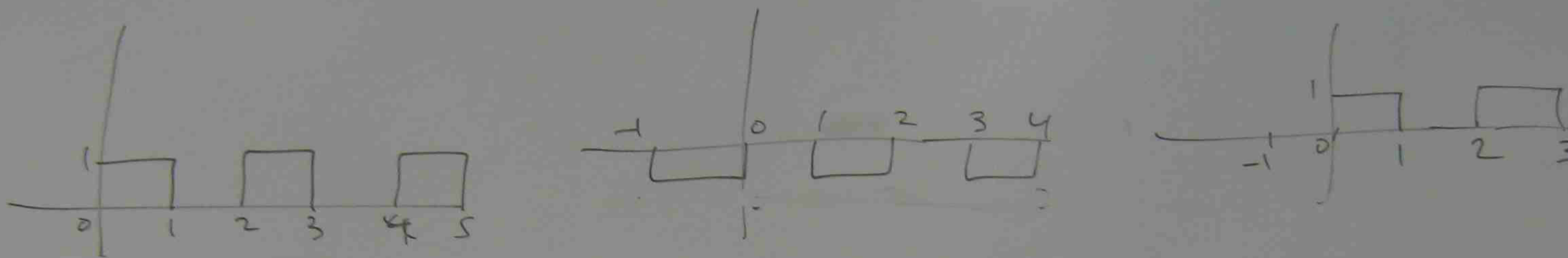
APPLICATION OF LEGENDRE POLYNOMIAL IN DIGITAL SIGNAL PROCESSING



$$f(x) = \sum_{k=0}^{\infty} A_k \, P_k(x) \qquad -1 < x < 1$$

$$A_k = \frac{2k+1}{2} \int_{-1}^{1} P_k(x) \, f(x) \, dx$$

$$P_k(x) = \frac{1}{2^k \, k!} \frac{d^k}{dx^k} (x^2-1)^k$$

pb EXPAND THE FUNCTION $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & -1 < x < 0 \end{cases}$ IN THE SERIES OF THE FORM $\sum_{k=0}^{1} A_k P_k$

$$\sum_{k=0}^{1} A_k P_k(x) = A_0 P_0(x) + A_1 P_1(x)$$

$$A_k = \frac{2k+1}{2} \int_{-1}^{1} P_k(x) f(x)\, dx = \frac{2k+1}{2} \left[\cancel{\int_{-1}^{0} P_k(x) f(x)\, dx}^{\,0} + \int_{0}^{1} P_k(x) f(x)\, dx \right]$$

$$\boxed{A_k = \frac{2k+1}{2} \int_{0}^{1} P_k(x) f(x)\, dx}$$

$$A_0 = \frac{2\times 0 + 1}{2} \left[\int_{-1}^{0} P_0(x) f(x)\, dx + \int_{0}^{1} P_0(x) f(x)\, dx \right]$$

$-1 \to 0 = f(x) = 0$, $0 \to 1 \Rightarrow f(x) = 1$

$$A_0 = \frac{1}{2} \left[\int_{-1}^{0} P_0(x) \cancel{\times 0}\, dx + \int_{0}^{1} P_0(x) \times 1\, dx \right]$$

$$= \frac{1}{2} \int_{0}^{1} P_0(x)\, dx$$

$0 < x < 1$

$-1 < x < 0$ IN THE SERIES OF THE FORM $\sum_{k=0}^{1} A_k P_k(x)$

$)$

$\left[\int_{-1}^{0} P_k(x) \cancel{f(x)\,dx} + \int_{0}^{1} P_k(x)\, f(x)\,dx \right]$

$f(x)\,dx \Big]$

$$\boxed{A_k = \frac{2k+1}{2} \int_0^1 P_k(x)\, f(x)\,dx}$$

$1\,dx$

$$P_k(x) = \frac{1}{2^k\, k!} \frac{d^k}{dx^k} (x^2-1)^k$$

$$P_0(x) = \frac{1}{2^0 \times 0!} \frac{d^0}{dx^0} (x^2-1)^0$$

$$= \frac{1}{1 \times 1} \times 1 = 1$$

$$A_0 = \frac{1}{2} \int_0^1 P_0(x)\,dx = \frac{1}{2} \int_0^1 1\,dx$$

$$= \frac{1}{2} [x]_0^1 = \frac{1}{2}(1-0)$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

$A_0 = 1/2$

$A_0 = 1/2, \quad P_0 = 1$

$$A_K = \frac{2K+1}{2}\int_0^1 P_K(x)\,dx$$

$$A_1 = \frac{2\times 1+1}{2}\int_0^1 P_1(x)\,dx$$

$$P_K(x) = \frac{1}{2^K\,K!}\,\frac{d^K}{dx^K}(x^2-1)^K$$

$$P_1(x) = \frac{1}{2^1\times 1!}\,\frac{d^1}{dx^1}(x^2-1)^1$$

$$= \frac{1}{2}\times\frac{d}{dx}(x^2-1)$$

$$= \frac{1}{2}\left[\frac{d}{dx}x^2 - \underbrace{\frac{d}{dx}1}_{0}\right]$$

$$= \frac{1}{2}\times 2x^{2-1}$$

$$= \frac{1}{2}\times 2x$$

$$= x$$

$$A_1 = \frac{2\times 1+1}{2}\int_0^1 x\,dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1^2}{2} - \frac{0^2}{2}\right]$$

