E050, Lesson 2, Logarithms

Logarithms

* $2^3 = 8$

  $3 = \log_2 8$

* $5^2 = 25$

  $2 = \log_5 25$

* $10^{-4} = \frac{1}{10000}$

  $-4 = \log(1/10000)$

Eg. 1. Write these exponential equations as logarithmic equations:

Note:

* See E0 50, lesson 2, logarithms.
* log Graph SUNP 0007 YouTube lessons

Eg 2. Write these logarithmic equations as exponential equations:

* $\log_6 \sqrt{6} = \frac{1}{2}$

  $6^{1/2} = \sqrt{6}$

* $\log_3 (9) = 2$

  $3^2 = 9$

Pb. 1. Write the exponential $4^2 = 16$ equation as a logarithmic equation.

$$\log_4 (16) = 2$$

Eg 3. Solve $\log_4 (x) = 2$ for x

Solution: $4^2 = x$

so $x = 16$

Eg 4. Solve $2^x = 10$ for x

solution: $x = \log_2(10)$

Common logarithms & Natural logarithms

- The common log is the logarithm with base 10, and is typically written $\log(x)$.
- The natural log is the logarithm with base e, and is typically written $\ln(x)$.

Eg 5. Evaluate $\log(1000)$ using the definition of the common log.

Values of common log

number	number as exponential	log(number)
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.01	10^{-2}	−2
0.001	10^{-3}	−3

To evaluate log (1000)

$x = \log(1000)$

common log base of 10. $10^x = 1000$

$10^x = 10^3$

$\therefore \; x = 3$

$\therefore$ The inverse property of logs, we can write

$\log_{10}(10^3) = 3$ #

(Pb.) Evaluate $\log(1000000)$

Sol: $\log^x = 1000000$

$\log^x = \log^6$

$x = 6$ #

Pb. Evaluate $\ln\sqrt{e}$

Sol: $\ln e^{1/2} = 1/2$

Pb. Evaluate $\log(500)$

Sol: $\log(500) = 2.69897$ #

Properties of logs : Exponent Property

Eg. $\log_b (A^r) = r \log_b (A)$

since the logarithmic and exponential functions are inverse.

$$b^{\log_b A} = A$$

Therefore, $A^r = (b^{\log_b A})^r$

Utilizing the exponential rule that states

$$(x^p)^q = x^{pq}$$

$$A^r = (b^{\log_b A})^r = b^{r \log_b A}$$

so then, $\log_b (A^r) = \log_b (b^{r \log_b A})$

Again, utilizing the inverse property on the right side yields the result

$$\log_b (A^r) = r \log_b A$$

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Eg: Rewrite $\log_3(25)$ using the exponent property for logs.

solution: $25 = 5^2$

$$\log_3(25) = \log_3(5^2) = 2\log_3 5$$

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Eg: Rewrite $4\ln(x)$ using the exponent property for logs.

solution: $4\ln(x) = \ln(x^4)$

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Problem. Rewrite using the exponent property for logs.

$$\ln\left(\frac{1}{x^2}\right)$$

solution: $2\ln(x)$

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Properties of logs : change of Base

Eg./ $\log_b(A) = \dfrac{\log_c(A)}{\log_c(b)}$

Solution: Let $\log_b(A) = x$

exponential $b^x = A$

$\log_c b^x = \log_c A$ (Taking $\log_c$ on both sides)

$x \log_c b = \log_c A$

By dividing, $x = \dfrac{\log_c A}{\log_c b}$

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Eg./ Evaluate $\underset{\text{base } 2}{\log_2(10)}$ using the change of base formula.

solution: For base 2, we choose natural logarithm.

$$\log_2(10) = \frac{\log_e 10}{\log_e 2} = \frac{\ln 10}{\ln 2}$$

By using calculator, $\dfrac{\ln 10}{\ln 2} \approx 3.3219$

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Eg: Evaluate $\log_5(100)$ using the change of base formula.

