E050, Lesson3, Circular Function

CIRCULAR FUNCTIONS

Page ①

* See Youtube

SUNP0011

- **Radians & degree**

  - The radian is another measure of angles
  - A circle with radius of 1 has a circumference of 2π - this is the basic of radian measure.
  - It is identified as a number with no unit.

(π means pi)

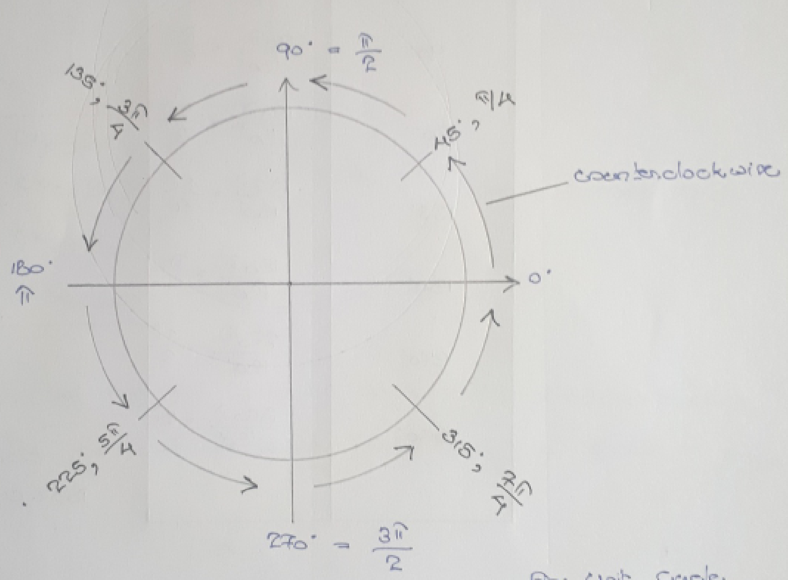

Fig: Unit Cycle

(pi) $\pi = 180°$

1 radian $= \frac{180°}{\pi}$

1 degree (1°) $= \frac{\pi}{180°}$

"Conversion from degrees to radians"
(°) (rad)

Degrees	Radians
30°	$\frac{30\pi}{180} = \frac{\pi}{6}$
45°	$\frac{45\pi}{180} = \frac{\pi}{4}$
60°	$\frac{60\pi}{180°} = \frac{\pi}{3}$
90°	$\frac{90\pi}{180°} = \frac{\pi}{2}$
120°	$\frac{120\pi}{180°} = \frac{2\pi}{3}$
135°	$\frac{135\pi}{180°} = \frac{3\pi}{4}$
180°	$\frac{180\pi}{180°} = \pi$
270°	$\frac{270\pi}{180°} = \frac{3\pi}{2}$
360°	$\frac{360\pi}{180°} = 2\pi$

Table (1)

Exercise: (1)

1. Plot degrees and radians on the unit circle. (See table 1)
2. Find the following degrees to radians.

   (a) 210° (b) 300° (c) 150°

3. Find the following radians to degrees.

   (a) $\frac{11\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{7\pi}{6}$

"Evaluation a trigonometric function using the unit circle."

* The unit circle is the circle with center at the origin and radius 1.

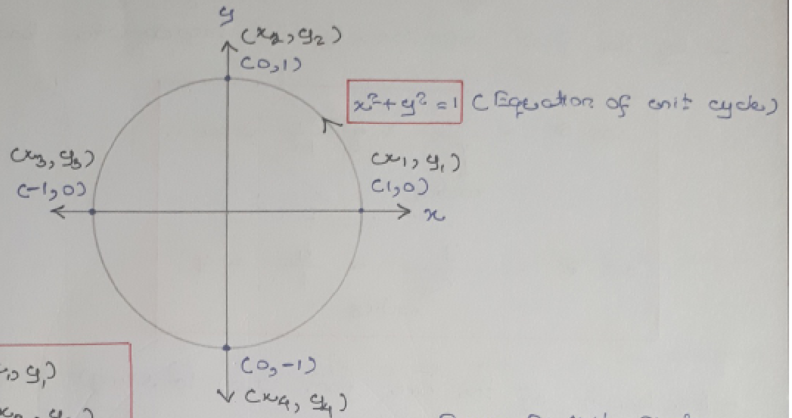

Figure 2: Unit Circle

* $(1,0) = (x_1, y_1)$
  $(0,1) = (x_2, y_2)$
  $(-1,0) = (x_3, y_3)$
  $(0,-1) = (x_4, y_4)$

* Suppose the terminal side of angle θ, in standard position, intersects the unit circle at point (x, y) as shown in figure 3.

