MATRICES

$$\begin{array}{c} \\ \text{Row (1)} \longrightarrow \\ (2) \longrightarrow \\ (3) \longrightarrow \\ \\ \\ \end{array}
\overset{\begin{array}{ccc} \text{column} \downarrow & \text{col2} & \text{col3} \end{array}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} & --- & a_{1m} \\ a_{21} & a_{22} & a_{23} & -- & a_{2m} \\ a_{31} & a_{32} & a_{33} & -- & a_{3m} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & — & a_{mm} \end{bmatrix}}$$

a_{11} = Row1, column 1

a_{12} = Row1, column2

a_{21} = Row2 column1

a_{22} = Row2 column 2

$\vdots$

pb(1)

$$A = \begin{bmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -5 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Row2, column 3 = 2 X 3 OR 2 BY 3 MATRIX

A & B ARE SAME SIZE

$$A + B = \begin{bmatrix} 2+3 & 1+(-5) & 4+1 \\ -3+2 & 0+1 & 2+3 \end{bmatrix} = \begin{bmatrix} 5 & -4 & 5 \\ -1 & 1 & 5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2-3 & 1-(-5) & 4-1 \\ -3-2 & 0-1 & 2-3 \end{bmatrix} = \begin{bmatrix} -1 & 6 & 3 \\ -5 & -1 & -1 \end{bmatrix}$$

$$4A = \begin{bmatrix} 4\times 2 & 4\times 1 & 4\times 4 \\ 4\times(-3) & 4\times 0 & 4\times 2 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 16 \\ -12 & 0 & 8 \end{bmatrix}$$

A^T = TRANSPOSE OF A MATRIX

A $\quad$ A^T

ROW 1 $\longrightarrow$ column 1 $\Rightarrow A^T = \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{bmatrix}$

ROW 2 $\longrightarrow$ column2

EXPRESSING MATRIX FORM FOR EQUATIONS

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 - - a_{1m}x_m = r_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 - - a_{2m}x_m = r_2$

$\ldots$

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 - - a_{mn}x_n = r_n$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & - - & a_{1m} \\ a_{21} & a_{22} & a_{23} & - - & a_{2m} \\ \cdot & - & - & - & - \\ a_{m1} & a_{m2} & a_{m3} & - - & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_m \end{bmatrix}$$

$5x_1 + 6x_2 + 7x_3 = 10$

$11x_1 + 13x_2 + 8x_3 = 12$

$4x_1 + 5x_2 + 9x_3 = 20$

$$\begin{bmatrix} 5 & 6 & 7 \\ 11 & 13 & 8 \\ 4 & 5 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 20 \end{bmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 7 \\ 11 & 13 & 8 \\ 4 & 5 & 9 \end{pmatrix}^{-1} \times \begin{bmatrix} 10 \\ 12 \\ 20 \end{bmatrix}$$

MULTIPLICATION MATRICES

pb 2

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 5 \\ 2 & -1 \\ 4 & 2 \end{bmatrix}$$

FIND → A × D

$$\begin{bmatrix} \sum_{\text{Row 1}}^{\text{1st matrix}} \times \text{2nd matrix column 1} & \sum_{\text{Row 1}}^{\text{1st matrix}} \times \text{2nd matrix column 2} \\ \sum_{\text{Row 2}}^{\text{1st matrix}} \times \text{2nd matrix column 1} & \sum_{\text{Row 2}}^{\text{1st matrix}} \times \text{2nd matrix column 2} \end{bmatrix}$$

1st MATRIX — 2 ROWS × 3 COLUMN

2nd MATRIX — 3 ROWS 2 COLUMNS

X MATRIX

SIZE OF MULTIPLIED MATRIX ⟹ (2 ROWS 2 COLUMN)

$$A \times D = \begin{bmatrix} (2\times3)+(1\times2)+(4\times4) & (2\times5)+(1\times -1)+(4\times2) \\ (-3\times3)+(0\times2)+(2\times4) & (-3\times5)+(0\times -1)+(2\times2) \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 17 \\ -1 & -11 \end{bmatrix}$$

