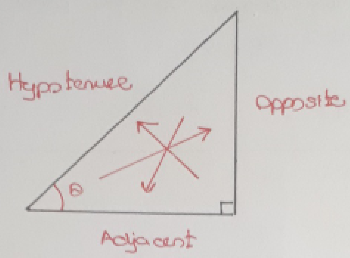
E050, Lesson4, Trigonometric functions

"Identifying the Trigonometric Functions"

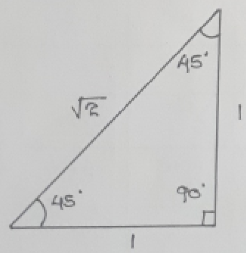
* Trigonometric functions: sin → sine, cos → cosine, tan → tangent, cot → cotangent, sec → sec, csc → co-sec



$$\sin(\theta) = \frac{\text{opposite (O)}}{\text{hypotenuse (H)}} \qquad \cos(\theta) = \frac{\text{adjacent (A)}}{\text{hypotenuse (H)}}$$

$$\tan(\theta) = \frac{\text{opposite (O)}}{\text{adjacent (A)}} = \frac{\sin\theta}{\cos\theta}$$

* Special angles

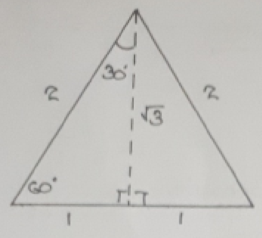


Definition:

$$\sin(45^\circ) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

$$\tan(45^\circ) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$$



Definitions:

$$\sin(30^\circ) = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

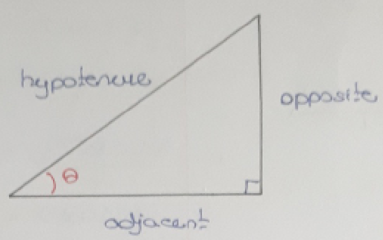
$$\tan(60^\circ) = \frac{\sqrt{3}}{1} = \sqrt{3}$$

"Values of $\sin\theta$, $\cos\theta$ & $\tan\theta$"

Special Angles (30°, 45°, 60°) — Trigonometric ratios of special angles

Angle		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degree	Radian			
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	----
120°	$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
135°	$3\pi/4$	$1/\sqrt{2}$	$1/-\sqrt{2}$	-1
180°	π	0	-1	0
270°	$3\pi/2$	-1	0	---
360°	2π	0	1	0

"The Trigonometric Ratio"



In all such triangles, corresponding pairs of sides are, in the same ratio. Depend on three sides, got three trigonometric ratios.

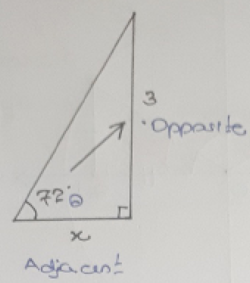
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

There are three reciprocal ratios, cosecant, secant and cotangent.

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite}}, \quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}, \quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Example: Find the value of x in the following triangle.



$$\cot\theta = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{x}{3}, \quad \theta = 72^\circ$$

$$\cot 72^\circ = \frac{x}{3}$$

$$\frac{1}{\tan 72^\circ} = \frac{x}{3}$$

$$x = \frac{3}{\tan 72^\circ} = 0.97$$

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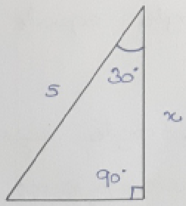
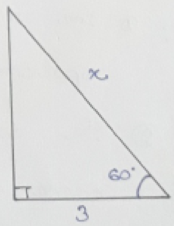
Exercise (3)

1. Find the value of x in the following triangles

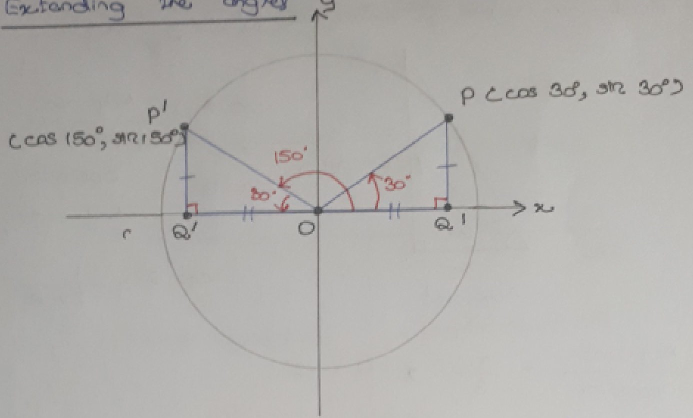
(a)



(b)



* Extending the angles



The triangles OPQ and OP'Q' are congruent, so the y-coordinates of P and P' are the same. Thus, $\sin 150° = \sin 30°$. Also, the x-coordinates of P and P' have the same magnitude but opposite sign, $\cos 150° = -\cos 150°$.

