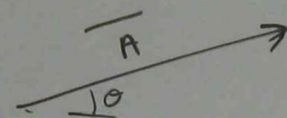
VECTOR ANALYSIS

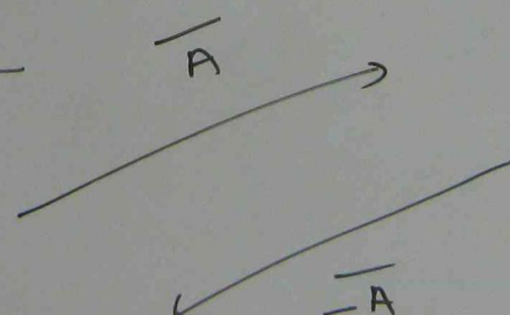
MAGNITUDE + DIRECTION = VECTOR



LENGTH OF ARROW = MAGNITUDE

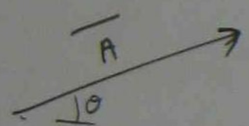
HEAD OF ARROW = DIRECTION

REVERSED VECTOR



VECTOR ANALYSIS

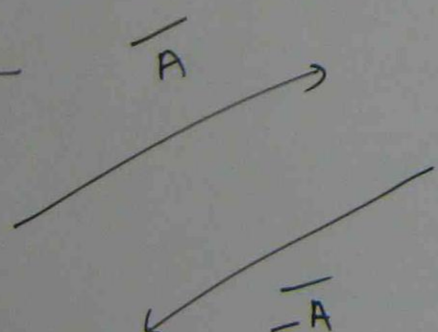
MAGNITUDE + DIRECTION = VECTOR



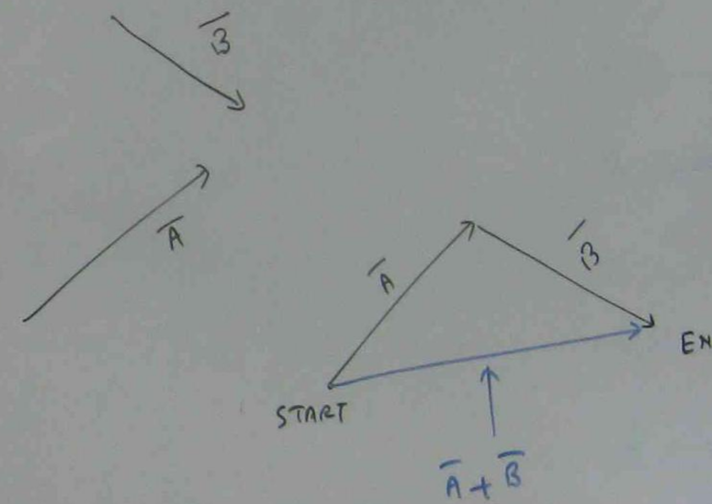
LENGTH OF ARROW = MAGNITUDE

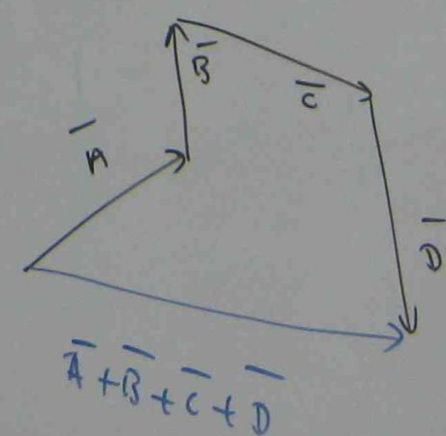
HEAD OF ARROW = DIRECTION

REVERSED VECTOR



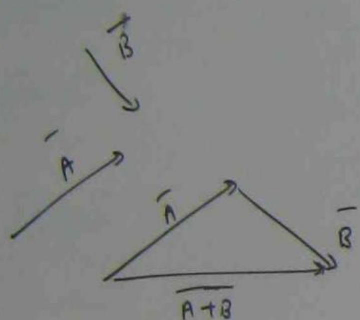
ADDITION OF VECTORS

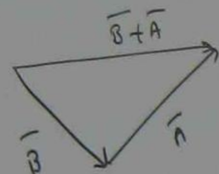




LAWS OF VECTOR ALGEBRA

$$A + B = B + A$$





$\overline{A+B}$ MAGNITUDE $=$ $\overline{B+A}$ MAGNITUDE

$\overline{A+B}$ DIRECTION $=$ $\overline{B+A}$ DIRECTION

$$A + (B + C) = (A + B) + C$$

