PARTIAL DIFFERENTIAL EQUATIONS

Q/ If $u = f(y-3x)$

Prove $\frac{\partial v}{\partial x} + 3\frac{\partial v}{\partial y} = 0$

$$\boxed{\frac{\partial v(x,y)}{\partial x} = v' \frac{\partial v}{\partial x}}$$

$$\boxed{\frac{\partial v(x,y)}{\partial y} = v' \frac{\partial v}{\partial y}}$$

$u = F(y-3x)$

$$\frac{\partial v}{\partial x} = f'(y-3x)\frac{\partial}{\partial x}(y-3x)$$

$$= F'(y-3x)\left[\frac{\partial y}{\partial x} - \frac{\partial 3x}{\partial x}\right]$$

$$= F'(y-3x)\left[0 - 3\cancel{\frac{\partial x}{\partial x}}\right]$$

$$= -3F'(y-3x) \quad \text{—(1)}$$

$u = y - 3x$

$$\frac{\partial u}{\partial y} = f'(y-3x)\,\frac{\partial}{\partial y}(y-3x)$$

$$= f'(y-3x)\left[\frac{\partial y}{\partial y} - \frac{\partial}{\partial y}3x\right]$$

$$= f'(y-3x)\,[1 - 0]$$

$$= f'(y-3x) \quad \text{--- } (2)$$

(1) & (2) SUBSTITUTE IN EQUATION

$$-3f'(y-3x) + 3f'(y-3x) = 0$$

$$\therefore \frac{\partial v}{\partial x} + 3\frac{\partial v}{\partial y} = 0$$

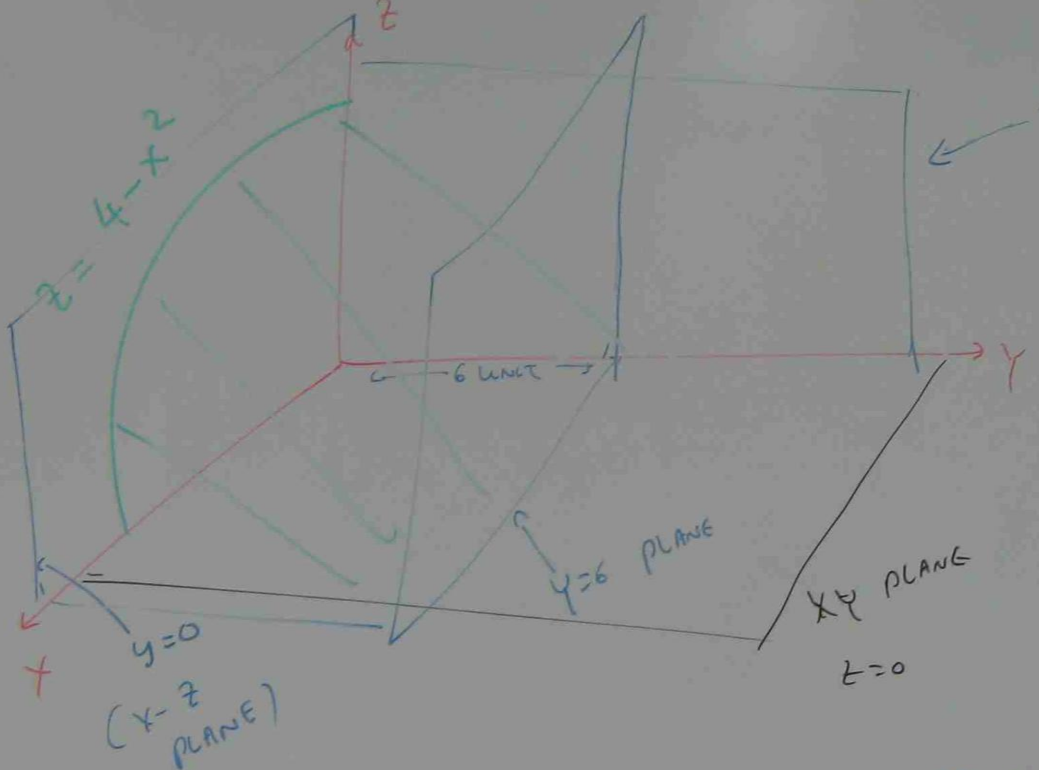
MULTIPLE INTEGRATION

$$\int f(y)\, dy = \text{AREA}$$

$$\iiint = \text{VOLUME}$$

Pb FIND THE VOLUME OF THE REGION R BOUNDED BY THE PARABOLIC CYLINDER $z = 4 - x^2$ AND THE PLANES $x = 0,\ y = 0,\ y = 6$