θ is any obtuse angle, supplement $180° - \theta$

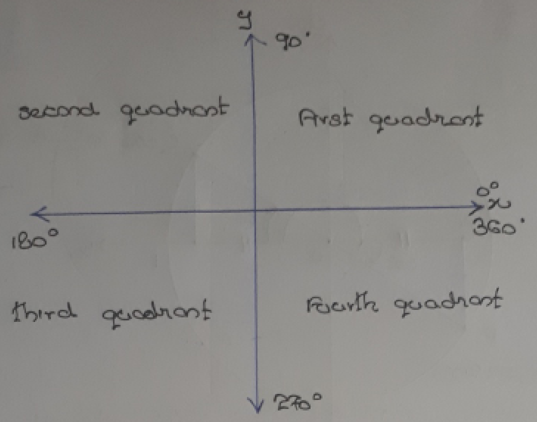
$$\sin\theta = \sin(180° - \theta), \quad 90° < \theta < 180°$$

$$\cos\theta = -\cos(180° - \theta), \quad 90° < \theta < 180°$$

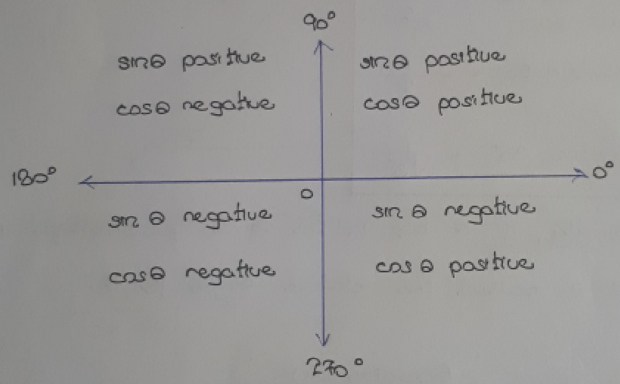
Note: ① the sine of an obtuse angle equals the sine of its supplement.

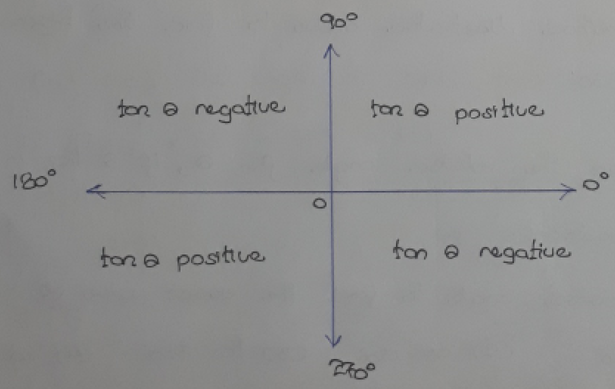
② the cosine of an obtuse angle equals minus the cosine of its supplement.

The four quadrants

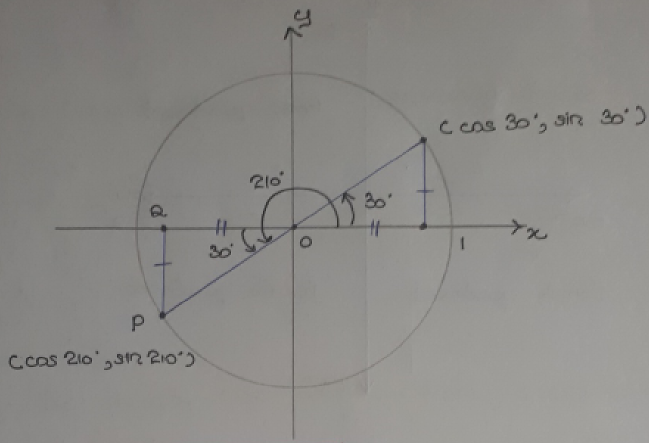


- identify the sign of each of the trigonometric ratios in a quadrant





Related Angles



$$\boxed{\begin{aligned} \cos 210^\circ &= -\cos 30^\circ = -\frac{\sqrt{3}}{2} \\ \sin 210^\circ &= -\sin 30^\circ = -\frac{1}{2} \\ \tan 210^\circ &= \tan 30^\circ = \frac{1}{\sqrt{3}} \end{aligned}}$$

Generally, if θ has in the third quadrant, then the acute angle $\theta - 180^\circ$ is called the related angle for θ.