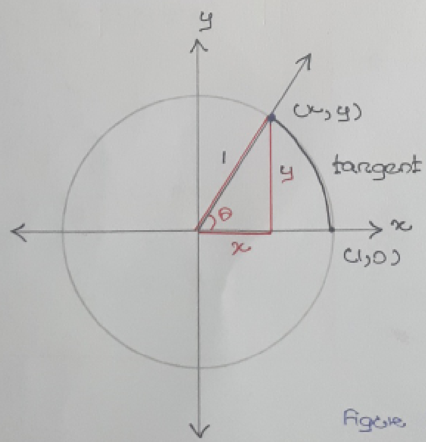

Figure 3. Taking θ

- Because the radius of the unit circle is 1, the distance from the origin to the point (x, y) is 1.

- By the first definition for the trigonometric functions,

$$\cos\theta = \underbrace{\frac{x}{r}}_{\text{radius}} = \frac{x}{1} = x$$

$$\sin\theta = \underbrace{\frac{y}{r}}_{\text{radius}} = \frac{y}{1} = y$$

- The length of the arc from $(1,0)$ to (x, y) is exactly the same as the radian measure of angle θ. Therefore,

$$\cos\theta = \cos t = x$$

$$\sin\theta = \sin t = y$$

* Circular functions depending on chord *

chord (θ) = AC = 2AB = $2\sin(\theta/2)$

Theta

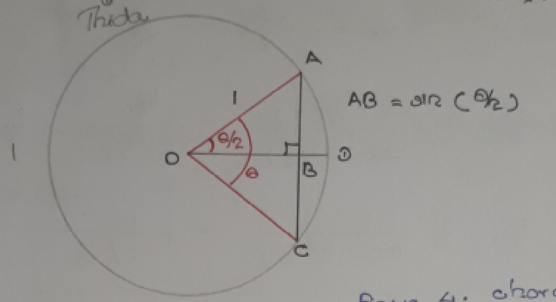

Figure 4: chord AC and θ

* If (x, y) is any point on the unit circle, and t is the distance from $(1, 0)$ to (x, y) along the circumference of the unit circle.

* Definition:

$\cos t = x$

$\sin t = y$

$\tan t = \frac{y}{x} \quad (x \neq 0)$

$\cot t = \frac{x}{y} \quad (y \neq 0)$

$\csc t = \frac{1}{y} \quad (y \neq 0)$

$\sec t = \frac{1}{x} \quad (x \neq 0)$

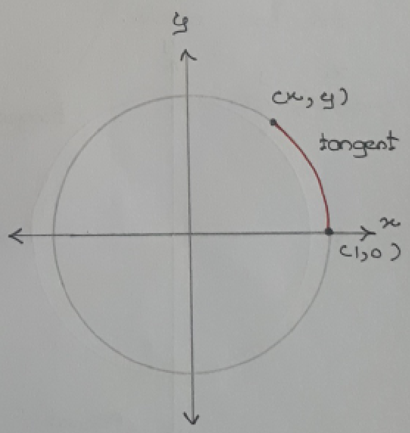

Figure 5: Tangent on unit cycle

<u>Examples:</u> Find the six trigonometric functions of $5\pi/6$.

1. $\sin \frac{5\pi}{6} = y = \frac{1}{2}$ ⇐

2. $\cos \frac{5\pi}{6} = x = -\frac{\sqrt{3}}{2}$ ⇐

3. $\tan \frac{5\pi}{6} = \frac{\sin(5\pi/6)}{\cos(5\pi/6)} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ ⇐

4. $\cot \frac{5\pi}{6} = \frac{1}{\tan(5\pi/6)} = \frac{1}{-1/\sqrt{3}} = -\sqrt{3}$ ⇐

5. $\sec \frac{5\pi}{6} = \frac{1}{\cos(5\pi/6)} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ ⇐

6. $\csc \frac{5\pi}{6} = \frac{1}{\sin(5\pi/6)} = \frac{1}{1/2} = 2$ ⇐

7. Find <u>tan t</u> if <u>t</u> corresponds to the point <u>$(-0.737, 0.675)$</u> on the unit circle.

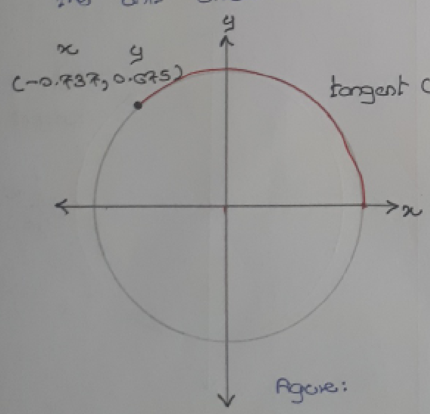

Figure:

<u>Solution:</u>

$$\tan t = \frac{y}{x} \quad \text{(definition)}$$
$$= \frac{0.675}{-0.737}$$
$$= -0.916$$

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Exercises (2)

1. Find the six trigonometric functions.

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{7\pi}{6}$ (d) $\frac{4\pi}{3}$ (e) $\frac{7\pi}{4}$

2. Find $\tan t$ if t corresponds to the point $(0.679, -0.789)$ on the unit circle.