Solution: For $\log_5$, we can choose common logarithms. common logrithms, base 10.

$$\log_5(100) = \frac{\log_{10}(100)}{\log_{10} 5} \approx 2.861$$

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Important Topics of This section

- The logarithmic function as the inverse of the exponential functio
- Writing logarithmic & exponential expressions
- Properties of logs
    - Inverse properties
    - Exponential properties
    - change of base
- common log
- Natural log

Properties of logs

* Inverse Properties:

$$\log_b (b^x) = x$$

$$b^{\log_b x} = x$$

* Exponential Properties:

$$\log_b (A^r) = r \log_b (A)$$

* Change of base:

$$\log_b (A) = \frac{\log_c (A)}{\log_c (b)}$$

* Sum of logs Property:

$$\log_b (A) + \log_c (C) = \log_b (AC)$$

* Difference of logs Property:

$$\log_b (A) - \log_b (C) = \log_b \left(\frac{A}{C}\right)$$

* $\log (A + B) \neq \log (A) + \log (B)$

Eg: Let $a = \log_b (A)$, $c = \log_b (C)$

$b^a = A$, $b^c = C$

$AC = b^a b^c$

Use exponent rules , $AC = b^{a+c}$

Taking logs on both sides,

$$\log_b (AC) \log_b (b^{a+c}) = a+c$$

Replacing $\log_b (AC) = \log_b A + \log_b C$

#

Eg: Write $\log_3 (5) + \log_3 (8) - \log_3 (2)$ as a single logarithm.

Sol: Using the sum of log property,

$$\log_3 (5) + \log_3 (8) = \log_3 (5 \times 8) = \log_3 (40)$$

Reduce from $\log_3 (40) - \log_3 (2)$

Using differences of property,

$$\log_3 (40) - \log_3 (2) = \log_3 \left(\frac{40}{2}\right) = \log_3 (20)$$

#

Eg: Evaluate $2 \log (5) + \log (4)$ using single logarithm.

Sol: Firstly, use exponent property of logs.

$$2 \log (5) = \log (5^2) = \log (25)$$

Expression reduced to a sum of two logs,

$$\log (25) + \log (4) = \log (4 \times 25)$$
$$= \log (100)$$

Since $100 = 10^2$,

$$\log (100) = \log (10^2)$$

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Problem:/ Calculate without calculator.

$$\log_2(8) + \log_2(4)$$ Ans: $3+2=5$

Eg:/ Rewrite $\ln\left(\frac{x^4 y}{7}\right)$ as a sum or difference of logs.

Solution: By notifying a quotient of two expressions, choose the difference property of logs,

$$\ln\left(\frac{x^4 y}{7}\right) = \ln(x^4 y) - \ln(7)$$

$$= \ln(x^4) + \ln(y) - \ln(7)$$

$$= 4\ln(x) + \ln(y) - \ln(7)$$

\#

* Using the definition of a Logarithm to solve Logarithmic Equations

Logarithmic equation

$\boxed{\log_b(x) = y}$ equal $\boxed{b^y = x}$

Eg:/ consider the equation $\log_2(2) + \log_2(3x-5) = 3$

Logarithmic Equation	$\log_2(2) + \log_2(3x-5) = 3$
Apply the product rule of logarithms	$\log_2(2(3x-5)) = 3$

Distribute	$\log_2 (6x - 10) = 3$
Apply the definition of a logarithms	$2^3 = 6x - 10$
Calculate 2^3	$8 = 6x - 10$
Add. 10 to both sides	$18 = 6x$
Divide by 6	$x = 3$

Eg:/ Using algebra to solve a logarithmic equation.

– Solve $2 \ln x + 3 = 7$

1. Subtract 3: $2 \ln x = 4$
2. Divide by 2: $\ln x = 2$
3. Exponential form: $x = e^2$

– Solve $2 \ln (6x) = 7$

1. divide by 2: $\ln (6x) = \frac{7}{2}$
2. Use the definition of ln: $6x = e^{7/2}$
3. Divide by 6: $x = \frac{1}{6} e^{7/2}$

– Solve $2 \ln (x+1) = 10$

1. Divide by 2: $\ln (x+1) = 5$
2. Use the defination of ln: $x+1 = e^5$
3. Subtract 1 from right side $x = e^5 - 1$

Using the one to one property of logarithms to solve Logarithmic Equations

For any real numbers, $x > 0$, $S > 0$, $T > 0$

For any positive numbers, b where $b \neq 1$

$\boxed{\log_b S = \log_b T}$ if and only if $S = T$

Eg:/ $\log_2 (x-1) = \log_2 (8)$

$$x - 1 = 8$$
$$x = 8 + 1 = 9$$

substitute original equation, $\log_2 (9-1) = \log_2 (8)$

$2^3 =$

$\log_2 (8) = 3$

Eg:/ $\log (3x-2) - \log (2) = \log (x+4)$

one to one property for expressions	$\log (3x-2) - \log (2) = \log (x+4)$
Apply the quotient rule of logarithms	$\log \left(\frac{3x-2}{2}\right) = \log (x+4)$
Apply one to one property of a logarithm	$\frac{3x-2}{2} = x+4$
multiply by 2 on both sides of equation	$3x - 2 = 2x + 8$
	$3x - 2x = 8 + 2$
Find x	$x = 10$