Exercise (4)

a. Use the method illustrated above to find the trigonometric ratios of 330°

b. Write down the related angle for θ, if θ lies in the fourth quadrant.

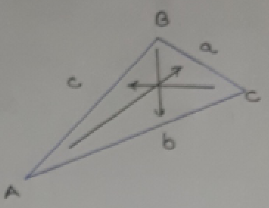
c. Use the related angle to find the exact value of

(1) $\sin 120^\circ$ (2) $\cos 150^\circ$ (3) $\tan 300^\circ$ (4) $\cos 240^\circ$

Sine and Cosine Rule

* The Sine Rule

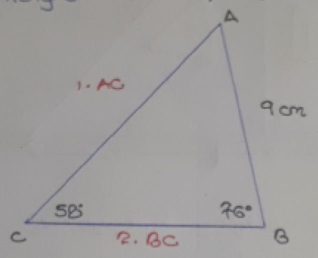
In the triangle ABC,



* $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

* $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Example: The triangle ABC has $AB = 9cm$, $\angle ABC = 76^\circ$ and $\angle ACB = 58^\circ$



Find the correct two decimal places.

1. AC
2. BC

Solution: (1) Applying the sine rule, sine = $\dfrac{\text{opposite}}{\text{hypotenuse}}$

$$\frac{AC}{\sin 76^\circ} = \frac{9}{\sin 58^\circ}$$

$$AC = \frac{9 \sin 76^\circ}{\sin 58^\circ} \approx 10.30 \text{ cm (two decimal places)}$$

(2) To find BC, the angle of CAB, then

$$\angle CAB + 58^\circ + 76^\circ = 180^\circ$$

$$\angle CAB = 180^\circ - 58^\circ - 76^\circ$$

$$\angle CAB = 46^\circ$$

By the sine rule, $\dfrac{BC}{\sin 46^\circ} = \dfrac{9}{\sin 58^\circ}$

$$\therefore BC \approx 7.63 \text{ cm}$$

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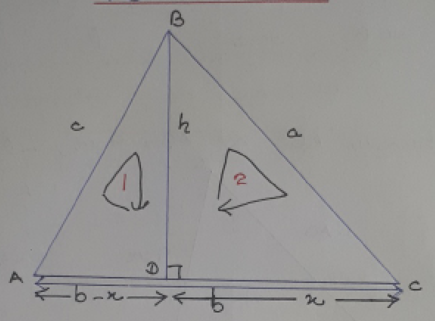
Exercise (5)

1. The triangle ABC has $AB = 7\,cm$, $\angle ACB = 50^\circ$, $\angle CAB = 30^\circ$

Find, correct to two decimal places.

1. AC   2. BC

The Cosine Rule



In the triangle ABD, applying Pythagoras' theorem,

$$c^2 = h^2 + (b-x)^2$$

In the triangle BCD, applying Pythagoras' theorem,

$$h^2 = a^2 - x^2$$

Substituting the expression of h^2. into the 1st equation,

$$c^2 = a^2 - x^2 + (b-x)^2$$

$$= a^2 - \cancel{x^2} - 2bx + b^2 + \cancel{x^2}$$

$$= a^2 + b^2 - 2bx$$

Triangle BCD, $x = a \cos C$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

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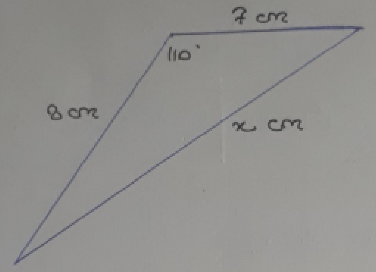
Exercise (6)

(1). Find the value of x to one decimal place. Use cosine rule.



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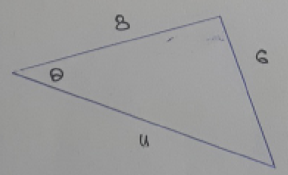
* Finding angle equation from cosine rule.

$$\boxed{\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos c \\ \cos c &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}}$$

Exercise (7)

(1) A triangle has side lengths 6 cm, 8 cm and 11 cm, Find the smallest angle in the triangle. (use finding angle equation)



The area of angle

sides a and b, acute angle θ

The area of triangle, $\boxed{\text{Area} = \frac{1}{2} ab \sin \theta}$

Exercise (8)

(1) A triangle has two sides of length 5cm and 4cm containing an angle, θ. It's area is $5cm^2$. Find the two possible exact values of θ and draw the two triangles that satisfy the given information.

(2) Write down two different expressions for the area of a triangle ABC and derive the sine rule from them.