TRACE OF MATRIX

$$\begin{bmatrix} 5 & 2 & 0 \\ 3 & 1 & -2 \\ -1 & 4 & 2 \end{bmatrix}$$

$5+1+2 = 8 =$ TRACE OF MATRIX

COFACTOR

$$\begin{bmatrix} 2 & -1 & 1 & 3 \\ -3 & 2 & 5 & 0 \\ 1 & 0 & -2 & 2 \\ 4 & -2 & 3 & 1 \end{bmatrix}$$

FIND COFACTOR 5

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \\ 4 & -2 & 1 \end{bmatrix} \leftarrow \text{COFACTOR "5"}$$

MATRIX OF COFACTOR $(A) = A_{jk}$

$$A = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$A_{jk} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

e "5"

$) = A_{jk}$

$A_{11} = (-1)^{1+1}$ [CROSS OUT ROW 1 & COLUMN 1]

$A_{21} = (-1)^{2+1}$ [CROSS OUT ROW 2, COLUMN 1]

$A_{32} = (-1)^{3+2}$ [CROSS OUT ROW 3, COLUMN 2]

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = (-1)^{2}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} = 1\left(2\times 2 + (-)(1\times(-3))\right)$$
$$= 1(4+3) = 7$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = (-1)^{3}\begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix} = -1\left[2 + (-)\,4(-3)\right] = -1(14) = -14$$

$$A_{13} = (-1)^{1+3}\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^{4}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = 1\left[1 - 4\times 2\right] = 1\left[1-8\right] = -7$$

$$A_{22} = (-1)^{2+2}\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^{4}\begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix} = 1\left[6 - 4\times 2\right] = -2$$

$A_{11} = (-1)^{1+1}$ [CROSS OUT ROW 1 & COLUMN 1]

$A_{21} = (-1)^{2+1}$ [CROSS OUT ROW 2, COLUMN 1]

$A_{32} = (-1)^{3+2}$ [CROSS OUT ROW 3, COLUMN 2]

— COFACTOR "5"

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = (-1)^2 \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} = 1\left(2\times 2 + (-)(1\times(-3))\right)$$

$$= 1(4+3) = 7$$

FOR $(A) = A_{jk}$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = (-1)^3 \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix} = -1\left[2+(-)\,4(-3)\right] = -1(14) = -14$$

$$A_{13} = (-1)^{1+3}\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^4 \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = 1\left[1-4\times 2\right] = 1\left[1-8\right] = -7$$

$$A_{22} = (-1)^{2+2}\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^4 \begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix} = 1\left[6-4\times 2\right] = -2$$

$$A_{21} = (-1)^{2+1}\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^3 \begin{pmatrix} -2 & 2 \\ 1 & 2 \end{pmatrix} = -1\left[-2\times 2+(-)1\times 2\right] = -1\left[-6\right] = 6$$

A_{13}

A_{23}

A_{33}

$$A_{23} = (-1)^{2+3} \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^5 \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} = -1\left[3 + (-)\,4\,(-2)\right] = -1\,[11] = -11$$

Row column

$$A_{31} = (-1)^{3+1} \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^4 \begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix} = 1\left[-2\times(-3) + (-)\,2\times 2\right] = 1\,[6-4] = 2$$

$$A_{32} = (-1)^{3+2} \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^5 \begin{bmatrix} 3 & 2 \\ 1 & -3 \end{bmatrix} = -1 \times \left[3\times -3 + (-)\,1\times(2)\right] = -\left[-9-2\right] = -1\,[-11] = 11$$

$$A_{33} = (-1)^{3+3} \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} = (-1)^6 \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix} = (1)\left[3\times 2 + (-)\,1\times(-2)\right] = [6+2] = 8$$

INVERSE

$$A = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & \end{vmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}^{-1}$$

$(-1)^5 \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} = -1\,[3 + (-)\,4\,(-2)] = -1\,[11] = -11$

