LINEAR DIFFERENTIAL EQUATION

IF $\frac{dy}{dx} + P(x)\,y = Q(x)$

THEN

ANSWER $\boxed{y = e^{-\int P\,dx} \int Q\, e^{\int P\,dx}\, dx + c\,e^{-\int P\,dx}}$

Pb SOLVE $x\frac{dy}{dx} - 2y = x^3 \cos 4x$

BOTH SIDES IS DIVIDED BY X $\frac{1}{x}\left(x\frac{dy}{dx} - 2y\right) = \frac{x^3 \cos 4x}{x}$

$$\frac{dy}{dx} - \frac{2y}{x} = x^2 \cos 4x$$

$$\frac{dy}{dx} - \frac{2}{x}\,y = x^2 \cos 4x$$

COMPARE WITH $\frac{dy}{dx} + P(x)\,y = Q(x)$

$\therefore\ P(x) = -\frac{2}{x}$ AND $Q(x) = x^2 \cos 4x$

$$y = e^{-\int P\,dx} \int Q\, e^{\int P\,dx}\, dx + c\, e^{-\int P\,dx}$$

$$= e^{-\int -\frac{2}{x}\,dx} \int x^2 \cos 4x \cdot e^{\int -\frac{2}{x}\,dx}\, dx + c\, e^{-\int -\frac{2}{x}\,dx}$$

$$= e^{2\int \frac{dx}{x}} \int x^2 \cos 4x \times e^{-2\int \frac{dx}{x}}\, dx + c\, e^{2\int \frac{dx}{x}}$$

$$\boxed{\int \frac{dx}{x} = \ln x}$$

$$= e^{2\ln x} \int x^2 \cos 4x \cdot e^{-2\ln x}\, dx + c\, e^{2\ln x}$$

$$\boxed{n \log m = \log m^n}$$

$$e^{\ln x^2} \int x^2 \cos 4x \cdot e^{\ln x^{-2}} \, dx + c \, e^{\ln x^2}$$

$$\boxed{e^{\ln x} = x} \quad \therefore e^{\ln x^2} = x^2$$

$$x^2 \int x^2 \cos 4x \times x^{-2} \, dx + c \times x^2$$

$$x^2 \int \cancel{x^2} \cos 4x \times \frac{1}{\cancel{x^2}} \, dx + c \times x^2$$

$$x^2 \int \cos 4x \, dx + c x^2$$

$$x^2 \int \frac{\cos 4x \, d4x}{4} + c x^2$$

$$x^2 \frac{\sin 4x}{4} + c x^2$$

$$\frac{x^2}{4} \sin 4x + c x^2$$

$) + c$