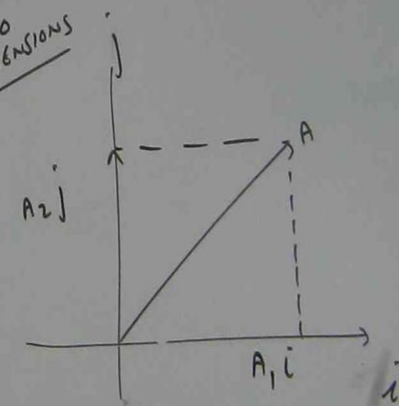
$$m(A) = mA$$

$$m(nA) = mnA$$

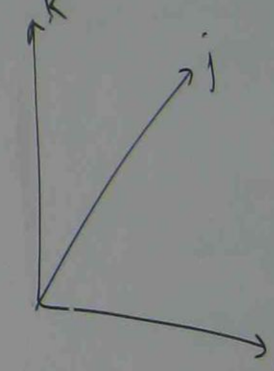
$$m(\bar{A} + \bar{B}) = m\bar{A} + m\bar{B}$$

MAGNITUDE of 3 DIMENSIONS VECTOR

TWO DIMENSIONS



THREE DIMENSIONS



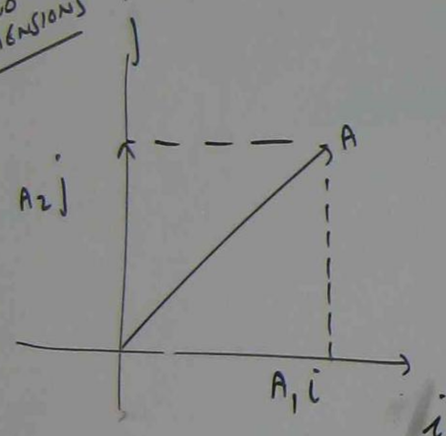
$$\bar{A} = A_1 i + A_2 j$$

$$|\bar{A}| = \sqrt{A_1^2 + A_2^2}$$

$$m(\bar{A} + \bar{B}) = m\bar{A} + m\bar{B}$$

MAGNITUDE of 3 DIMENSIONS VECTOR

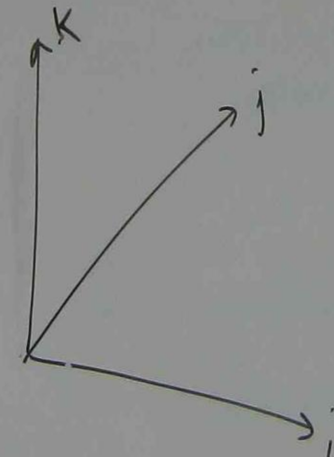
TWO DIMENSIONS

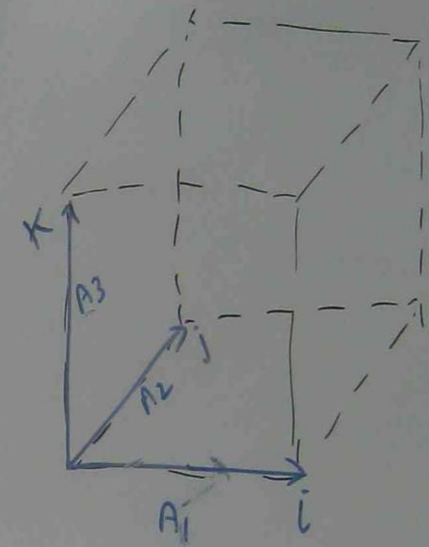


$$\bar{A} = A_1 i + A_2 j$$

$$|\bar{A}| = \sqrt{A_1^2 + A_2^2}$$

THREE DIMENSIONS





$$\bar{A} = A_1 i + A_2 j + A_3 k$$

$$|\bar{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$$

TUDE

ON

CROSS VECTOR PRODUCT (VECTOR MULTIPLICATION)