$(-1)^4 \begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix} = 1\,[-2\times(-3) + (-)\,2\times 2] = 1\,[6-4] = 2$

$= (-1)^5 \begin{bmatrix} 3 & 2 \\ 1 & -3 \end{bmatrix} = -1 \times [3\times -3 + (-)\,1\times(2)] = -[-9-2] = -1[-11] = 11$

$= (-1)^6 \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix} = (1)\,[3\times 2 + (-)\,1\times(-2)] = [6+2] = 8$

$A_{jk} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix}$

$(A_{jk})^{\text{TRANSPOSE}} = (A_j$

$[A]\cdot[x] = [B] \longrightarrow [x]$

INVERSE MATRIX

$A^{-1} = \dfrac{(A_j}{d}$

$A = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}^{-1}$

$\det A = \begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} -$

$= 3\,[2\times 2 + (-)\,1$

$= 3\,[4+3] + 2\,[$

$$\left[-9-2\right] = -1\left[-11\right] = 11$$

$$+2\,\Big] = 8$$

$$A_{jk} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 7 & -14 & -7 \\ 6 & -2 & -11 \\ 2 & 11 & 8 \end{bmatrix}$$

$$\left(A_{jk}\right)^{\text{TRANSPOSE}} = \left(A_{jk}\right)^{T} = \begin{bmatrix} 7 & 6 & 2 \\ -14 & -2 & 11 \\ -7 & -11 & 8 \end{bmatrix}$$

Row1 $\longrightarrow$ column 1
Row2 $\longrightarrow$ column 2
Row 3 $\longrightarrow$ column 3

$$[A]\cdot[x] = [B] \longrightarrow [x] = [A]^{-1} \times [B]$$

$$A^{-1} = \frac{\left(A_{jk}\right)^{T}}{\det A}$$

INVERSE MATRIX

$$A^{-1} = \frac{\left(A_{jk}\right)^{T}}{\det A}$$

$$\det A = \begin{vmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} = 3\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -3 \\ 4 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 3\left[2\times 2 + (-)1\times(-3)\right] + 2\left[1\times 2 + (-)4(-3)\right] + 2\left[1\times 1 + (-)(4)2\right]$$

$$= 3(4+3) + 2\left[2+12\right] + 2\left[1-8\right] = 3\times 7 + 2\times 14 + 2\times(-7) = 35$$

-7
-11
8

Row 1 → column 1
Row 2 → column 2
Row 3 → column 3

$$A^{-1} = \frac{(A\,jn)^T}{\det A} = \frac{\begin{bmatrix} 7 & 6 & 2 \\ -14 & -2 & 11 \\ -7 & -11 & 8 \end{bmatrix}}{35}$$

$$= \begin{bmatrix} \frac{7}{35} & \frac{6}{35} & \frac{2}{35} \\ -\frac{14}{35} & -\frac{2}{35} & \frac{11}{35} \\ -\frac{7}{35} & -\frac{11}{35} & \frac{8}{35} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{6}{35} & \frac{2}{35} \\ -\frac{2}{5} & -\frac{2}{35} & \frac{11}{35} \\ -\frac{1}{5} & -\frac{11}{35} & \frac{8}{35} \end{bmatrix}$$

$$\frac{7}{35} = \frac{7/7}{35/7} = 1/5$$

$1 \quad 2$
$4 \quad +1.$

$)] + 2[1\times1 + (-)(4).2\ )$

$7 + 2\times14 + 2\times(-7) = 35$

EX

REFERRING

MATRIX

$3x_1$
x_1
$4x_1$

$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\frac{7}{35} = \frac{7/7}{35/7} = 1/5$$

$$\begin{bmatrix} \frac{1}{5} & \frac{6}{35} & \frac{2}{35} \\ -\frac{2}{5} & -\frac{2}{35} & \frac{11}{35} \\ -\frac{1}{5} & -\frac{11}{35} & \frac{8}{35} \end{bmatrix}$$

