DIFFERENTIATION

INTEGRATION

CALCULUS (7759P / UEENEEE050) ← FLEXIBLE DELIVERY

WK 13 → WK 18 FACE TO FACE SUPPORT

DIFFERENTIAL EQUATION

ORDINARY DIFFERENTIAL EQUATIONS

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$$

Pb

$$\frac{dy}{dx} = 3x^2$$

FIND $y = ?$

$$dy = 3x^2\,dx$$

INTEGRATE BOTH SIDES

$$\int dy = \int 3x^2\,dx$$

$$y = 3\int x^2\,dx$$

$$= 3\,\frac{x^{2+1}}{2+1} + c$$

$$y = 3 \times \frac{x^3}{3} + c$$

$$y = x^3 + c$$

pb ② SOLVE $y'' = 3x-2$, $y(0)=2$, $y'(1) = -3$

$y' = \dfrac{dy}{dx}$ ← FIRST ORDER DIFFERENTIAL EQUATION

$y'' = \dfrac{d^2y}{dx^2}$ ← SECOND ORDER DIFFERENTIAL EQUATION

$y'' = 3x-2$

$$\boxed{\frac{d^2y}{dx^2} = 3x-2}$$

$$\int y''\,dy = \int 3x\,dx - \int 2\,dx + c_1$$

$$\boxed{\int y''\,dy = y'(x)}$$

$y'(x) = 3\int x\,dx - 2\int dx + c_1$

$y'(x) = 3\dfrac{x^{1+1}}{1+1} - 2x + c_1$

$y'(x) = 3\dfrac{x^2}{2} - 2x + c_1$ —— 1

$y'(1) = 3\dfrac{1^2}{2} - 2\times1 + c_1$

$-3 = \dfrac{3}{2} - 2 + c_1$

$c_1 = -3 - \dfrac{3}{2} + 2 = -1 - \dfrac{3}{2} = -\dfrac{5}{2}$

+c

$\therefore \; y'(x) = 3\frac{x^2}{2} - 2x - \frac{5}{2}$

$\int y'(x)\, dy = \frac{3}{2}\int x^2\,dx - 2\int x\,dx - \frac{5}{2}\int dx$

$y(x)$

$y = \frac{3}{2} \times \frac{x^{2+1}}{2+1} - 2\frac{x^{1+1}}{1+1} - \frac{5}{2}x + c_2$

$y = \frac{3}{2} \times \frac{x^3}{3} - 2\frac{x^2}{2} - \frac{5}{2}x + c_2$

$y = \frac{x^3}{2} - x^2 - \frac{5}{2}x + c_2 \quad \text{—— } (2)$

x

$y(0) = \frac{0^3}{2} - 0^2 - \frac{5}{2}(0) + c_2$

$2 = c_2$

$\therefore c_2 = 2$

$y = \frac{x^3}{2} - x^2 - \frac{5}{2}x + 2$

y (OR) $y(x)$

Pb. SOLVE

$\frac{dy}{dx} + 3y = 8, \; y(0) = 2$

FIND EQUATION

$$\frac{dy}{dx} + 3y = 8$$

$$\frac{dy}{dx} = 8 - 3y$$

$$dy = (8 - 3y)\,dx$$

$$\frac{dy}{8-3y} = dx \quad \text{—— (1)}$$

$$\boxed{\int \frac{1}{y}\,dy = \ln y + c}$$

$$\boxed{\int \frac{1}{8-3y}\,d(8-3y) = \ln(8-3y) + c}$$

$$d(8-3y) = d8 - d3y = -d3y = -3\,dy$$

$$-3\,dy = d(8-3y)$$

$$dy = \frac{d(8-3y)}{-3} = -\frac{1}{3}d(8-3y) \quad //$$

SUBSTITUTE IN EQUATION (1)