→ MULTIPLY MAGNITUDE
&
→ MULTIPLY DIRECTION

$$A = A_1 i + A_2 j + A_3 k$$

$$B = B_1 i + B_2 j + B_3 k$$

$$A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$= i \begin{vmatrix} A_2 & A_3 \\ B_2 & B_3 \end{vmatrix} - j \begin{vmatrix} A_1 & A_3 \\ B_1 & B_3 \end{vmatrix} + k \begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix}$$

$$= i\left[A_2 B_3 - A_3 B_2\right] - j\left[A_1 B_3 - B_1 A_3\right] + k\left[A_1 B_2 - B_1 A_2\right]$$

$\bar{A}$

$\bar{B}$

$A \cdot$

$\bar{i} \times$

$\bar{j} \times$

k

$$\boxed{j = \sqrt{-1}}$$

Y MAGNITUDE

LY DIRECTION

DOT VECTOR PRODUCT (SCALOR MULTIPLICATION) ⟶ MULTIPLY MAGNITUDE

$$\overline{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 k$$

$$\overline{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 k$$

$$A \cdot B = (A_1 \hat{i} + A_2 \hat{j} + A_3 k) \cdot (B_1 \hat{i} + B_2 \hat{j} + B_3 k)$$

$$= (A_1 \times B_1 + A_2 \times B_2 + A_3 \times B_3)$$

$$\hat{i} \times \hat{i} = 1 \qquad \hat{i} \times \hat{j} = \hat{j} \times k = k \times \hat{i} = 0$$

$$\hat{j} \times \hat{j} = 1$$

$$k \times k = 1$$

$$\boxed{j = \sqrt{-1} \qquad \hat{j} \times \hat{j} = -1}$$

pb

If $A = A_1 i + A_2 j + A_3 k$

$B = B_1 i + B_2 j + B_3 k$

PROVE $A.B = A_1B_1 + A_2B_2 + A_3B_3$

$$A.B = (A_1 i + A_2 j + A_3 k) \cdot (B_1 i + B_2 j + B_3 k)$$

$$A_1 i (B_1 i + B_2 j + B_3 k) + A_2 j (B_1 i + B_2 j + B_3 k) + A_3 k (B_1 i + B_2 j + B_3 k)$$

$$A_1B_1 \underbrace{i \cdot i}_{1} + A_1B_2 \underbrace{i \cdot j}_{0} + A_1B_3 \underbrace{i \cdot k}_{0} + A_2B_1 \underbrace{j \cdot i}_{0} + A_2B_2 \underbrace{j \cdot j}_{1} + A_2B_3 \underbrace{j \cdot k}_{0} + A_3B_1 \underbrace{k \cdot i}_{0} + A_3B_2 \underbrace{k \cdot j}_{0} + A_3B_3 \underbrace{k \cdot k}_{1}$$

$$A.B = A_1B_1 * 1 + A_2B_2 * 1 + A_3B_3 * 1$$

$$A.B = A_1B_1 + A_2B_2 + A_3B_3$$

Pb

If $A = 3i - j + 2k$, $B = 2i + 3j - k$

FIND (i) $A \cdot B$

(ii) $A \times B$

(i) $A \cdot B = (3i - j + 2k) \cdot (2i + 3j - k)$

$= 3 \cdot 2 + (-1)(3) + (2 * (-1))$

$= 6 - 3 - 2 = 1$

(ii) $A \times B = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix}$

$= i \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} - j \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$

$i\left[(-1 * -1) - 3 * 2\right] - j\left[3 * (-1) - 2 * 2\right]$

$+ k\left[3 * 3 - (-1) * 2\right]$

$i\left[1 - 6\right] - j\left[-3 - 4\right] + k\left[9 + 2\right]$

$-5i - j \times (-7) + k \times 11$

$-5i + 7j + 11k$

pb/ IF $A = i + j$, $B = 2i - 3j + k$, $C = 4j - 3k$

FIND (a) $(A \times B) \times C$ (b) $A \times (B \times C)$

(a) $A = i + j + 0k$ $\quad B = 2i - 3j + 1k$ $\quad C = 0i + 4j - 3k$

$= 1i + 1j + 0k$

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 2 & -3 & 1 \end{vmatrix} = i \begin{vmatrix} 1 & 0 \\ -3 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = i(1-0) - j(1-0) + k(-3-2\cdot1)$$

$$= i - j - 5k$$

$$= 1i - 1j - 5k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ 1 & -1 & -5 \\ 0 & 4 & -3 \end{vmatrix} = i \begin{vmatrix} -1 & -5 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} 1 & -5 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 0 & 4 \end{vmatrix} = i(3-(-20)) - j(-3-0) + k(4-0)$$