EX

REFERRING THE ABOVE PROBLEM, SOLVE THE FOLLOWING EQUATION BY MATRIX METHOD

$$3x_1 - 2x_2 + 2x_3 = 10$$
$$x_1 + 2x_2 - 3x_3 = -1$$
$$4x_1 + x_2 + 2x_3 = 3$$

$$\begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}^{-1} \times \begin{bmatrix} 10 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & \frac{6}{35} & \frac{2}{35} \\ -\frac{2}{5} & -\frac{2}{35} & \frac{11}{35} \\ -\frac{1}{5} & -\frac{11}{35} & \frac{8}{35} \end{bmatrix} \times \begin{bmatrix} 10 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \times 10 + \frac{6}{35} \times (-1) + \\ -\frac{2}{5} \times 10 + \left(-\frac{2}{35}\right)(-1) + \\ -\frac{1}{5} \times 10 + \left(-\frac{11}{35}\right)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 - \cancel{\frac{6}{35}} + \frac{6}{35} \\ -4 + \frac{2}{35} + \frac{33}{35} \\ -2 + \frac{11}{35} + \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 + \frac{2+33}{35} \\ -2 + \frac{11+24}{35} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4+1 \\ -2+1 \end{bmatrix}$$

THE FOLLOWING EQUATION BY

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5}\times 10 + \frac{6}{35}\times(-1) + \frac{2}{35}\times 3 \\ -\frac{2}{5}\times 10 + \left(\frac{-2}{35}\right)(-1) + \frac{11}{35}\times 3 \\ -\frac{1}{5}\times 10 + \left(\frac{-11}{35}\right)(-1) + \frac{8}{35}\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - \cancel{\frac{6}{35}} + \cancel{\frac{6}{35}} \\ -4 + \frac{2}{35} + \frac{33}{35} \\ -2 + \frac{11}{35} + \frac{24}{35} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 + \frac{2+33}{35} \\ -2 + \frac{11+24}{35} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4+1 \\ -2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

$\therefore x_1 = 2$

$x_2 = -3$

$x_3 = -1$

//

$$\begin{bmatrix} 10 \\ -1 \\ 3 \end{bmatrix}$$

SOME EQUATIONS HAVE NO SOLUTION

PROOF BY DETERMINANT

pb SOLVE

$$2x_1 + 5x_2 - 3x_3 = -3 \quad \text{—— } (1)$$
$$x_1 - 2x_2 + x_3 = 2 \quad \text{—— } (2)$$
$$7x_1 + 4x_2 - 3x_3 = -4 \quad \text{—— } (3)$$

$$(1) \rightarrow 2x_1 + 5x_2 - 3x_3 = -3$$
$$(2) \times 3 \Rightarrow 3x_1 - 6x_2 + 3x_3 = 6$$
$$5x_1 - x_2 = 3 \quad \text{—— } (\alpha)$$

$$(2) \times 3 \Rightarrow 3x_1 - 6x_2 + 3x_3 = 6$$
$$+$$
$$(3) \Rightarrow 7x_1 + 4x_2 - 3x_3 = -4$$
$$10x_1 - 2x_2 = 2 \quad \text{—— } (\beta)$$

Solution

$-3x_3 = -3$ —— ①

$+x_3 = 2$ —— ②

$_2 - 3x_3 = -4$ —— ③

$-3x_3 = -3$

$+3x_3 = 6$

$= 3$ —— α

$+3x_3 = 6$

$_2 - 3x_3 = -4$

$x_2 = 2$ —— β

$\alpha \Rightarrow 5x_1 - x_2 = 3$

$\beta \Rightarrow \cancel{10x_1} - \cancel{2x_2} = 2$

$- \quad +$

$\alpha \times 2 \Rightarrow \cancel{10x_1} - \cancel{2x_2} = 6$

$0 = 8$

$$\Delta = \begin{bmatrix} 2 & 5 & -3 \\ 1 & -2 & 1 \\ 7 & 4 & -3 \end{bmatrix}$$

$$= 2\begin{vmatrix} -2 & 1 \\ 4 & -3 \end{vmatrix} - 5\begin{vmatrix} 1 & 1 \\ 7 & -3 \end{vmatrix} + (-3)\begin{vmatrix} 1 & -2 \\ 7 & 4 \end{vmatrix}$$

