E050, Lesson 7, Differentiation

Differentiation

$y = x^n$, $\frac{dy}{dx} = n x^{n-1} \frac{dx}{dx} = n x^{n-1}$

$y = u^n$, $\frac{dy}{dx} = n u^{n-1} \frac{du}{dx}$

$y = \sin\theta \rightarrow \frac{dy}{d\theta} = \frac{d \sin\theta}{d\theta} = \cos\theta$

$y = \sin 3\theta \rightarrow \frac{dy}{d\theta} = \frac{d \sin 3\theta}{d\theta} = \cos 3\theta \frac{d\, 3\theta}{d\theta}$

$= 3 \cos 3\theta$

$y = \cos 3\theta \rightarrow \frac{dy}{d\theta} = \frac{d \cos 3\theta}{d\theta} = -\sin 3\theta \frac{d\, 3\theta}{d\theta}$

$= -3 \sin 3\theta$

Pb $y = 5x^3 + 6x^2 + 7$ Differentiate.

$\frac{dy}{dx} = \frac{d\, 5x^3}{dx} + \frac{d\, 6x^2}{dx} + \frac{d7}{dx}$

$= 5 \times 3 x^{3-1} + 6 \times 2 x^{2-1} + 0$

$= 15 x^2 + 12x$

Pb. $y = \sin x + \cos x$, Differentiate.

$\frac{dy}{dx} = \frac{d \sin x}{dx} + \frac{d \cos x}{dx}$

$= \cos x - \sin x$

$$\frac{d \tan\theta}{d\theta} = \sec^2\theta \quad , \quad \frac{d \sec\theta}{d\theta} = \sec\theta \tan\theta$$

$$\frac{d \cot\theta}{d\theta} = -\operatorname{cosec}^2\theta \quad , \quad \frac{d \operatorname{cosec}\theta}{d\theta} = -\operatorname{cosec}\theta \cot\theta$$

Attach: Stamp card

$$y = e^u \longrightarrow \frac{dy}{dx} = e^u \frac{du}{dx}$$

$$y = \ln u \longrightarrow \frac{dy}{dx} = \frac{1}{u} \frac{du}{dx}$$

Problem. Differentiate $y = \log_e x^2$

solution : $y = \log_e x^2$

$$\frac{dy}{dx} = \frac{d}{dx} \ln x^2 = \frac{1}{x^2} \frac{dx^2}{dx} = \frac{1}{x^2} \times 2x^{2-1} \frac{dx}{dx}$$

$$= \frac{2x}{x^2} = \frac{2}{x}$$

Problem. Differentiate $y = \log_e x^2$

Solution: $y = \log_e x^2$

$$\frac{dy}{dx} = \frac{d}{dx} \ln x^2$$

$$= \frac{1}{x^2} \frac{d x^2}{dx}$$

$$= \frac{1}{x^2} \times 2x \frac{dx}{dx}$$

$$= \frac{2}{x}$$

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Successive Differentiation

$y = x^3 + 3x^2 + 4$ Find $\frac{dy}{dx}, \frac{dy^2}{dx^2}, \frac{dy^3}{dx^3}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 3x^2 + 4)$$

$$= \frac{d x^3}{dx} + 3 \frac{dx^2}{dx} + \frac{d4}{dx}$$

$$= 3x^{3-1} + 3 \times 2 x^{2-1} + 0$$

$$= 3x^2 + 6x$$

$$\frac{dy^2}{dx^2} = \frac{d}{dx}(3x^2 + 6x)$$

$$= 3 \frac{dx^2}{dx} + 6 \frac{dx}{dx}$$

$$= 3 \times 2 x^{2-1} + 6$$

$$= 6x + 6$$

$$\frac{dy^3}{dx} = \frac{d(6x+6)}{dx}$$

$$= 6\frac{dx}{dx} + \frac{d6}{dx}$$

$$= 6 + 0$$

$$= 6$$

Differentiation of implicit Functions.

