8}{315}$

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