$2[-2(-3) +$

$2[6 - 4$

$2 \times 2 -$

$4 +$

54

$$2\left[-2(-3)+(-)4\times1\right]-5\left[1\times-3+(-)7\times1\right]+(-3)\left[1\times4+(-)7(-2)\right]$$

$$2\left[6-4\right]-5\left[-3-7\right]-3\left[4+14\right]$$

$$2\times2-5\times(-10)-3\times18$$

$$4+50-54$$

$$54-54=0$$

AS $\Delta=0$, THERE IS NO SOLUTION FOR

THE GIVEN EQUATIONS

$$+(-3)\begin{vmatrix}1 & -2\\ 7 & 4\end{vmatrix}$$

ph GIVEN THAT $P_0(x) = 1$, $P_1(x) = x$

FIND (a) $P_2(x)$ (b) $P_3(x)$

$$P_{n+1}(x) = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} P_{n-1}(x)$$

$P_2(x) = P_{1+1}(x)$ COMPARE WITH P_{n+1}

$\therefore n = 1$

$$P_{1+1}(x) = \frac{2(1)+1}{1+1} x P_1(x) - \frac{1}{1+1} P_{1-1}(x)$$

$$= \frac{3}{2} x P_1(x) - \frac{1}{2} P_0(x)$$

$$= \frac{3}{2} x \times x - \frac{1}{2} \times 1$$

$$= \frac{3}{2} x^2 - \frac{1}{2}$$

(b) $P_3(x) = ?$

$P_3(x) = P_{(2+1)}(x)$ COMPARE WITH $P_{n+1}(x)$

$\therefore n = 2$

$$P_{2+1}(x) = \frac{2\times 2+1}{2+1}\, x\, P_2(x) - \frac{2}{2+1} P_{2-1}(x)$$

$$= \frac{5}{3} x P_2(x) - \frac{2}{3} P_1(x)$$

$$= \frac{5}{3} x \left[\frac{3}{2} x^2 - \frac{1}{2} \right] - \frac{2}{3} x$$

$$= \frac{5}{3} x \times \frac{3}{2} x^2 - \frac{1}{2} \times \frac{5}{3} x - \frac{2}{3} x$$

$$= \frac{5}{2} x^3 - \frac{5}{6} x - \frac{2}{3} x$$

$$= \frac{5}{2} x^3 - \frac{(5+2\times 2)}{6} x$$

$$= \frac{5}{2} x^3 - \frac{9}{6} x \quad = \frac{5}{2} x^3 - \frac{3}{2} x$$

BINOMIAL THEOREM

$$(p+v)^n = 1 + pv + \frac{p(p-1)}{2!} v^2 + \frac{p(p-1)(p-2)}{3!} v^3 + \cdots$$

GENERATIVE FUNCTION

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)\, t^n$$

$$\left(\frac{1}{\sqrt{1-2xt+t^2}}\right)^2 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P_m(x)\, P_n(x)\, t^{m+n}$$

$$(10+x)^5 = 1 + 10x + \frac{10(10-1)}{2!} x^2 + \frac{10(10-1)(10-2)}{3!} x^3 + \frac{10(10-1)(10-2)(10-3)}{4!} x^4 + \frac{10(10-1)(10-2)(10-3)(10-4)}{5!} x^5$$

$$= 1 + 10x + \frac{10 \times 9}{2 \times 1} x^2 + \frac{10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\cancel{3} \times \cancel{2} \times 1} x^3 + \frac{10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} x^4 + \frac{10 \times 9 \times \overset{2}{\cancel{8}} \times 7 \times \overset{2}{\cancel{6}}}{5 \times \cancel{4} \times \cancel{3} \times 2 \times 1} x^5$$