$$\frac{-\frac{1}{3}d(8-3y)}{8-3y} = dx$$

$$\int -\frac{1}{3}\frac{d(8-3y)}{(8-3y)} = \int dx + c$$

$$-\frac{1}{3}\int \frac{d(8-3y)}{(8-3y)} = x + c$$

$$-\frac{1}{3}\ln(8-3y) = x + c$$

$$y(0) = 2 \rightarrow \begin{cases} y = 2 \\ x = 0 \end{cases}$$

$$-\frac{1}{3}\ln(8-3\times 2) = 0 + c$$

$$-\frac{1}{3}\ln 2 = c$$

$$-\frac{1}{3}\ln(8-3y) = x - \frac{1}{3}\ln 2$$

//

Pb. SOLVE $\frac{dy}{dx} = \sec y \tan x$

MAIN	$\sin$	$\cos$	$\tan$
	$\downarrow$	$\downarrow$	$\downarrow$
ADDITIONAL	cosec	$\sec$	$\cot$

$\tan = \frac{\sin}{\cos}$

(i) CONVERT TO MAIN, BASIC TRIGO RATIOS

$$\frac{dy}{dx} = \frac{1}{\cos y} \times \frac{\sin x}{\cos x}$$

(ii)

$$\cos y \, dy = \frac{\sin x \, dx}{\cos x} \leftarrow$$ SEPARATION

$\boxed{\int \cos y \, dy = \sin y}$

$\boxed{d\cos x = -\sin x \, dx} \rightarrow \boxed{-(\sin x \, dx) = d\cos x}$

$\sin x \, dx = -d\cos x$

$$\int \cos y \, dy = \int \frac{\sin x \, dx}{\cos x} + c$$

$$\sin y = \int \frac{-d \cos x}{\cos x} + c$$

$$\sin y = -\ln \cos x + c$$

$$\int \frac{1}{u} du = \ln u + c$$

$$\int \frac{1}{\cos x} d\cos x = \ln \cos x + c$$

$$\int \frac{1}{\sin x} d\sin x = \ln \sin x + c$$

pb/ FIND THE GENERAL SOLUTION OF

$$(4x + x y^2) dx + (y + x^2 y) dy = 0$$

$$x(4 + y^2) dx + y(1 + x^2) dy = 0$$

BOTH SIDES OF EQUATION IS DIVIDED BY

$$(4+y^2)(1+x^2)$$

$$\frac{x(4+y^2)dx + y(1+x^2)dy}{(4+y^2)(1+x^2)} = 0$$

$$\frac{x\cancel{(4+y^2)}\,dx}{\cancel{(4+y^2)}(1+x^2)} + \frac{y\cancel{(1+x^2)}\,dy}{(4+y^2)\cancel{(1+x^2)}} = 0$$

$$\frac{x\,dx}{(1+x^2)} + \frac{y\,dy}{4+y^2} = 0$$

$$dx^n = n x^{n-1} dx$$

$$dx^2 = 2x^{2-1} dx = 2x\,dx$$

$$d(1+x^2) = d1 + dx^2 = 0 + 2x\,dx$$

$$d(1+x^2) = 2x\,dx$$

$$dx = \frac{d(1+x^2)}{2x}$$

$$d(4+y^2) = d4 + dy^2 = 0 + 2y\,dy$$

$$d(4+y^2) = 2y\,dy$$

$$dy = \frac{d(4+y^2)}{2y}$$

$$\int \frac{1}{u} du = \ln u + c$$

$$\int \frac{1}{(1+x^2)} d(1+x^2) = \ln(1+x^2) + c$$

$$x \, dx \, \frac{1}{(1+x^2)} + y \, dy \, \frac{1}{(4+y^2)} = 0$$

$$\cancel{x} \, \frac{d(1+x^2)}{2\cancel{x}} \times \frac{1}{(1+x^2)} + \cancel{y} \, \frac{d(4+y^2)}{2\cancel{y}} \times \frac{1}{(4+y^2)} = 0$$

$$\frac{1}{2} \frac{d(1+x^2)}{(1+x^2)} + \frac{1}{2} \frac{d(4+y^2)}{(4+y^2)} = 0$$