$z = 0$



$z = 4 - x^2$ IS

$$\int_{z=0}^{z=4-x^2} f(z)\, dz$$

$x = -$

OTHER LIMIT

$z = 0$

& $z = 4 - x^2$

THEREFORE $0 = 4 - x^2$

$x^2 = 4 \rightarrow x = \pm 2$

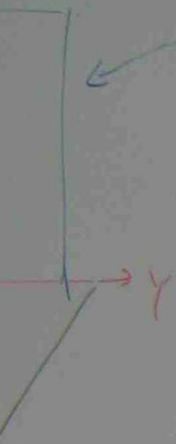
X LIMIT = $\int_{-2}^{2} dx$

$= 2 \int_{0}^{2} dx$

Y LIMIT (GIV

$0 \longrightarrow 6$

$$\int_{0}^{6} d$$



$x = 0$
(y-z plane)

... plane

$... = 0$

$z = 4 - x^2$ is given

$$\int_{z=0}^{z=4-x^2} f(z)\,dz$$

$$\int_{x=-2}^{x=2}\int_{y=0}^{6}\int_{z=0}^{z=4-x^2} f(z)\,dz\;dy\;dx = 2\int_{x=0}^{x=2}\int_{y=0}^{6}\int_{z=\ldots}$$

$$= 2\int_{x=0}^{x=2}\int_{y=0}^{y=6}\left[(4-x^2)-0\right]dy\;dx$$

$$= 2\int_{x=0}^{x=2}\int_{y=0}^{y=6}(4-x^2)\,dy\;dx$$

$$= 2\int_{x=0}^{x=2}(4-x^2)\left[6-0\right]dx$$

OTHER LIMIT

$z = 0$

& $z = 4 - x^2$

THEREFORE $0 = 4 - x^2$

$x^2 = 4 \rightarrow x = \pm 2$

X LIMIT $= \int_{-2}^{2} dx$

$= 2\int_{0}^{2} dx$

Y LIMIT (GIVEN)

$0 \longrightarrow 6$

$$\int_0^6 dy$$

2

$2[$

2

$=$

$$\int_{y=0}^{6} \int_{z=0}^{z=4-x^2} f(z)\,dz \;\; dy \;\; dx = 2 \int_{x=0}^{x=2} \int_{y=0}^{6} \int_{z=0}^{z=4-x^2} f(z)\,dz \; dy \; dx$$

$$= 2 \int_{x=0}^{x=2} \int_{y=0}^{y=6} \left[(4-x^2) - 0 \right] dy \;\; dx$$

$$= 2 \int_{x=0}^{x=2} \int_{y=0}^{y=6} (4-x^2)\, dy \;\; dx$$

$$= 2 \int_{x=0}^{x=2} (4-x^2) \left[6-0 \right] dx$$

$$2 \int_{x=0}^{x=2} (24 - 6x^2)\, dx$$

$$2 \left[\int_0^2 24\, dx - 6 \int_0^2 x^2\, dx \right]$$

$$2 \left[24(2-0) - 6 \left(\frac{x^3}{3} \right)_0^2 \right]$$

$$= 2 \left[48 - 6 \frac{\left[2^3 - 0^3 \right]}{3} \right]$$

$$= 2 \left[48 - 6 \times 8/3 \right]$$

$$2 \left[48 - 16 \right]$$

$$2 \times 32 = 64$$

UNIT

ph FIND PARTIAL DIFFERENTIATION OF THE FOLLOWINGS

(i) $\frac{\partial}{\partial y} 3x^2 \cos y$

(ii) $\frac{\partial}{\partial x} 4x^3 \times \tan y \sin\theta$

(iii) $\frac{\partial}{\partial y} 3(x+2y)^3$

(iv) $\frac{\partial}{\partial z} 3(x+2y)^3$

(i) $\frac{\partial}{\partial y} 3x^2 \cos y = 3x^2 \frac{\partial}{\partial y}$

(ii) $\frac{\partial}{\partial x} 4x^3 \tan y \sin\theta =$

(iii) $\frac{\partial}{\partial y} 3(x+2y)^3 =$

(iv) $\frac{\partial}{\partial z} 3(x+2y)^3 =$

(i) $\frac{\partial}{\partial y} 3x^2 \cos y = 3x^2 \frac{\partial}{\partial y} \cos y = 3x^2 (-\sin y) = -3x^2 \sin y$