$$= 10 + 10x + 45x^2 + 120x^3 + 210x^4 + 2520\, x^5$$

ph $(3+x)^5 = ?$

ph $(3+2x)^4 = ?$

ph $(3+x)^5 = (p+v)^n \quad \therefore p=3, \; v=x, \; n=5$

$$(3+x)^5 = 1 + 3x + \frac{3(3-1)}{2!}x^2 + \frac{3(3-1)(3-2)}{3!}x^3 + \frac{3(3-1)(3-2)(3-3)}{4!}x^4 \to 0$$

$$\boxed{\begin{array}{l} 2! = 2\times 1 \\ 3! = 3\times 2\times 1 \\ 4! = 4\times 3\times 2\times 1 \end{array}}$$

$$+ \frac{3(3-1)(3-2)(3-3)(3-4)}{5!}x^5$$

$$= 1 + 3x + \frac{3\times 2}{2\times 1}x^2 + \frac{3\times 2\times 1}{3\times 2\times 1}x^3 + 0 + \frac{3\times 2\times 1\times 0\times -1}{5!}x^5$$

$$= 1 + 3x + 3x^2 + x^3$$

$$(3+2x)^4 = \left(2\left(\tfrac{3}{2}+x\right)\right)^4 = 2^4\left(\tfrac{3}{2}+x\right)^4$$

$$2^4\left(\tfrac{3}{2}+x\right)^4 = 2^4\left[1+\tfrac{3}{2}x+\frac{3/2(3/2-1)}{2!}x^2+\frac{3/2(3/2-1)(3/2-2)}{3!}x^3+\frac{3/2(3/2-1)(3/2-2)(3/2-3)}{4!}x^4\right]$$

$$=16\left[1+\tfrac{3}{2}x+\tfrac{3}{2}\times\tfrac{1}{2}\times\frac{1}{2\times1}x^2+\tfrac{3}{2}\times\tfrac{1}{2}\times\left(-\tfrac{1}{2}\right)\times\frac{1}{3\times2\times1}x^3+\frac{\tfrac{3}{2}\times\tfrac{1}{2}\left(-\tfrac{1}{2}\right)\left(-3/2\right)}{4\times3\times2\times1}x^4\right]$$

$$=16\left[1+\tfrac{3}{2}x+\frac{3}{8}x^2-\frac{x^3}{16}+\frac{9}{16}\times\frac{1}{4\times3\times2}x^4\right]$$

$$=16\left(1+\tfrac{3}{2}x+\frac{3}{8}x^2-\frac{x^3}{16}+\frac{3}{128}x^4\right]$$

$$=16+16\times\tfrac{3}{2}x+16\times\tfrac{3}{8}x^2-16\times\frac{x^3}{16}+16\times\frac{3}{128}x^4\right]$$

$$=16+24x+6x^2-x^3+\frac{3}{7}x^4$$

$\cancel{\cancel{x}}$

$$(6-3x)^3 = ?$$

$$(p+v)^n = 1 + pv + \frac{p(p-1)}{2!}v^2 + \frac{p(p-1)(p-2)}{3!}v^3 + \cdots$$

$$(p+2v)^n = 1 + p(2v) + \frac{p(p-1)(2v)^2}{2!} + \frac{p(p-1)(p-2)(2v)^3}{3!} + \cdots$$

$$(p-2v)^n = 1 + p(-2v) + \frac{p(p-1)(-2v)^2}{2!} + \frac{p(p-1)(p-2)(-2v)^3}{3!} + \cdots$$

$$(6-3x)^3 = 1 + 6(-3x) + \frac{6(6-1)}{2!}(-3x)^2 + \frac{6(6-1)(6-2)}{3!}(-3x)^3$$

$$= 1 - 18x + \frac{6\times5}{2\times1}\times 9x^2 + \frac{6\times5\times\cancel{4}^{2}}{\cancel{3}\times\cancel{2}\times1}\times\cancel{9}^{3}x^3$$

$$= 1 - 18x + 135x^2 + 180x^3$$