$$\frac{1}{2} \int \frac{d(1+x^2)}{(1+x^2)} + \frac{1}{2} \int \frac{d(4+y^2)}{(4+y^2)} = 0$$

$$\frac{1}{2}\ln(1+x^2)+\frac{1}{2}\ln(4+y^2)=c$$

pb IN ABOVE PROBLEM, FIND THE PARTICULAR EQUATION FOR

$y(1)=2$

$x=1 \rightarrow y=2 \longrightarrow$ EQUATION

$$\frac{1}{2}\ln(1+x^2)+\frac{1}{2}\ln(4+y^2)=c$$

$$\frac{1}{2}\left[\ln(1+x^2)+\ln(4+y^2)\right]=c$$

$$\ln(1+x^2)+\ln(4+y^2)=2c=c_1$$

$$\boxed{\log m+\log N=\log mN}$$

$$\ln(1+x^2)(4+y^2)=c_1$$

$$(1+x^2)(4+y^2)=e^{c_1}=c_2$$

WHEN $x=1,\ y=2$

$$(1+1^2)(4+2^2)=c_2$$

$$2\times 8=c_2 \quad \therefore c_2=16$$

$$\therefore (1+x^2)(4+y^2)=16$$

$$4+y^2=\frac{16}{1+x^2}$$

$$y^2=\frac{16}{1+x^2}-4$$

$$y=\sqrt{\frac{16}{1+x^2}-4}$$

LINEAR DIFFERENTIAL EQUATION

IF $\frac{dy}{dx} + P(x)\,y = Q(x)$

THEN

$$\boxed{y = e^{-\int P dx} \int Q\, e^{\int P dx}\, dx + c e^{-\int P dx}}$$

ANSWER

Pb SOLVE $x\frac{dy}{dx} - 2y = x^3 \cos 4x$

BOTH SIDES IS DIVIDED BY X

$$\frac{1}{x}\left(x\frac{dy}{dx} - 2y\right) = \frac{x^3 \cos 4x}{x}$$

$$\frac{dy}{dx} - \frac{2y}{x} = x^2 \cos 4x$$

$$\frac{dy}{dx} - \frac{2}{x}\,y = x^2 \cos 4x$$

COMPARE WITH $\frac{dy}{dx} + P(x)\,y = Q(x)$

$\therefore\ P(x) = -\dfrac{2}{x}$ AND $Q(x) = x^2 \cos 4x$

$$y = e^{-\int P\,dx} \int Q\, e^{\int P\,dx}\, dx + c\, e^{-\int P\,dx}$$

$$= e^{-\int -\frac{2}{x}\,dx} \int x^2 \cos 4x \cdot e^{\int -\frac{2}{x}\,dx}\, dx + c\, e^{-\int -\frac{2}{x}\,dx}$$

$$= e^{2\int \frac{dx}{x}} \int x^2 \cos 4x \times e^{-2\int \frac{dx}{x}}\, dx + c\, e^{2\int \frac{dx}{x}}$$

$$\boxed{\int \frac{dx}{x} = \ln x}$$

$$= e^{2\ln x} \int x^2 \cos 4x \cdot e^{-2\ln x}\, dx + c\, e^{2\ln x}$$

$$\boxed{n \log m = \log m^n}$$

$$e^{\ln x^2} \int x^2 \cos 4x \cdot e^{\ln x^{-2}}\, dx + c\, e^{\ln x^2}$$

$$\boxed{e^{\ln x} = x} \quad \therefore e^{\ln x^2} = x^2$$

$$x^2 \int x^2 \cos 4x \times x^{-2}\, dx + c \times x^2$$

$$x^2 \int \cancel{x^2} \cos 4x \times \frac{1}{\cancel{x^2}}\, dx + c \times x^2$$

$$x^2 \int \cos 4x\, dx + c\, x^2$$

$$x^2 \int \frac{\cos 4x\, d4x}{4} + c\, x^2$$

$$x^2\, \frac{\sin 4x}{4} + c\, x^2$$

$$\frac{x^2}{4} \sin 4x + c\, x^2 \quad \#$$

)+c