$$= i(3+20) - j(-3) + k(4)$$

$$= 23i + 3j + 4k$$

$$B \times C = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix} = i \begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= i\,[+9-4] - j\,[-6-0] + k\,[8-0]$$

$$= 5i + 6j + 8k$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix}$$

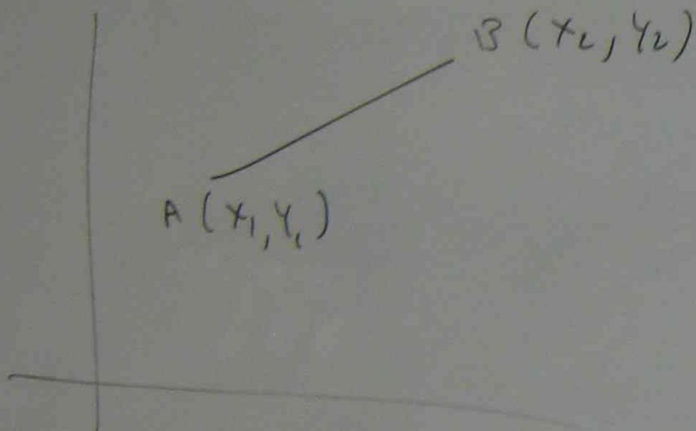
$$= i \begin{vmatrix} 1 & 0 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 1 & 1 \\ 5 & 6 \end{vmatrix}$$

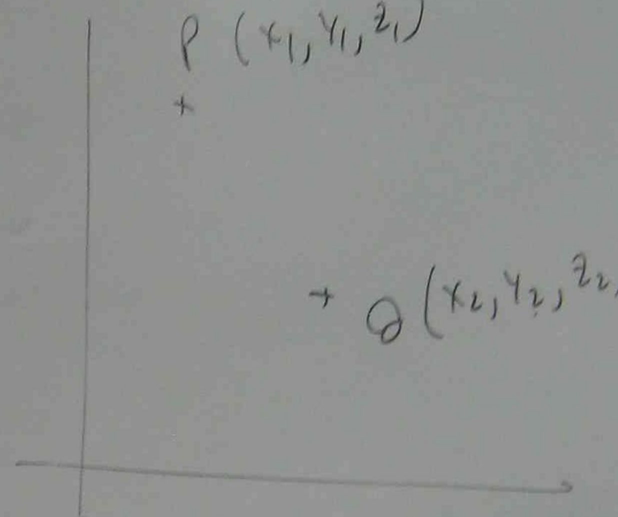
$$= i\,[8-0] - j\,[8-0] + k\,[6-5]$$

$$= 8i - 8j + k$$

VECTOR DIFFERENCE

pb





$$AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$\overline{PQ} = (x_2-x_1)\, i + (y_2-y_1)\, j + (z_2-z_1)\, k$$

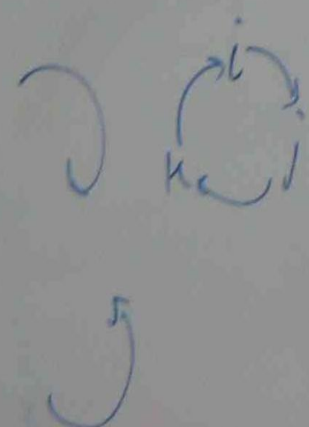
$$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