(ii) $\frac{\partial}{\partial x} 4x^3 \tan y \sin\theta = 4 \tan y \sin\theta \frac{\partial}{\partial x} x^3 = 4 \tan y \sin\theta \times 3 x^{3-1} \frac{\partial x}{\partial x} = 12$

(iii) $\frac{\partial}{\partial y} 3(x+2y)^3 = 3 \frac{\partial}{\partial y} (x+2y)^3 = 3 \times 3 (x+2y)^{3-1} \frac{\partial}{\partial y}(x+2y) = 9(x+2y)^2$

$= 9(x+2y)^2$

(iv) $\frac{\partial}{\partial z} 3(x+2y)^3 = 0$

$\cos y = 3x^2(-\sin y) = -3x^2 \sin y$

$$4 \tan y \sin\theta \frac{\partial}{\partial x} x^3 = 4 \tan y \sin\theta \times 3 x^{3-1} \frac{\partial x}{\partial x} = 12 \tan y \sin\theta \, x^2$$

$$\frac{\partial}{\partial y}(x+2y)^3 = 3 \times 3 (x+2y)^{3-1} \frac{\partial}{\partial y}(x+2y) = 9(x+2y)^2 \left[\frac{\partial x}{\partial y} + \frac{\partial}{\partial y} 2y\right]$$

$$= 9(x+2y)^2 [0 + 2] = 18(x+2y)^2$$

FIND THE INTEGRAL OF THE FOLLOWINGS

(a) $$\int_{y=1}^{y=3}\int_{x=0}^{x=2} x^2 \times 3y \; dx\, dy$$

(b) $$\int_{1}^{2}\int_{2}^{3}\int_{0}^{\pi/4} \sec\theta \tan\theta \, x^3 \, y^4 \; d\theta \, dx \, dy$$

(c) $$\int_{1}^{2}\int_{0}^{1} e^{x} \; dx \; dy$$

(a)

$$\int_1^3 \left[x^3\right]_0^2 \times 3y \, dy$$

$$\int_1^3 \left[\frac{2^3 - 0^3}{3}\right] \times 3y \, dy$$

$$\int_1^3 \frac{8}{\cancel{3}} \times \cancel{3}y \, dy$$

$$8 \int_1^3 y \, dy$$

$$8 \left(\frac{y^{1+1}}{1+1}\right)_1^3$$

$$8 \left[\frac{y^2}{2}\right]_1^3$$

$$8 \times \frac{3^2 - 1^2}{2}$$

$$4 \times [9 - 1]$$

$$4 \times 8$$

$$= 32$$

(b)

$$\frac{d}{d\theta} \tan\theta = \sec\theta \tan\theta$$

$$\tan\theta = \int \sec\theta \tan\theta$$

$$\int_1^2 \int_2^3 \left[\tan\theta\right]_0^{\pi/4} x^3 \, dx \, dy$$

$$\int_1^2 \int_2^3 \left[\tan\frac{\pi}{4} - \tan 0\right] x^3 \, dx \, dy$$

$$\int_1^2 \int_2^3 x^3 \, dx \; y^4 \, dy$$

$$\int_1^2 \left[\frac{x^4}{4}\right]_2^3 y^4 \, dy$$

(b)

$$\frac{d}{d\theta}\tan\theta = \sec\theta\tan\theta$$

$$\tan\theta = \int \sec\theta\tan\theta$$

$$\int_1^2 \int_2^3 \left[\tan\theta\right]_0^{\pi/4} x^3 \, dx \, dy$$

$$\int_1^2 \int_2^3 \left[\tan\frac{\pi}{4} - \tan 0\right] x^3 \, dx \, dy$$

$$\int_1^2 \int_2^3 x^3 \, dx \; y^4 \, dy$$

$$\int_1^2 \left[\frac{x^4}{4}\right]_2^3 y^4 \, dy$$

$$\int_1^2 \frac{3^4 - 2^4}{4} \, y^4$$

$$\frac{81-16}{4}\left[\frac{y^5}{5}\right]_1^2$$

$$\frac{65}{4} \times \frac{2^5 - 1^5}{5}$$

$$\frac{65}{4} \times \frac{32-1}{\cancel{5}}$$

$$\frac{13 \times 31}{4}$$

$$\frac{403}{4}$$