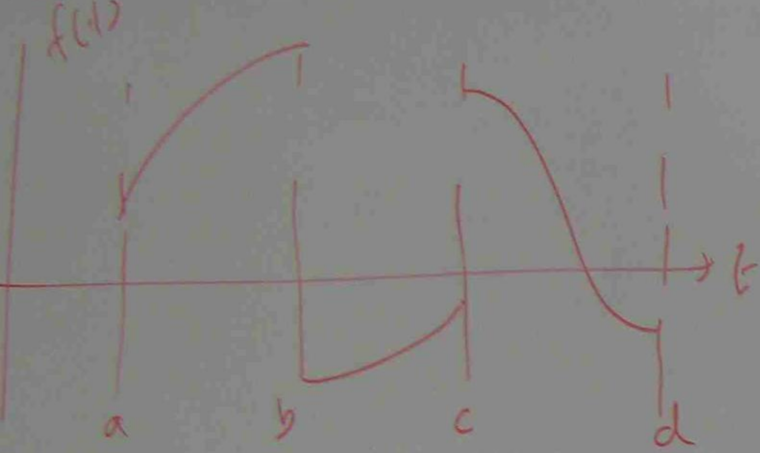
PIECEWISE CONTINUITY

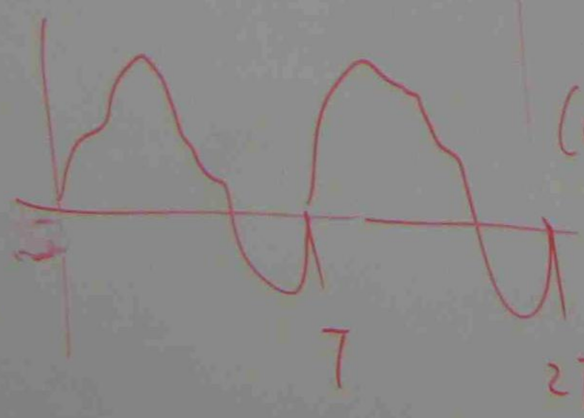


$a \longrightarrow b$

$b \longrightarrow c$

$c \longrightarrow d$

PIECE WISE CONTINUITY



SOME SPECIAL THEOREM FOR LAPLACE TRANSFORM

(1) $\mathcal{L} f(t) = F(s)$

(2) $\mathcal{L} e^{at} f(t) = F(s-a)$

(3) $\mathcal{L} u(t-a) f(t-a) = e^{-as} F(s)$

(4) $\mathcal{L} f(at) = \frac{1}{a} F\left(\frac{s}{a}\right)$

(5) $\mathcal{L} t^n f(t) = (-1)^n \frac{d^n f(s)}{ds^n}$

(6) PERIODIC FUNCTION $p>0$

$$\mathcal{L} f(t) = \int_0^p \frac{e^{-st} f(t)\,dt}{1-e^{-st}}$$

INTEGRATION

$$\mathcal{L}\int_0^t f(u)\,du = \frac{F(s)}{s}$$

Pb 5

FIND (a) $\mathcal{L}^{-1} \dfrac{5}{s+2}$ (b) $\mathcal{L}^{-1} \dfrac{4s-3}{s^2+4}$

(c) $\mathcal{L}^{-1} \dfrac{2s-5}{s^2}$

$$\boxed{\mathcal{L}^{-1} \frac{1}{s-a} = e^{at}}$$

$$\mathcal{L}^{-1} \frac{1}{s+a} = e^{-at}$$

(a) $\mathcal{L}^{-1} \dfrac{5}{s+2} = 5\, \mathcal{L}^{-1} \dfrac{1}{s+2}$

USE $\mathcal{L}^{-1} \dfrac{1}{s+a} = e^{-at}$

$= 5 \times e^{-2t}$

(b) $\mathcal{L}^{-1}\dfrac{4s-3}{s^2+4} = \mathcal{L}^{-1}\dfrac{4s}{s^2+4} - \mathcal{L}^{-1}\dfrac{3}{s^2+4}$

$= 4\,\mathcal{L}^{-1}\dfrac{s}{s^2+4} - 3\,\mathcal{L}^{-1}\dfrac{1}{s^2+4}$

$$\boxed{\mathcal{L}^{-1}\frac{s}{s^2+w^2} = \cos wt \quad , \quad \mathcal{L}^{-1}\frac{w}{s^2+w^2} = \sin wt}$$

$= 4\,\mathcal{L}^{-1}\dfrac{s}{s^2+2^2} - 3\,\mathcal{L}^{-1}\dfrac{1}{s^2+2^2}$

$= 4\,\mathcal{L}^{-1}\dfrac{s}{s^2+2^2} - \dfrac{3}{2}\,\mathcal{L}^{-1}\dfrac{2}{s^2+2^2} = 4\cos 2t - \dfrac{3}{2}\sin 2t$

(c) $\mathcal{L}^{-1}\dfrac{2s-5}{s^2}$

$= \mathcal{L}^{-1}\left(\dfrac{2s}{s^2} - \dfrac{5}{s^2}\right)$

$= \mathcal{L}^{-1}\dfrac{2}{s} - \mathcal{L}^{-1}\dfrac{5}{s^2}$

$$\boxed{\mathcal{L}^{-1}\frac{n!}{s^{n+1}} = t^n}$$

$\dfrac{2}{s} = \dfrac{2}{s^{0+1}}$ $\quad n=0$

(c) $\mathcal{L}^{-1} \dfrac{2s - 5}{s^2}$

$= \mathcal{L}^{-1}\left(\dfrac{2s}{s^2} - \dfrac{5}{s^2} \right)$

$= \mathcal{L}^{-1} \dfrac{2}{s} - \mathcal{L}^{-1} \dfrac{5}{s^2}$

$$\boxed{\mathcal{L}^{-1} \dfrac{n!}{s^{n+1}} = t^n}$$

$\dfrac{2}{s} = \dfrac{2}{s^{0+1}}$ $\quad n=0$

$\mathcal{L}^{-1} \dfrac{0!}{s^{0+1}} = t^0$

$\mathcal{L}^{-1} \dfrac{1!}{s^{1+1}} = t^1$

$\mathcal{L}^{-1} \dfrac{0!}{s^{0+1}} = t^0$

$\mathcal{L}^{-1} \dfrac{2}{s} = \dfrac{2 \quad 0!}{s^{0+1}} = 2 \times t^0$

$= 2$

$$\mathcal{L}^{-1}\frac{5}{s^2} = 5\,\mathcal{L}^{-1}\frac{1}{s^2} = 5\,\mathcal{L}^{-1}\frac{1}{s^{1+1}} =$$

$n=1$

$$= \frac{5}{1!}\,\mathcal{L}^{-1}\frac{1!}{s^{1+1}} = \frac{5}{1!}\,t^1 = 5t$$

$$\therefore \mathcal{L}^{-1}\frac{2}{s} - \mathcal{L}^{-1}\frac{5}{s^2} = 2 - 5t$$