pb/ $A(1,2,3)$, $B:(4,7,10)$

FIND $|AB|$

$$\overline{AB} = (4-1)i + (7-2)j + (10-3)k$$
$$= 3i + 5j + 7k$$

$$|\overline{AB}| = \sqrt{3^2 + 5^2 + 7^2}$$
$$= \sqrt{9+25+49} = \sqrt{83}$$



pb/ IF $A = A_1 i + A_2 j + A_3 k$

$B = B_1 i + B_2 j + B_3 k$

PROVE THAT

$$A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

DOT

$i \cdot i = 1 \quad j \cdot j = 1 \quad k \cdot k = 1$

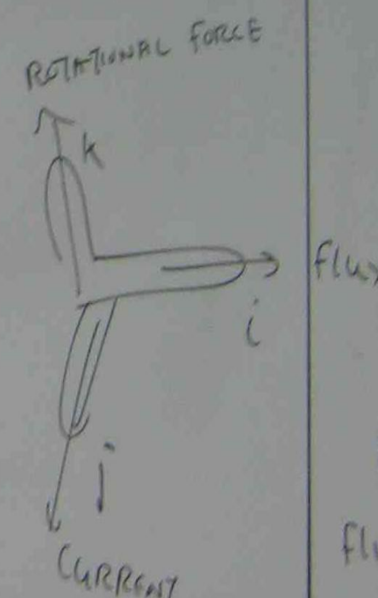
$i \cdot j = j \cdot k = k \cdot i = 0$

CROSS

$i \times i = j \times j = k \times k = 0$

$i \times j = k$, $j \times k = i$, $k \times i = j$

$j \times i = -k$, $k \times j = -i$, $i \times k = -j$



RHS

HS

$$\begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = i\begin{bmatrix} A_2 & A_3 \\ B_2 & B_3 \end{bmatrix} - j\begin{bmatrix} A_1 & A_3 \\ B_1 & B_3 \end{bmatrix} + k\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix} = (A_2B_3 - A_3B_2)i - (A_1B_3 - A_3B_1)j + (A_1B_2 - A_2B_1)k$$

WE NEED TO PROVE $A \times B$

$$A \times B = \left((A_1 i + A_2 j + A_3 k\right) \times \left(B_1 i + B_2 j + B_3 k\right)$$

$$= A_1 i \times (B_1 i + B_2 j + B_3 k) + A_2 j \times (B_1 i + B_2 j + B_3 k) + A_3 k \times (B_1 i + B_2 j + B_3 k)$$

$$= (A_1B_1 \underbrace{i \times i}_{0} + A_1B_2 \underbrace{i \times j}_{k} + A_1B_3 \underbrace{i \times k}_{-j}) + (A_2B_1 \underbrace{j \times i}_{-k} + A_2B_2 \underbrace{j \times j}_{0} + A_2B_3 \underbrace{j \times k}_{i}) + (A_3B_1 \underbrace{k \times i}_{j} + A_3B_2 \underbrace{k \times j}_{-i} + A_3B_3 \underbrace{k \times k}_{0})$$

$$= A_1B_2k - A_1B_3j - A_2B_1k + A_2B_3i + A_3B_1j - A_3B_2i$$

$$= (A_2B_3i - A_3B_2i) + (-A_1B_3j + A_3B_1j) + (A_1B_2k - A_2B_1k)$$

$$= (A_2B_3 - A_3B_2)i - (A_1B_3 - A_3B_1)j + (A_1B_2 - A_2B_1)k$$

$\times j = k$

$\times$ CURRENT = FORCE

THUS

$$A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

PROVED

RHS $\begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = i\begin{bmatrix} A_2 & A_3 \\ B_2 & B_3 \end{bmatrix} - j\begin{bmatrix} A_1 & A_3 \\ B_1 & B_3 \end{bmatrix} + k\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix} = (A_2B_3 - A_3B_2)i - (A_1B_3 - A_3B_1)j + (A_1$

WE NEED TO PROVE $A \times B$

$A \times B = \left(A_1 i + A_2 j + A_3 k \right) \times \left(B_1 i + B_2 j + B_3 k \right) = A_1 i \times (B_1 i + B_2 j + B_3 k) + A_2 j \times (B_1 i +$

$= (A_1 B_1 \underbrace{\cancel{i \times i}}_{0} + A_1 B_2 \underbrace{i \times j}_{k} + A_1 B_3 \underbrace{i \times k}_{-j}) + (A_2 B_1$

$= A_1 B_2 k - A_1 B_3 j - A_2 B_1 k + A_2$

flux

$= (A_2 B_3 i - A_3 B_2 i) + (-A_1 B_3 j + A_3$

$i \times j = k$

$= (A_2 B_3 - A_3 B_2) i - (A_1 B_3 - A_3 B_1) j$

flux × CURRENT = FORCE

THUS $A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$ PROVED

$$-j\begin{bmatrix}A_1 & A_3\\ B_1 & B_3\end{bmatrix} + k\begin{bmatrix}A_1 & A_2\\ B_1 & B_2\end{bmatrix} = \underbrace{(A_2B_3 - A_3B_2)i - (A_1B_3 - A_3B_1)j + (A_1B_2 - A_2B_1)k}$$