Problem: Differentiate $x^2 + y^2 = 4$

Solution: $\frac{d(x^2+y^2)}{dx} = \frac{d4}{dx}$

$$\frac{dx^2}{dx} + \frac{dy^2}{dx} = 0$$

$$2x^{2-1}\frac{dx}{dx} + 2y^{2-1}\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Differention. (Ex)

No.1 $y = 2x + 3$, Differentiate.

No. 2 $y = 4x^3 \ln x$, Differentiate

No. 3. Differentiate $y = \frac{x+4}{2x+5}$

No. 4 Differentiate $y = 3x^2 + 4x + 1$.

$y = -6x^3 + 8x^2 - 5x + 2$

$y = x^2(2x - 1)$

$y = x^4 + x^{-4} + 4$ with respect to x

No. 5 Differentiate $y = 15x^{100} - 3x^{18} + 5x - 46$.

$y = 8x^3 - \frac{1}{3}x^{-5} + x - 23$

$y = \sqrt[3]{x^2}(2x - x^2)$

$y = 2x^3 + \frac{300}{x^3} + 4$

No. 6. Differentiate $y = \frac{1}{\sin x}$

Differention

$y = x^m$

$\frac{dy}{dx} = n\, x^{n-1} \frac{dx}{dx}$ (Power Rule)

$= n\, x^{n-1}$

$y = u^n$

$\frac{dy}{dx} = n\, u^{n-1} \frac{du}{dx}$

$y = \sin\theta \rightarrow \frac{dy}{d\theta} = \frac{d \sin\theta}{d\theta} = \cos\theta$

$y = \sin 3\theta \rightarrow \frac{dy}{d\theta} = \frac{d \sin 3\theta}{d\theta} = \cos 3\theta \frac{d 3\theta}{d\theta}$

$= 3 \cos 3\theta$

$y = \cos 3\theta \rightarrow \frac{dy}{d\theta} = \frac{d \cos 3\theta}{d\theta} = -\sin 3\theta \frac{d 3\theta}{d\theta}$

$= -3 \sin 3\theta$

Bo. $y = 5x^3 + 6x^2 + 7$

$\frac{dy}{dx} = \frac{d\, 5x^3}{dx} + \frac{d\, 6x^2}{dx} + \frac{d\, 7}{dx}$

$= 5 \times 3\, x^{3-1} + 6 \times 2x + 0$

$= 15\, x^2 + 12x$

Pb. $y = \sin x + \cos x$

$$\frac{dy}{dx} = \frac{d \sin x}{dx} + \frac{d \cos x}{dx}$$

$$= \cos x - \sin x$$

$$\boxed{y = u.v} \quad (\text{Product Rule})$$

$$\boxed{\frac{dy}{dx} = \frac{d\, uv}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}}$$

Pro. $y = (x+1)^2 (x+3)^3$, Differentiate.

$$y = (x+1)^2 (x+3)^3$$

$$\frac{dy}{dx} = (x+1)^2 \frac{d(x+3)^3}{dx} + (x+3)^3 \frac{d(x+1)^2}{dx}$$

$$= (x+1)^2 \times 3 (x+3)^{3-1} \frac{d(x+3)}{dx} +$$

$$(x+3)^3 \times 2 (x+1)^{2-1} \frac{d(x+1)}{dx}$$

$$= 3 (x+1)^2 (x+3)^2 \left(\frac{dx}{dx} + \frac{d3}{dx}\right) +$$

$$2 (x+3)^3 (x+1) \left(\frac{dx}{dx} + \frac{d(1)}{dx}\right)$$

$$= 3 (x+1)^2 (1+0) + 2 (x+3)^3 (x+1) (1+0)$$

$$(x+3)^2$$

$$= 3 (x+1)^2 (x+3)^2 + 2 (x+3)^3 (x+1)$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Ex. Differentiate $\frac{x}{x+1}$

$$\frac{dy}{dx} = \frac{(x+1)\frac{dx}{dx} - x\frac{d(x+1)}{dx}}{(x+1)^2}$$

$$= \frac{(x+1) - x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$