To check the result,

$$\log(3x-2) - \log(2) = \log(x+4)$$

$$\log(3\cdot 10 - 2) - \log(2) = \log(10+4)$$

$$\log(28) - \log(2) = \log(14)$$

$$\log\left(\frac{28}{2}\right) = \log(14)$$

#

Problems: (1) Using the one-to-one property of logarithms,

solve

$$\ln(x^2) = \ln(2x+3)$$

Problems: (2) Use the one-to-one property of the logarithm:

Solve $x^2 = 2x + 3$

Exercises

1. Find the value of the logarithm:

(i) $\log_4 16$

(ii) $\log_2 16$

(iii) $\log_{27} 3$

(iv) $\log_{1/9} 81$

(v) $\ln e^2$

(vi) $\ln e^2 + \ln e^3$

(vii) $\log 20 + \log 50$

(viii) $\log 500 - \log 5$

(ix) $\log_{1/8} \sqrt{128}$

2. Solve the logarithmic equation:

(i) $\log_x 2 = 10$

(ii) $\log x^2 = 2$

(iii) $\log_x 25 = 2$

(iv) $\log_x 4 + \log_x 2 = 1$

(v) $\log (x-5) - \log (1-x) = \frac{1}{3}$

(vi) $\log_6 (4x+3) = 2$

(vii) $\log_3 (x^2 - 8x) = 2$

(viii) $\dfrac{\log (x^3+13)}{\log (x+5)} = 2$

3. Solve the logarithmic equation:

(i) $\dfrac{\log x}{\log x + 1} = 1$

(ii) $\log_3 [1 + \log_3 (2^x - 7)] = 1$

(iii) $\log x + \dfrac{1}{\log x} = 2$

(iv) $\dfrac{2 \log x}{\log (5x-4)} = 1$

(v) $\log_3 (1-x) = \log_3 (x+16-x^2)$

Solving Logarithmic Inequalities

A logarithmic inequality is an inequality that involves logarithms.

Properties of Inequality for logarithmic functions	If $b>1$, $x>0$, and $\log_b x > y$, then $x > b^y$ If $b>1$, $x>0$, and $\log_b x < y$, then $0 < x < b^y$ If $b>1$, then $\log_b x > \log_b y$ if and only if $x>y$, and $\log_b x < \log_b y$ if and only if $x<y$.

Eg: (1) $\log_5 (4x-3) < 3$

Solve:

$\log_5 (4x-3) < 3$ $\Longrightarrow$ original Inequality

$0 < 4x-3 < 5^3$ $\Longrightarrow$ property of inequality for logarithmic functions

$3 < 4x < 125+3$ $\Longrightarrow$ simplify

$\frac{3}{4} < x < 32$ $\Longrightarrow$ simplify

The solution set is $\{x \mid \frac{3}{4} < x < 32\}$

\#

Eg ②. $\log_3 (3x-4) < \log_3 (x+1)$

Solution:

$\cancel{\log_3} (3x-4) < \cancel{\log_3} (x+1)$ → original inequality

$3x-4 < x+1$ → property of inequality

$2x-4 < 1$ → subtract x from each side

$2x < 5$ → Add 4

$x < 5/2$ → divide by 2

In this case, $(3x-4)$ and $(x+1)$ must both be positive numbers. Use this information to constrain ~~the~~ solution.

$3x-4 > 0$

$3x > 4$

$x > \frac{4}{3}$

$x+1 > 0$

$x > -1$

The solution is $\{x \mid \frac{4}{3} < x < \frac{5}{2}\}$

\#

Exercises

\# Solve each inequality.

① $\log_2 2x > 2$

②. $\log_2 (3x+1) < 4$

③. $\log_3 (x+3) < 3$

④. $\log_{10} 5x < \log_{10} 30$

⑤. $\log_{10} x < \log_{10} (2x-4)$

⑥. $\log_2 (8x+5) > \log_2 (9x-18)$

⑦. $\log_2 (3x-4) < \log_2 (2x+7)$

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