$$= \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$$

$$A_1 = 3/4,\quad P_1(x) = x$$

$$= \frac{1}{2} \times 2x^{2-1}$$

$$= \frac{1}{2} \times 2x$$

$$= x$$

$$(x^2-1)^K$$

$$A_1 = \frac{2\times1+1}{2} \int_0^1 x \, dx$$

$$(x^2-1)^1$$

$$= \frac{3}{2} \left[\frac{x^2}{2} \right]_0^1$$

$$-1)$$

$$= \frac{3}{2} \left[\frac{1^2}{2} - \frac{0^2}{2} \right]$$

$$\left. x \right]_0^1$$

$$= \frac{3}{2} \times \frac{1}{2} = 3/4$$

$$A_1 = 3/4, \quad P_1(x) = x$$

$$\sum_{K=0}^{1} A_K P_K(x) = A_0 P_0(x) + A_1 P_1(x)$$

$$= \frac{1}{2} \times 1 + \frac{3}{4} x$$

$$= \frac{1}{2} + \frac{3}{4} x$$

$$A_k = \frac{2k+1}{2}\int_0^1 P_k(x)\,dx$$

$$A_1 = \frac{2\times1+1}{2}\int_0^1 P_1(x)\,dx$$

$$P_k(x) = \frac{1}{2^k\,k!}\,\frac{d^k}{dx^k}(x^2-1)^k$$

$$P_1(x) = \frac{1}{2^1\times 1!}\,\frac{d^1}{dx^1}(x^2-1)^1$$

$$= \frac{1}{2}\times\frac{d}{dx}(x^2-1)$$

$$= \frac{1}{2}\left[\frac{d}{dx}x^2 - \frac{d}{dx}1\right]$$

$$= \frac{1}{2}\times 2x^{2-1}$$

$$= \frac{1}{2}\times 2x$$

$$= x$$

$$A_1 = \frac{2\times1+1}{2}\int_0^1 x\,dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1^2}{2} - \frac{0^2}{2}\right]$$

$$= \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$$

$$A_1 = 3/4,\quad P_1(x) = x$$

$$= \frac{1}{2} \times 2x^{2-1}$$

$$= \frac{1}{2} \times 2x$$

$$= x$$

$$A_1 = \frac{2\times1+1}{2} \int_0^1 x \, dx$$

$$= \frac{3}{2} \left[\frac{x^2}{2} \right]_0^1$$

$$= \frac{3}{2} \left[\frac{1^2 - 0^2}{2} \right]$$

$$= \frac{3}{2} \times \frac{1}{2} = 3/4$$

$A_1 = 3/4$, $P_1(x) = x$

$$\sum_{K=0}^{1} A_K P_K(x) = A_0 P_0(x) + A_1 P_1(x)$$

$$= \frac{1}{2} \times 1 + \frac{3}{4} x$$

$$= \frac{1}{2} + \frac{3}{4} x$$

$(x^2-1)^K$

$(x^2-1)^1$

$P_1(x)$

IN ABOVE PROBLEM, FIND $A_3(x)$ AND $P_3(x)$

$$P_k(x) = \frac{1}{2^k\,k!}\,\frac{d^k}{dx^k}(x^2-1)^k$$

$$P_3(x) = \frac{1}{2^3 \times 3!}\,\frac{d^3}{dx^3}(x^2-1)^3$$

$$= \frac{1}{8\times 3\times 2\times 1}\,\frac{d^3}{dx^3}(x^2-1)^3$$

$$= \frac{1}{48} \times \frac{d^3}{dx^3}(x^2-1)^3$$

$$\frac{d}{dx}(x^2-1)^3 = 3(x^2-1)^{3-1}\,\frac{d}{dx}(x^2-1)$$

$$= 3(x^2-1)^2\;2x^{2-1}$$

$$= 6x(x^2-1)^2$$

$$= 6x(x^4-2x^2+1)$$

$$= 6x^5-12x^3+6x$$

$$\frac{d^2x}{dx^2} = \frac{d}{dx}(6x^5-12x^3+6x)$$

$$= 6\times 5\,x^{5-1} - 12\times 3\times x^{3-1} + 6$$

$$= 30x^4 - 36x^2 + 6$$

$$\frac{d^3x}{dx^3} = \frac{d}{dx}(30x^4-36x^2+6)$$

$$= 30\times 4\times x^{4-1} - 36\times 2\times x^{2-1} + 0$$

$$= 120x^3 - 72x$$

$$P_3(x) = \frac{1}{48}\times(120x^3-72x)$$

$$= \frac{5}{2}x^3 - 3x$$

APPLICATION of LEGENDRE POLYNOMIAL IN DIGITAL SIGNAL PROCESSING

$A_3(x) = ?$

$$A_k(x) = \frac{2k+1}{2} \int_0^1 P_k(x)\,dx$$

$$A_3(x) = \frac{2\times 3+1}{2} \int_0^1 P_3(x)\,dx$$

$$= \frac{7}{2} \int_0^1 \left(\frac{5}{2}x^3 - 3x\right) dx$$

$$= \frac{7}{2} \times \left[\frac{5}{2}\int_0^1 x^3\,dx - 3\int_0^1 x\,dx\right]$$

$$= \frac{7}{2}\left[\frac{5}{2}\times\left[\frac{x^4}{4}\right]_0^1 - 3\times\left[\frac{x^2}{2}\right]_0^1\right]$$

$$\frac{7}{2}\left[\frac{5}{2}\times\frac{1}{4} - 3\times\frac{1}{2}\right]$$

$$\frac{7}{2}\left[\frac{5}{8} - \frac{3}{2}\right]$$

$$\frac{7}{2}\left[\frac{5-12}{8}\right]$$

$$\frac{7}{2}\times\frac{-7}{8}$$

$$= -\frac{49}{16}$$