WE NEED TO PROVE A×B

$$(B_1 i + B_2 j + B_3 k) = A_1 i \times (B_1 i + B_2 j + B_3 k) + A_2 j \times (B_1 i + B_2 j + B_3 k) + A_3 k \times (B_1 i + B_2 j + B_3 k)$$

$$= (A_1B_1 \underbrace{\cancel{i \times i}}_{0} + A_1B_2 \underbrace{i \times j}_{k} + A_1B_3 \underbrace{i \times k}_{-j}) + (A_2B_1 \underbrace{j \times i}_{-k} + A_2B_2 \underbrace{\cancel{j \times j}}_{0} + A_2B_3 \underbrace{j \times k}_{i}) + (A_3B_1 \underbrace{k \times i}_{j} + A_3B_2 \underbrace{k \times j}_{-i} + A_3B_3 \underbrace{\cancel{k \times k}}_{0})$$

$$= A_1B_2k - A_1B_3j - A_2B_1k + A_2B_3i + A_3B_1j - A_3B_2i$$

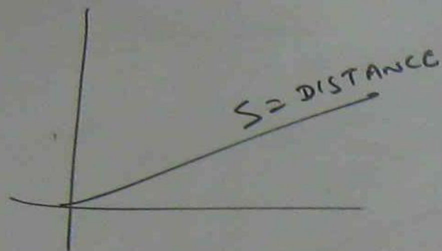
$$= (A_2B_3i - A_3B_2i) + (-A_1B_3j + A_3B_1j) + (A_1B_2k - A_2B_1k)$$

$$= (A_2B_3 - A_3B_2)i - (A_1B_3 - A_3B_1)j + (A_1B_2 - A_2B_1)k$$

THUS

$$A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

PROVED



V = VELOCITY

a = ACCELERATION

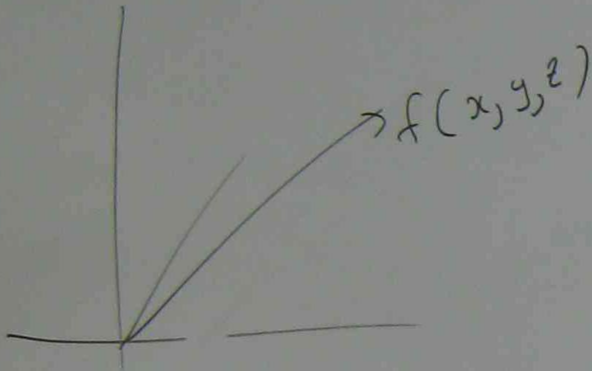
$$\frac{ds}{dt} = v \quad , \quad \frac{d^2 s}{dt^2} = a$$

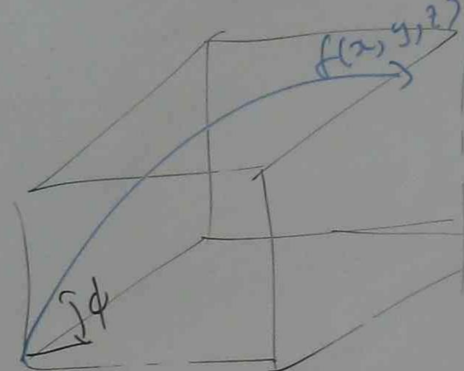
REPRESENTATING DIFFERENTIALS IN VECTOR FORM

deL (∇) VECTOR

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\bar{A} = A_1 i + A_2 j + A_3 k$$





$$f(x,y,z) = 3\,x^2 y^3 z^{1.5}$$

$$\nabla f(x,y,z) = i\frac{\partial}{\partial x} f(x,y,z) + j\frac{\partial}{\partial y} f(x,y,z) + k\frac{\partial}{\partial z} f(x,y,z)$$

GRADIENT $\phi \Rightarrow \nabla\phi = i\frac{\partial}{\partial x}\phi(x,y,z) + j\frac{\partial}{\partial y}\phi(x,y,z) + k\frac{\partial}{\partial z}\phi(x,y,z)$

DIVERGENCE

$$\mathrm{div}\,.\,A = \frac{\partial}{\partial x}A_1 + \frac{\partial}{\partial y}A_2 + \frac{\partial}{\partial z}A_3$$

$+A_3 k$

$$
\operatorname{CURL} A = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}
$$